MATH 4030 – MIDTERM #1

Your Name

- You have 80 minutes to do this exam.
- No calculators!
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- If not otherwise stated, assume all matrices are $n \times n$.
- Good luck!

Problem	Total Points	Score
1	10	
2	8	
3	12	
4	10	
5	10	
6	10	
EC	3	
Total	60	

Number 1.

- (5 points) Use the Euclidean algorithm to compute $\gcd(272, 1479)$ (Hint: your answer should be > 1)

- (5 points) What is the repeating decimal for $1/13$? (Show your work)

Numero 2. You are given that

$$3 = 84 \cdot 7 - 117 \cdot 5.$$

Using this information, you should be able to (without computation) answer the following questions. Hint: If you find yourself doing anything complicated, there's an easier way! No need to show your work (2 points each)

- Find an integer solution (x, y) to the equation $30 = 84x + 117y$.

  $x =$

  $y =$

- Find an integer solution (w, z) to the equation $87 = 84w + 117z$.

  $w =$

  $z =$

- Find an integer a so that $[5a] = [3]$ modulo 84.

  $a =$

3.

(1) (6 points) Prove by induction that the sum of the first n odd numbers is n^2. I will check that you clearly set this problem up.

(2) (4 points) Consider the following set $S = \{\frac{n}{7},\ n \in \mathbb{Z}\}$. Are the following statements true or false? Justify your answer.

- If $a, b \in S$ then $a + b \in S$
- If $a, b \in S$ then $ab \in S$

No. 4.

(1) What does it mean for a fraction $\frac{a}{b}$ to be in **lowest terms**? (4 points)

(2) The following are the 9 equivalence classes in $\mathbb{Z}/9\mathbb{Z}$.

- Draw a circle around the additive identity (2 points)
- Draw an X through the multiplicative identity. (2 points)
- Draw an arrow between pairs of elements that are multiplicative inverses of each other (you may need to draw an arrow from an object to itself) (4 points)

(Hint: You might want to start by first writing each class in the familiar way as $[x]$ with $0 \leq x < 9$)

$$[-3], \quad [17], \quad [4], \quad [3], \quad [19]$$

$$[-2], \quad [5], \quad [9], \quad [2]$$

5.

(1) (5 points) Does every nonempty subset of the rational numbers have a least element? Justify your answer.

(2) (5 points) (In this problem a, b, and k are integers)
Suppose that ab is divisible by k^2. Does this mean that at least one of a or b is divisible by k? If yes, then prove it. If no, then provide an example.

VI.

(1) (5 points) If $\gcd(a, b) = 6$ then explain why a must be even.

(2) (5 points) Explain in words how you could convince someone that the decimal expansion of a rational number is a repeating decimal.

Extra Credit. (3 points) Let n be a positive integer that is NOT a perfect square. Prove that there is no rational number a/b such that $(a/b)^2 = n$.