Worksheet 7 - Double Integrals
Due Wednesday October 28th, 2015

You are greatly encouraged to work on this worksheet in groups - in fact, write down the names of your group members with contact information, so you can get in touch with them after class to finish up this worksheet:

When working on this worksheet go slowly - make sure every member of your group understands what is going on - it's not a race!

1. **Describing regions:** In this problem you will get practice describing regions in the plane and setting up the integrals. Remember, the bounds of integration have nothing to do with the function you are integrating.

A. (3 points) Let S be the region inside the triangle with vertices $(0,0)$, $(0,4)$ and $(1,4)$. Write down an iterated integral (with order $dy\ dx$) that computes

$$\int\int_S f(x,y)dA.$$

B. (3 points) Now switch the order of integration and write down an expression (with order $dx\ dy$) that computes $\int\int_S f(x,y)dA$.

C. (3 points) Work out both integrals in the case that the function is $f(x, y) = x + y$ and verify that you get the same answer.

D. (6 points) Compute

$$\int\int_S \frac{2}{1+x^2} dA$$

where S is the triangular region with vertices at $(0,0)$, $(2,2)$, and $(0,1)$. (Hint: Before setting it up, think about whether you want to integrate dy or dx first.)

E. Consider the solid in the first octant bounded by the surface $9x^2 + 4y^2 = 36$ and the plane $9x + 4y - 6z = 0$. Set up an integral $\int\int_S f(x, y)dA$ that computes the volume in the following way:

(1 point) First, what type of shape is the surface $9x^2 + 4y^2 = 36$? Notice it doesn't involve a z-coordinate.

(3 points) Draw a picture of the region S in the xy plane over which you should integrate to compute the volume.

(6 points) Finally, write down an integral that computes the volume and evaluate it. (Your answer should be an integer.)

2. **Rewriting Iterated Integrals** Rewrite the following integrals with the order of integration reversed. (3 points each)

A. $\int_0^2 \int_{y^2}^{2y} f(x,y)dxdy$

B. $\int_{-1}^0 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} f(x,y)dxdy.$

C. $\int_0^1 \int_{x^2}^{x^{1/4}} f(x,y)dydx.$

3. **More practice** For each problem, sketch the indicated solid and find its volume by iterated integration. (5 points each)

A) Solid in the first octant bounded by the cylinder $y = x^2$ and the planes $x = 0$, $z = 0$, and $y + z = 1$.

B) Solid in the first octant bounded by the surface $z = e^{x-y}$, the plane $x + y = 1$ and the coordinate planes.

4. **Using Symmetry** (5 points)
Evaluate $\int\int_S \sin(xy^2)\, dA$ where S is the annulus

$$\{(x,y) : 1 \leq x^2 + y^2 \leq 4\}.$$

(Hint: Done without thinking, this problem is hard, but using symmetry it is trivial.) Your writeup should split this integral into pieces and explain what is going on.