Worksheet 4 - Functions of Several Variables
Due Wednesday Sept. 23, 2015

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You are greatly encouraged to work on this worksheet in groups - in fact, write down the names of your group members' names, and contact information, so you can get in touch with them after class to finish up this worksheet:

When working on this worksheet go slowly - make sure every member of your group understands what is going on - it's not a race!

1) VISUALIZING 3D GRAPHS

Draw the level sets for the following functions for 4 different values of c that you choose. (Be sure to label the values on your graph.)
(3 points each)

a) $f(x, y) = y - 3x^2$, b) $g(x, y) = xy$, c) $h(x, y) = (x - 1)(y - 1)$.

(Hint: For b) and c) think about how the functions are related. You should be able to draw the contours immediately for c) once you have done part b).) After you're done, try $Plot[y - 3x^2]$, etc. in Wolfram Alpha to see some pictures.

2) **REPARAMETRIZATIONS** Consider the curve

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), (2/3)t^{3/2} \rangle, \quad 0 \le t \le a.$$

First see what it looks like by using the function

```
ParametricPlot3D[{Cos[t], Sin[t], (2/3)*t^(3/2)}, {t, 0, 3*Pi}]
```

a) (1 point) Be very careful to use the same sorts of parentheses as I do. Try changing the `3*Pi` to another value. What are you changing?

b) (5 points) Now compute the arclength of the curve, using the integral we learned in class. Your answer should be a function of a. (Hint: You won't need trig substitution or anything fancy. You might want to start writing on a separate sheet of paper)

Recall that there are many different ways to parametrize a curve. For instance, on the last quiz, you wrote down two different parametrizations of a line. The parametrizations

$$\mathbf{r}(t) = (\cos t, \sin t), \quad 0 \le t \le 2\pi$$
$$\mathbf{p}(t) = (\cos(2t+1), \sin(2t+1))$$

both trace out a circle.

c) (2 points) What would the range for t be in the second example?

d) (4 points) Verify by integrating, that in the second parametrization, $p(t)$ that the arclength of the curve is still 2π.

Fact: It is true that no matter what parametrization we choose, as long as the same curve is traced, the arclength formula will always yield the same answer.

3) SOME MORE VISUALIZATIONS

We can plot in Wolfram Alpha. Look at the following equations, can you guess what they look like before you plot them?

```
Plot[sin(x*y)]
Plot[x^2 - y^2]
Plot[(x*y)/(x^2+y2)]
```

a) (5 points) One of the above should look like a saddle (the normal kind that you might see on a horse). Which one is it? Draw a sketch of its contour plot, (with labels)

Now we will draw what is called the monkey saddle:

$$z = x(x^2 - 3y^2)$$

b) (5 points total) Before typing it into a computer, determine where the z-value is 0, and use this to start a contour plot. (This is the part of the xy plane where the function achieves a value of 0.)

Your contour graph should now have 6 regions. Add to your contour plot by labeling a + or − to indicate whether the z coordinate will be positive or negative in that region.

This shape is called a monkey-saddle, because it has places for two legs and a tail. Normal saddles would be uncomfortable for monkeys.

Finally look at the last page of this worksheet. It contains a map of the area around Lake Blanche, with a summit and saddle labeled around Sundial peak. Talk about why it's called a saddle. Last weekend I attempted to climb to the summit, but had to turn around because I ran out of time - (fyi, I think 6-7 hours is the right time round-trip for the peak). Later when we know more about second derivatives, we can learn how to detect saddles.

c) (6 points) On the figure are points labeled A,B and C. Fill in the following table with a $+$, $-$ or 0 to indicate the sign of the partial derivatives, of the function

$$f(x, y) = \text{height above sea level at a point } (x, y)$$

assume normal orientation of x and y.

Remember that f_x denotes the rate of change when moving in the (positive) x-direction, and f_y denotes the rate of change when moving in the y-direction. (The subject of Wednesday's lecture, but rate of change is all you need to think about to do this problem.)

	f_x	f_y
A		
B		
C		

4) A CURVE WITH INFINITE LENGTH

In this example, we will see a curve that has infinite length.

a) (3 points) Consider the curve defined by $y = f(x)$ where $f(x) = x\cos(\frac{\pi}{2x})$ if $x \neq 0$ and $f(0) = 0$. Use Wolfram Alpha to sketch this graph. It should look chaotic.

b) (5 points) One way to estimate the length of a curve is by using what's called an **inscribed polygon**. For example, consider the values

$$\{x = 0,\ x = 1/4,\ x = 1/3, x = 1/2,\ x = 1\}.$$

Find the corresponding y values and plot them. Go ahead and join consecutive points with straight lines (you should have a zig-zig pattern). By virtue of the fact that the shortest path between any two points is a straight line, the sum of the lengths of these line segments should be smaller than the length of the curve. What is the sum above for the 4 line segments you drew? (No need to simplify)

