Math 1260 - Quiz 1

1. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ be vectors in $\mathbb{R}^3$, let $\mathbf{e}, \mathbf{f}$ be vectors in $\mathbb{R}^2$, and let k be a real number. Draw a circle around the following expressions that do **NOT** make sense (i.e. those that are not defined):

$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

$(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

$\mathbf{a} \cdot \mathbf{b} + k$

$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} \cdot$

$\mathbf{u} \cdot (\mathbf{e} \times \mathbf{f})$

$\mathbf{u} + (\mathbf{v} \times \mathbf{w})$

$(\mathbf{a} + \mathbf{b}) + k$

$(\mathbf{a} + \mathbf{b}) \times (\mathbf{c} + \mathbf{f})$

$\|k\mathbf{c}\|$

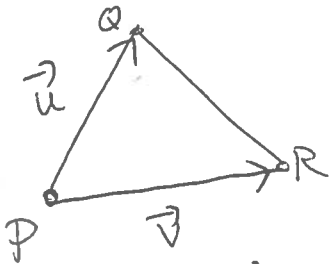
$(k\mathbf{u}) \times \mathbf{v}$

2. Let $P = (1, 2, -5), Q = (-2, 3, 0), R = (3, 0, 2)$ be three points in $\mathbb{R}^3$. Let $\mathbf{L}$ be the plane containing P, Q, R.

a) Find a vector perpendicular to $\mathbf{L}$.

b) What is the equation of the plane $\mathbf{L}$.

c) Compute the area of the triangle PQR. (Hint: Using a) should help!)



a) In the picture, $\vec{u} \times \vec{v}$ will be $\perp$ to the plane. $\vec{u} = \vec{Q} - \vec{P} = (-3, 1, 5)$.

$\vec{v} = \vec{R} - \vec{P} = (2, -2, 7)$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 1 & 5 \\ 2 & -2 & 7 \end{vmatrix} = (17, 31, 4) \leftarrow \text{normal vector.}$$

b) $17(x-1) + 31(y-2) + 4(z+5) = 0$

c) $\text{Area}_{\triangle} = \frac{1}{2}\text{Area}_{\square} = \frac{1}{2}\|\vec{u} \times \vec{v}\| = \frac{1}{2}\sqrt{17^2 + 31^2 + 4^2}$