

Math 1260 - Quiz 3

KEY

1. (5 points) Find the slope of the tangent line at $t = \pi/6$ to the curve $\mathbf{r}(t) = (x(t), y(t))$ with:

$$x = 2 \sin^2 t, \quad y = 2 \sin t \cos t$$

The values of sin and cos will be written on the board, should you need them.

$$\vec{r}'(t) = \langle 4 \sin t \cdot \cos t, 2 \cos^2 t - 2 \sin^2 t \rangle$$

$$\begin{aligned} \text{@ } t = \pi/6, \quad \vec{r}'(t) &= \left\langle 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}, 2 \cdot \frac{3}{4} - 2 \cdot \frac{1}{4} \right\rangle \\ &= \langle \sqrt{3}, 1 \rangle \end{aligned}$$

so $\boxed{\text{slope} = \frac{1}{\sqrt{3}}}$

2. (4 points) A bug is moving in \mathbb{R}^3 so that its speed at time t is given by

$$\text{speed}(t) = \sin(t) + \cos(t), \quad 0 \leq t \leq 1.$$

- a) Compute the length of the path the bug travels during this time.

- b) (1 point) The range for t was important for this problem to make sense. What would be wrong with this setup if I allowed, $0 \leq t \leq 4$ for instance?

$$\text{a) } L = \int_0^1 \text{speed } dt = \int_0^1 \sin t + \cos t \, dt$$

$$= -\cos t + \sin t \Big|_0^1 = -\cos 1 + \sin 1 - (-\cos 0 + \sin 0)$$

$$= \boxed{\sin 1 - \cos 1 + 1}$$

- b) speed should always be ≥ 0 . At $t = \pi$, speed(t) would be negative in this problem.