Worksheet 10 - Introduction to Vector Fields
Due Wednesday November 18th, 2015

1. (4 points) A) Draw a sketch of the vector field given by $\mathbf{F}(x, y) = \langle 1, y \rangle$. Draw arrows for all points with integer coordinates in the rectangle $-3 \leq x, y \leq 3$. You might be wise to notice a pattern rather than write down 16 values....

(4 points) B) Is $\mathbf{F} = \nabla f$ for some f? If so, find such an f. If not, explain why.

2. (8 points) A) Compute the divergence and curl of the following vector fields

$$\mathbf{F}(x, y, z) = x^2\mathbf{i} - 2xy\mathbf{j} + yz^2\mathbf{k}.$$

$$\mathbf{G}(x, y, z) = \cos x\mathbf{i} + \sin y\mathbf{j} + 3\mathbf{k}.$$

B) (1 point) We defined curl by using cross products. Remember **We only defined cross products for vectors in $\mathbb{R}^3$. So curl only makes sense for vector fields in $\mathbb{R}^3$.** Please write "Yes" to indicate that you've read this statement.

3. **Parametrizing Curves** You're going to become an expert at parametrizing curves over the course of the next few days, so let's get to work.

The easiest shape to parameterize is a straight line from a point $\mathbf{P}$ to $\mathbf{Q}$. Use the formula

$$\mathbf{r}(t) = (1-t)\mathbf{P} + t\mathbf{Q}, \quad 0 \leq t \leq 1.$$

A) (1 point) Double check that $\mathbf{r}(0) = \mathbf{P}$ and $\mathbf{r}(1) = Q$. Write "I have checked this, and I understand that writing $0 \leq t \leq 1$ is part of the parametrization."

B) (2 points) Write down the parametrization for a line segment that goes from $(0,0,0)$ to $(3,2,1)$.

C) (2 points) Now write down the parametrization for a line segment that goes from $(3,2,1)$ to $(0,0,0)$. This is DIFFERENT than the previous question, and this distinction is oh so very important.

D) (2 points) Now write down a parametrization for the line segment that goes from $(3,2,1)$ to $(4,6,-3)$.

D) (2 points) Now finally write down a parametrization for the line segment that goes from $(4,6,-3)$ to $(0,0,0)$.

You have just written down the parameterization for a triangle with vertices $(0,0,0)$, $(3,2,1)$, $(4,6,-3)$. It has three parts.

E) (2 points) Write down a parameterization for the circle in $\mathbb{R}^2$ of radius 3 centered at the origin. Have $\mathbf{r}(0) = (3,0)$.

You should be able to do these parameterizations in your sleep!

4) (12 points) Prove the following formula for a general vector field $\mathbf{F} = (M, N, P)$. Remember, this just means that M, N, P are the first, second, and third components of $\mathbf{F}$.

i) $\text{div}(\text{curl } \mathbf{F}) = 0$;

ii) $\text{curl}(\text{grad } f) = \mathbf{0}$;

iii) $\text{div}(f\mathbf{F}) = (f)(\text{div } \mathbf{F}) + \nabla f \cdot \mathbf{F}$.