Worksheet 11 - Green's Theorem
Due Wednesday Dec 2nd, 2015

1. We're going to get some practice with Green's Theorem. Use Green's Theorem to compute the following line integrals. Begin each problem by drawing the curve C and the region S.

A) $\oint 2xy\,dx + y^2\,dy$ where C is the closed curve formed by $y = x/2$ and $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$.

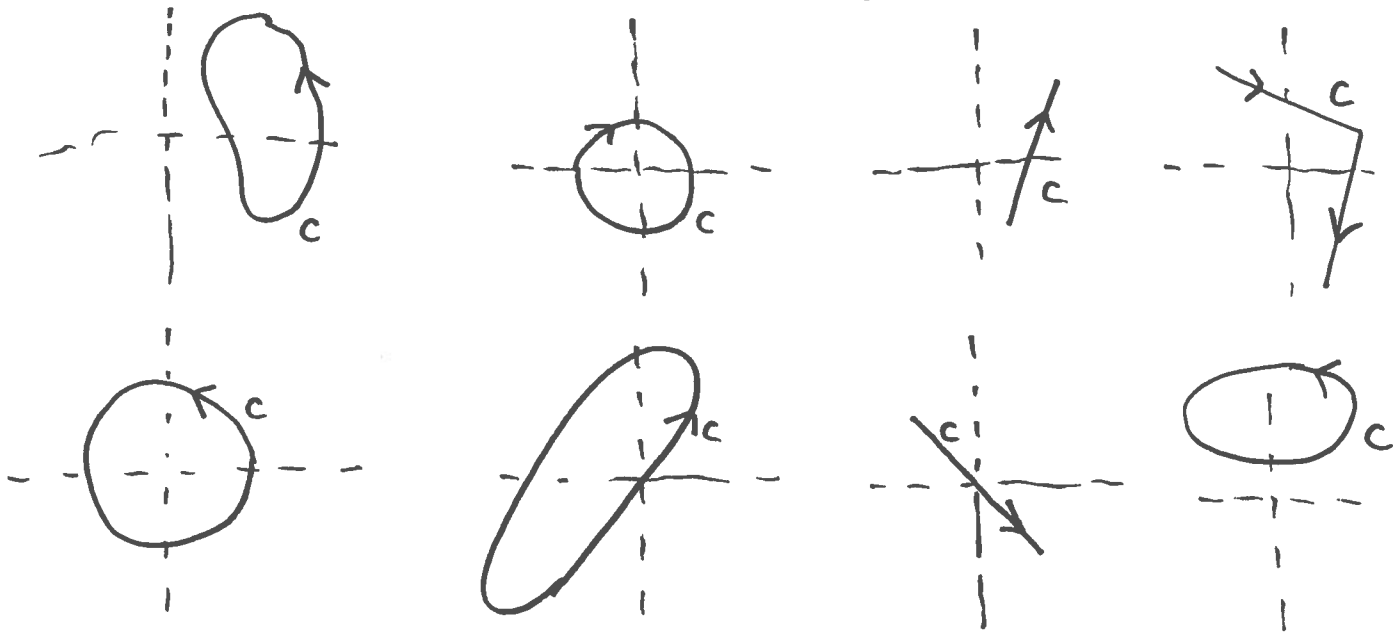
B) $\oint (e^{3x} + 2y)\,dx + (x^2 + \sin y)\,dy$, where C is the rectangle with vertices $(2,1)$, $(6,1)$, $(6,4)$, and $(2,4)$.

2. **Cut and Paste** We're going to learn a strong version of Green's Theorem during the course of this problem. But first, a reality check:

