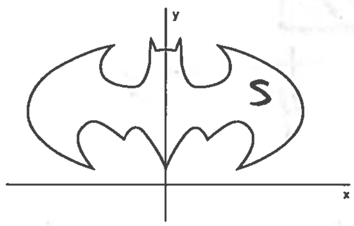
Math 1260 - Quiz 9

1. Let C be the curve in the figure below, oriented counterclockwise. Compute

$$\int_C xy\,dx + e^y\,dy.$$



By Green's Thm

$$= \iint_S \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = \iint_S x\,dA = \bar{x} \cdot \cancel{\text{Mass}}\,\text{Area}(S)$$

$$= 0 \text{ by Symmetry}$$

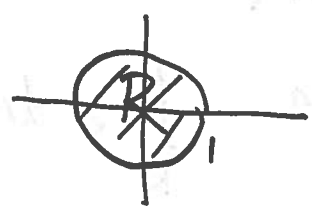
2. This problem has two parts. In neither part do you need to compute an integral. Let G be the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

a) Write down a surface integral that represents the surface area of G. What I'm looking for is an expression of the form $\iint_G g(x, y, z)\, dS$.

$$\boxed{\iint_G 1\, dS}$$

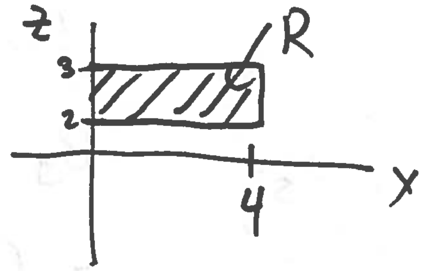
b) Now write your answer from part a) as a double integral $\iint_R h(x, y) dA$. Indicate with a picture what the region R is.

$$\iint_R \sqrt{y^2 + x^2 + 1}\; dA$$



3. If $\mathbf{F} = \langle xy, yz, z^2\rangle$ and G is the square with vertices $(0, 1, 2)$, $(0, 1, 3)$, $(4, 1, 2)$, $(4, 1, 3)$, and $\mathbf{n}$ is the normal that points in the positive y direction, compute $\iint_G \mathbf{F} \cdot \mathbf{n}\, dS$. (Reprinted on the back for your convenience).

3. If $\mathbf{F} = \langle xy, yz, z^2 \rangle$ and G is the ~~square~~ rectangle with vertices $(0, 1, 2)$, $(0, 1, 3)$, $(4, 1, 2)$, $(4, 1, 3)$, and $\mathbf{n}$ is the normal that points in the positive y direction, compute $\iint_G \mathbf{F} \cdot \mathbf{n}\, dS$. (Reprinted on the back for your convenience).



The ~~square~~ rectangle has $y=1$.

$\vec{n} = \langle 0,1,0 \rangle$.

$\vec{F} \cdot \vec{n} = yz = 1 \cdot z$ since $y=1$.

So $\displaystyle\iint_G \vec{F} \cdot \vec{n}\, ds = \iint_R z\, dA$

$$= \int_0^4 \int_2^3 z\, dz\, dx = \int_0^4 \tfrac{1}{2} z^2 \Big|_2^3 =$$

$$= \tfrac{1}{2}(5) \cdot 4 = \boxed{10}$$

Alternatives $\displaystyle\iint_R z\, dA = \bar{z} \cdot \text{Area}(R) = (2.5) \cdot 4 = \boxed{10}$