MATH 1260 – MIDTERM #1

Your Name

- You have 50 minutes to do this exam.
- No calculators!
- You may have one 3×5-card of notes
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	-	
2	-	
3	-	
4	-	
Total	-	

1. Short answer questions (4 points each).

(a) Setup, but *do not* evaluate, an integral which computes the arclength of $t \mapsto \langle e^t, \sin(t^2), \cos(t)\rangle$ for t from 3 to 5.

(b) What is the scalar projection of the vector $\langle 1, 2, 3\rangle$ in the direction of the normal vector to the plane $x + y + z = 0$?

(c) Find a vector perpendicular to both $\langle 0, 1, -1\rangle$ and $\langle 1, 0, 0\rangle$.

(d) If an ant is climbing down a hill whose height is given by $z = x^2 + y + 3x\cos(y^2)$ and is at position $(0, 1)$, what direction should the ant climb to descend the hill fastest?

(e) Find the equation of the line through the point $(2, 0, -3)$ that points in the direction of the vector $\langle 7, 8, 4\rangle$

(f) Write an equation of a tangent plane at the point $(1, 1, 1)$ of the surface $z = 3x^2 - 4y^2 + xy + 1$.

2. (10 Points) A) Say whether the following statements are true or false

I) The dot product of two unit vectors is non-negative.

II) If $f : \mathbb{R}^2 \to \mathbb{R}^3$ and $g : \mathbb{R}^3 \to \mathbb{R}^4$ then the composition $g \circ f$ is defined.

III) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ then $\mathbf{b} = \mathbf{c}$.

IV) If $f(x, y)$ is continuous on a closed bounded set S then f attains a maximum value on S.

V) If $\mathbf{r}(t)$ is a parametrized curve in $\mathbb{R}^3$ then

$$\frac{d}{dt}\left(\mathbf{r}(t) \times \mathbf{r}'(t)\right) = \mathbf{r}(t) \times \mathbf{r}''(t)$$

B) (10 points) Suppose that P, Q, R are three points in $\mathbb{R}^3$. Write down what steps you would take to determine if they were collinear. If they are not collinear, describe how you could find the equation of the plane they span.

3. Find and classify[1] the critical points of the surface $z = f(x, y) = -2x^2 + x^4 + 6y^2 - 12y$. (16 points)

[1]Local max, local min, or saddle point

4. (10 points) B) Let $f(x, y) = x^2 + y^2$. At what point is the tangent plane to the graph of f parallel to the plane

$$x - y + z = 3?$$

(10 points) A) Suppose S is an unknown surface in $\mathbb{R}^3$. You know that the point $(1, 0, 0)$ lies on the surface and you know also that the curves

$$\vec{r}_1(t) = \langle \cos(t), \sin(t), 1 - e^t \rangle$$

$$\vec{r}_2(t) = \langle 1 - t, 5e^t \sin(t), 1 - \frac{1}{1-t} \rangle$$

both lie in the surface. Compute the equation for the tangent plane to S at $(1, 0, 0)$.

5. BONUS PRACTICE PROBLEM Our intrepid bug Antoine (get it? Ant-oine! Like an ant!) has fallen in love! While crawling along the parametrized line given by

$$x = 3t - 1, \quad y = t + 3$$

he collided with the great 56 year old Andean condor named Andy who lives in Liberty Park in SLC, and it was ♡ at first sight. This collision occurred at time t_0 while Andy was walking at a constant velocity in a straight line starting at $(0, 0)$ and walking toward $(3, 6)$.

A) (3 points) What is the speed of Antoine?

B) (5 points) At what point (x_0, y_0) did these star-crossed lovers meet?

C) (7 points) Determine the time t_0 when our friends first met, and write down parametric equations for Andy's walk. Make sure Andy and Antoine are at the same point at time t_0, or else all is lost.

Hint: The correct answer to part C certainly cannot be found by looking on Tracy Aviary's website. In fact, all you'll find is "You might get to go for a walk with a Andy, our "celebirdy" Andean Condor! These walks are not scheduled, but occur almost daily. If you happen to stumble across a 3-foot tall vulture, you've found a Condor Walk!" This is pretty lame, I think, and makes it hard for bugs and people alike to plan their days.

D) (10,000 points)[2] Which pun is better: Antoine or celebirdy? (Hint: This isn't even close. Clearly the best pun on this page is Andy!)

[2]Sadly I will keep extended jokes and 10,000 point problems to a minimum on the actual exam.

General Study Tips

The exam may have some true/false or multiple choice questions. In general, these are meant to be quick conceptual questions. The exam will cover the following topics:

Chapter 11: (Sections 11.1-11.6) Vectors in $\mathbb{R}^n$, their magnitude, direction, dot products, how to find the angle between two vectors, the projection formula, cross products, the right hand rule, equations of planes, lines, vector-valued functions, tangent lines, velocity vectors, arclength,

Chapter 12: (Sections 12.1-12.9) Contour plots and level sets, partial derivatives, definition of partial derivative, limits (including definition), open sets, the tangent plane to a differentiable function, gradient and its meaning, directional derivatives, chain rule, maxima and minima, critical points, saddle points, the second derivative test, Lagrange multipliers.

At the end of each chapter in the book there is a "Chapter Review" that contains a lot of good problems to help you prepare for the test. There are also several true and false questions. The answers to the odd questions are in the back of the book.