Worksheet 9 - Applications of Integration
Due Wednesday November 11th, 2015

0. The goal of this worksheet is to get lots of practice setting up and evaluating triple integrals.

1. **Cylindrical and Spherical Coordinate Review** We went over cylindrical coordinates in class on Monday. The general mantra is this: If we want to describe points in a cylinder, then we can use the coordinates (r, θ, z) with the relationship that

$$(x, y, z) = (r\cos\theta, r\sin\theta, z)$$
$$dV = r\ dzdrd\theta.$$

We didn't quite finish up with spherical coordinates the other day, but this is a system for describing points inside a solid sphere. We use the letters ρ, θ, ϕ. Using the powers of teamwork, books, etc write down the corresponding two lines for spherical coordinates (4 points)

You will use cylindrical or spherical coordinates when you are integrating over (part of) a cylinder or sphere. For the following integrals, describe (with words or pictures) the region over which you are integrating. (12 points)

$$A)\ \int_0^{2\pi}\int_0^3\int_0^{12} r\ dzdrd\theta \qquad B)\ \int_0^{\pi}\int_0^{2\pi}\int_0^{a} \rho^2\sin\phi\ d\rho\ d\theta\ d\phi$$

$$C)\ \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a} \rho^2\cos^2\phi\sin\phi\ d\rho\ d\theta\ d\phi \quad D)\ \int_0^{\pi/4}\int_0^{3}\int_0^{9-r^2} zr\ dz\ dr\ d\theta$$

(Remember, the region over which you are integrating can ALWAYS be determined by looking at the bounds. The function you are integrating has no bearing on this.)

2. **General Practice:** (30 points) Below are some problems that can be solved with integration. What I'd like you to do while in your group is to go through each problem and discuss whether you expect to use cylindrical or spherical coordinates (or neither). Then solve the problem. For the center of mass problems, use as much symmetry as you can! The same rules apply as in double integrals, for instance

$$\overline{z} = \frac{\iiint_E z\delta(x, y, z)dV}{\iiint_E \delta(x, y, z)}$$

A) Find the volume of the solid bounded above by the sphere centered at the origin having radius 5 and below by the plane $z = 4$.

B) Find the mass of a solid inside a sphere of radius $2a$ and outside a circular cylinder of radius a whose axis is a diameter of the sphere. Assume the density is proportional to the square of the distance from the center of the sphere. (This is the same as saying that the center of the sphere is at the center of the cylinder)

C) Center of mass of a solid hemisphre of radius a is the density is proportional to the distance from the center of the sphere.

D) Center of mass of the homogeneous solid bounded above by $z = 12 - 2x^2 - 2y^2$ and below by $z = x^2 + y^2$.

E) Find the volume of the solid inside both of the spheres $\rho = 2\sqrt{2}\cos\phi$ and $\rho = 2$. What is the centers of these two spheres?

F) Set up an integral that computes the volume of the tetrahedron with vertices

$$(0, 0, 0), (3, 2, 0), (0, 3, 0), (0, 0, 2).$$

3. **Extra Credit:** Naturally this will concern goats. Surprisingly, it won't involve triple integrals. Solve problem 34 in Section 10.7 in the book. If you are "exceedingly ambitious" try solving it by using polar coordinates rather than their Riemann sum suggestion. (I've failed to do so many times) Finally, write down a parametrization of the curve that the goat traces out if it keeps the rope taut throughout.