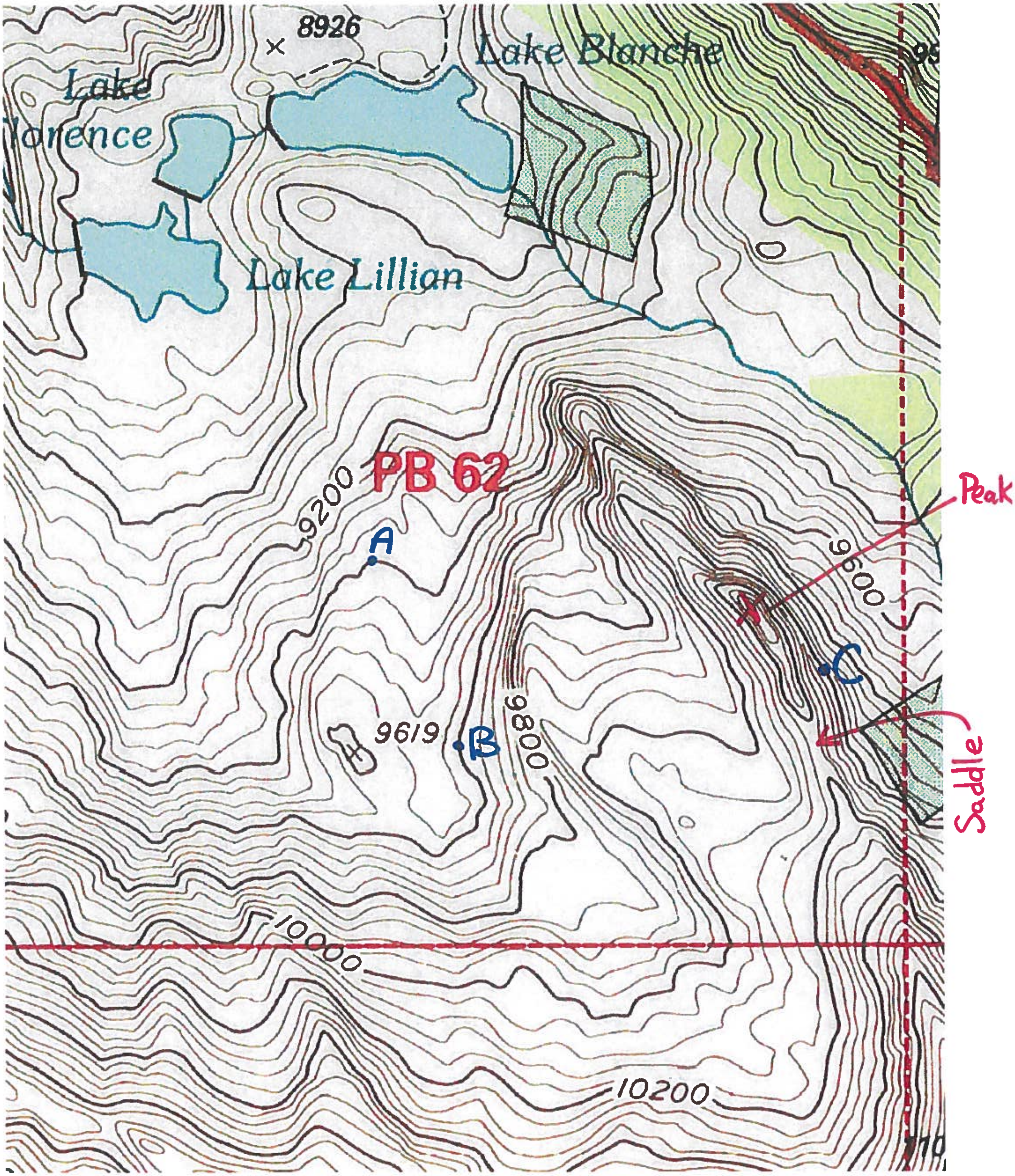
c) (Harder - 8 points) Now consider the set of points on the curve with x-coordinates equal to

$$\{x = 0,\ x = \frac{1}{2n},\ x = \frac{1}{2n-1},\ \ldots,\ x = \frac{1}{3}, x = \frac{1}{2},\ x = 1\}.$$

Compute the sum S of the lengths of the line segments connecting each pair of consecutive points, and show that

$$S > 1 + 1/2 + 1/3 + \ldots + 1/2n. \quad (\star)$$

(Hint: Your sum should have $2n$ terms, and if you are careful, you can see that each one of them should be bigger than one of the terms in H. I'd recommend starting from the right - with the segment from the points with $x = 1$ and $x = 1/2$.) Now we are done, since the right hand side of $(\star)$ diverges. (the Harmonic series - remember this from Calc 1-2?)



Mercator Projection
WGS84
USNG Zone 12TVK
CalTopo.com

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 km
0.1 0.2 0.3 0.4 0.5 mi
Scale **1:8234** 1 inch = 686 feet

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