

## Russell "On Denoting": Logical vocabulary, translation, and the Present King of France

~	Negation	"not," "It is not the case that"
•	Conjunction	"and"
∨	Disjunction	"or"
⊃	Conditional	"if-then," "entails," "implies"
≡	Biconditional	"if and only if"
(x)	Universal Quantifier	"For all x..."
(∃x)	Existential Quantifier	"There exists an x..."

Predicates: A, B, ..., Z

Names: a, b, ...

Variables: x, y, z

How do I say...

Socrates is a man. Hs  
s is human

There is a man. (∃x)Hx  
There exists an x such that x is human

There is a bald man. At least one man is bald (∃x)(Hx • Bx)  
There exists an x such that x is human and x is bald.

There is exactly one man (∃x)(Hx • (y)(Hx ⊃ y=x))  
There exists an x such that x is human and, for any y, if y is human then y=x

Notice the underlined part expresses *uniqueness*, the idea that there is only one thing that's human. The idea is this: suppose I say there's just one man on that island, that guy x. You say now wait a minute, I met a guy, y, on the other side of the island. OK I say, there's just one man there—y is identical to x, "they" are the same guy.

So, this language gives us a way to express what Russell takes to be the true form of what appear to be subject-predicate sentences:

"The King of France is bald" breaks down to the conjunction of three claims:

- |  |                       |
|--|-----------------------|
| (1) There is a (at least one) King of France | $(\exists x)Kx$       |
| (2) There is no more than one King of France | $(y)(Ky \supset y=x)$ |
| (3) That guy x is bald                       | $Bx$                  |

Putting it altogether

(4) The King of France is bald.  $(\exists x)(Kx \cdot (y)(Ky \supset y=x) \cdot Bx)$

(4) is just the *conjunction* (and-statement) of (1), (2) and (3). No big deal

Now let's consider the idea of "scope"

This is familiar from arithmetic. Consider the following:

(5)  $- 2 + 3$

(6)  $-(2 + 3)$

In (5) the scope of the minus sign is narrow—it only applies to the 2—so the answer is 1. In (6) however, the scope of the minus sign is the whole expression in parentheses so when we do the arithmetic we evaluate the expression in the parentheses *first* to get 5 and *then* stick on the minus sign to get  $- 5$ . Big difference.

