Math 1260 - Quiz 5

1. (5 Points) Find a formula for the tangent plane to the function $f(x, y) = 3x^2 + x^3y + y\cos(\pi x)$ at the point $(1, 3, f(1, 3))$. Does the point $(2, 4, 12)$ lie on this tangent plane? (5 points)

Solution: Note first that $f(1, 3) = 3+3+3\cos(\pi) = 3$. $f_x = 6x + 3x^2y - y\pi\sin(\pi x)$ and $f_y = x^3 + \cos(\pi x)$. Plugging in $x = 1$ and $y = 3$ yields

$$f_x(1, 3) = 6 + 6 - 3\pi\sin(\pi) = 9 \text{ and } f_y(1, 3) = 1^3 + \cos(\pi) = 1 - 1 = 0.$$

Hence the formula for the tangent place is

$$z = 3 + 15(x - 1) + 0(y - 3) \text{ or } z = 15x - 12.$$

We plug in $(2, 4, 12)$ and see if we get equality. But we don't since $12 \neq 15(2) - 12$.

2. Suppose our intrepid bug has once again found himself on a hot stove. The temperature at position (x, y) is given by the formula

$$T(x, y) = 200 + x^2 - y^3 + 4y.$$

Suppose our bug is walking in a direction so as to cool off his tarsal claws (his feet) as quickly as possible. It so happens that this direction is $\mathbf{j} = \langle 0, 1\rangle$, the direction of the y-axis. Which of the following locations might the ant be at? (More than one might be possible, and at least one is correct). Justify your answer. (5 points).

(a) $(0, 0)$
(b) $(0, 3)$
(c) $(-1, 1)$
(d) $(0, 1)$

Solution: The partial derivatives are $T_x = 2x$ and $T_y = -3y^2 + 4$. So our gradiant is $\langle 2x, -3y^2 + 4\rangle$. We want the negative of our gradiant $\langle -2x, 3y^2 - 4\rangle$ to be $\langle 0, K\rangle$, $K > 0$. Hence x must be zero. Thus we can rule out (d). For (a) the negative gradiant is $\langle 0, -4\rangle$ so that's out. For (b) the negative gradiant is $\langle 0, 23\rangle$ which is ok. Finally (d) gives negative gradiant $\langle 0, -1\rangle$.

Thus the only possible answer is (b).