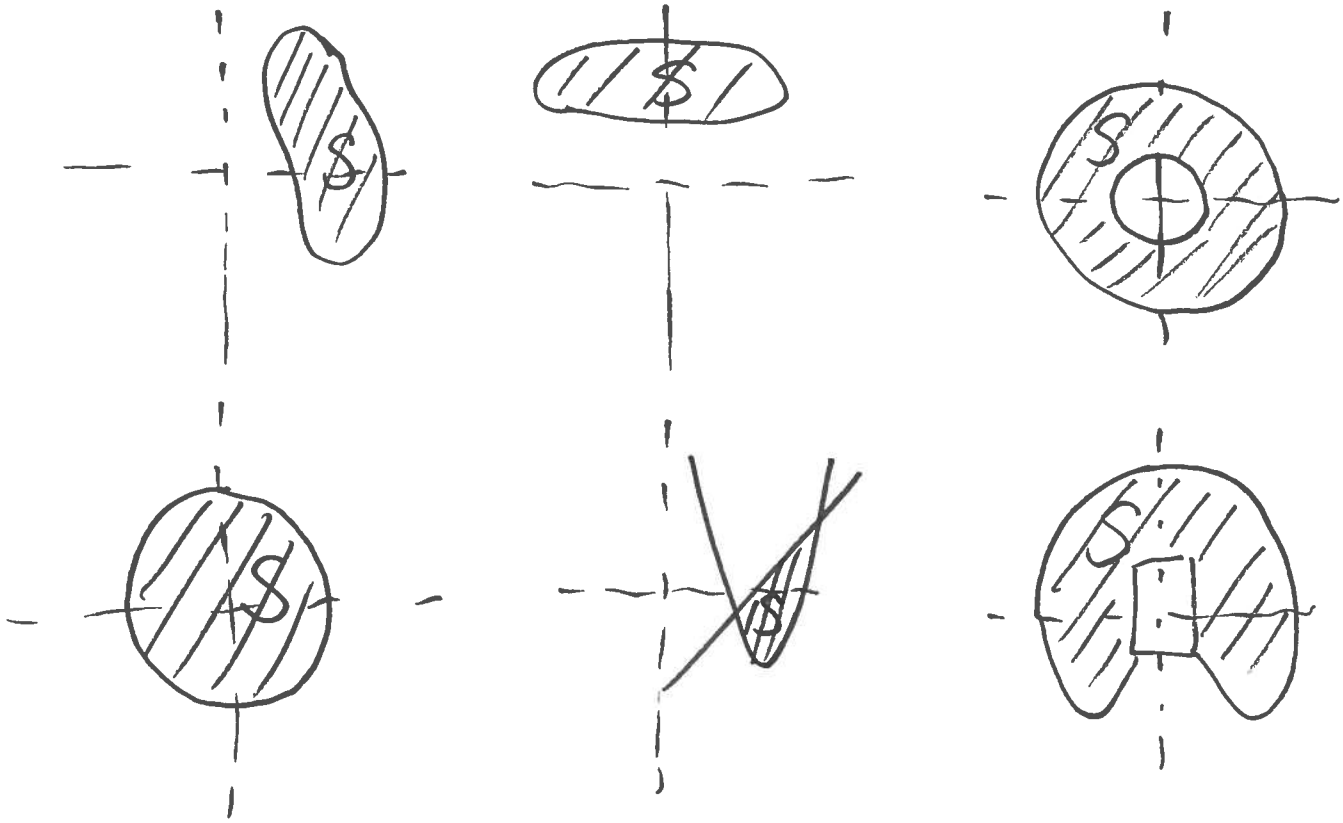
Let $\mathbf{F}(x,y) = \frac{y\mathbf{i}-x\mathbf{j}}{x^2+y^2}$. $\mathbf{F}$ is not defined at $(0,0)$. Let $\mathbf{F} = \langle M, N\rangle$.

A) What are M and N? Verify that if $(x,y) \neq (0,0)$ then $\partial M/\partial y = \partial N/\partial x$.

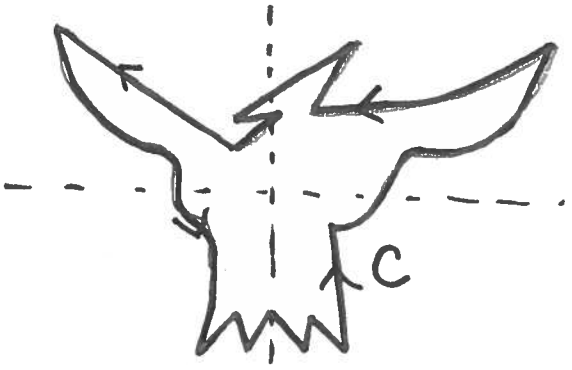
B) Still checking on the reality - for which curves below would $\int_C \mathbf{F} \cdot\, dr$ be defined?



C) For which of the regions below would $\iint_S \partial N/\partial x - \partial M/\partial y$ be defined? Next to each region where it is defined, write down the value. (Hint: you shouldn't be doing a computation here)



D) Now your task is to compute $\int_C \mathbf{F} \cdot \, dr$ where C is an artist's[1] depiction of the Mockingjay. (Hint: The answer is not zero. You will need to be creative with your use of Green's Theorem - you might want to think about cutting up the region into pieces. There's a figure in the book that might be useful...)



Book Problems Do: 14.4 2,3,4,5, 13, 15, 23
14.5 1,3,5,7,9,11,13,17,19

[1] A. Booché