MATH 1260 – MIDTERM #1

Your Name

- You have 50 minutes to do this exam.
- No calculators!
- You may have one 3×5-card of notes
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	15	
2	15	
3	10	
4	15	
EC	5	
Total	55	

1. A) True or False (3 points each)

(a) The cross product of two unit vectors in $\mathbb{R}^3$ is a unit vector.

(b) If $\mathbf{u}$ and $\mathbf{v}$ are nonparallel vectors in $\mathbb{R}^3$ then up to scalar multiplication, $\mathbf{u} \times \mathbf{v}$ is the ONLY vector perpendicular to both u and v.

(c) The function $f(x, y) = \sin xy + y^2$ is an example of a function $f : \mathbb{R}^1 \to \mathbb{R}^3$.

(d) If $f(x, y)$ is differentiable and $\nabla f(a, b) = \mathbf{0}$, then the graph of $z = f(x, y)$ has a horizontal tangent plane at (a, b).

(e) The function $f(x, y) = \sqrt[3]{x^2 + y^4}$ has a global minimum value at the origin. (Hint: No Computation Needed)

2) Short answer (5 points each)

(a) Write an equation of a tangent plane at the point $(1, 1, 1)$ of the surface $z = 3x^2 - 4y^2 + xy + 1$.

(b) In what direction is $f(x, y) = 9x^4 + 4y^2$ increasing most rapidly at $(1, 2)$? (You can just give a vector - it need not be a unit vector)

(c) Does $\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$ exist? Explain your reasoning.

3 a) (5 points) Suppose a differentiable function satisfies

$$\frac{\partial f}{\partial x} = e^{-x^2}, \quad \frac{\partial f}{\partial y} = \cos y$$

and that $x = r\cos\theta$ and $y = r\sin\theta$. Calculate $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ in terms of x, y, r and θ.

b) (5 points) Explain why there is no differentiable function $f(x, y)$ with continuous second partial derivatives that satisfies

$$\frac{\partial f}{\partial x} = \sin y, \quad \frac{\partial f}{\partial y} = \cos x.$$

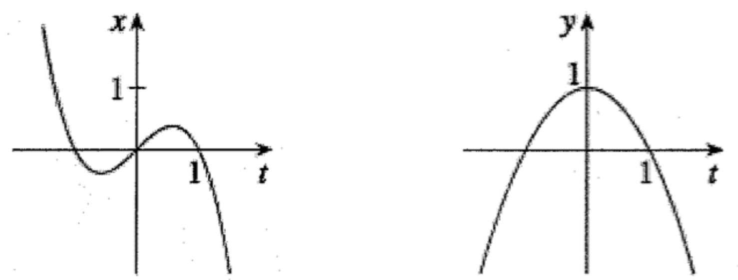
If you tried to find the function $f(x, y)$ in part (a), I bet you failed because e^{-x^2} does not have an antiderivative in terms of elementary functions. However, it's true that

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}.$$

We'll prove this later in the course.

4. A) (7 points) Find the minimum of $f(x, y, z) = 4x - 2y + 3z$ subject to the constraint $2x^2 + y^2 - 3z = 0$.

4. B) (8 points) Antoine (the bug) is walking around the xy-plane. His position is given by $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ with $x = f(t)$ and $y = g(t)$ as indicated in the graphs below. (Note that the labels on the axes!)



Use these graphs to **draw a sketch** of the curve $\mathbf{r}(t)$ in $\mathbb{R}^2$ and indicate with arrows the direction in which the curve is traced as t increases. What is the velocity vector of the bug at the time $t = 0$? Plotting points might help you.

Extra Credit (5 points possible) Let S be the surface defined by $x^2 + xy - y^3 + ax + z = 2$. Suppose that S contains a curve C whose tangent line at the point $(0, 0, -2)$ is parametrized by $(0, 0, -2) + t(1, 2, 3)$. Compute a.