MATH 4030 – MIDTERM #2

Your Name

- You have 80 minutes to do this exam.
- No calculators!
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	9	
2	11	
3	10	
4	10	
5	10	
6	10	
EC	3	
Total	60	

Number 1.

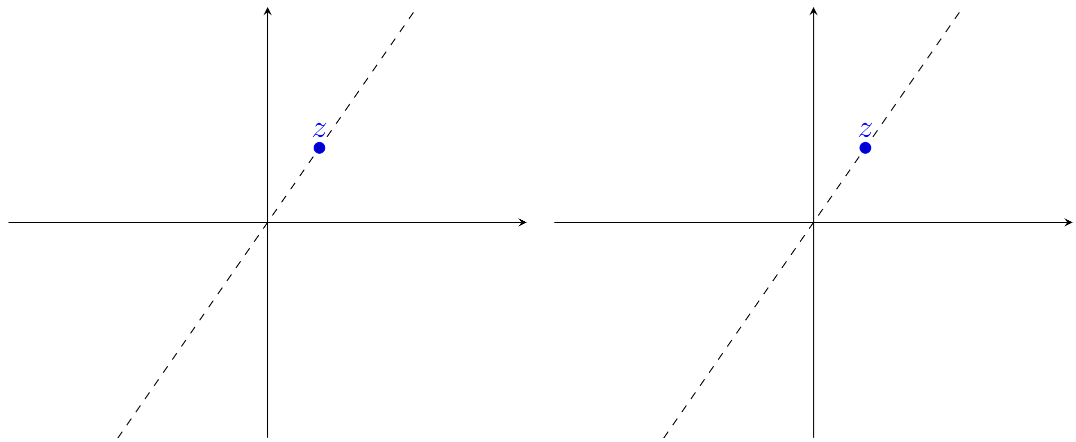
- (3 points) Let $z = 2 - 3i$ and let $w = (8, 30^o)$. (w is given in polar form). Compute $z + w$ and write the answer in the form $a + bi$.

- (3 points) What are all solutions x to the equation $x^3 = (8, 30^o)$? (You may write them in polar coordinates)

- (3 points) Let $w = (8, 30^o)$. Compute $1/w$ write your answer in both polar and standard coordinates.

Numero 2.

(1) (7 points) Consider the following drawing (reproduced twice):



You are given that $|z| = 2$. Using this information, draw (and label) approximate locations for the following points on the LEFT picture:

$$z^2, \quad \overline{z}, \quad z + i, \quad 2z, \quad 1/z.$$

Then on the RIGHT picture, draw all of the 4th roots of z.

(2) (4 points) Let $z = (r, \theta)$ and $w = (s, \phi)$. Write $(-1)(z/w)$ in polar coordinates. (Remember, in polar coordinates, the length is always a positive number!)

3.

(1) (5 points) Explain why every degree 1 polynomial over a field can be written in the form $c(x - r)$. (Hint: Your proof should begin "Let $f(x) = ax + b \ldots$" and should conclude by clearly saying "here's what $c =$ and here's what $r =$")

(2) (5 points) In the following list, circle the fields. Draw an X through those sets that have zero divisors:

$$\mathbb{N}, \quad \mathbb{Z}, \quad \mathbb{Q}, \quad \mathbb{R}, \quad \mathbb{C}, \quad \mathbb{Z}/2\mathbb{Z},$$

$$\mathbb{Z}/3\mathbb{Z}, \quad \mathbb{Z}/4\mathbb{Z}, \quad \mathbb{Z}/6\mathbb{Z}, \quad \mathbb{Q}[x], \quad \mathbb{C}[x], \quad \mathbb{Z}[i]$$

No. 4.

(1) (3 points) Let F be a field. Define what it means for a polynomial $f(x) \in F[x]$ of degree d to be prime.

(2) (5 points) Let $s = 3 + 4i$. Find a number r so that $(x - s)(x - r)$ is a polynomial with **REAL** coefficients.

(3) (2 points) In the ring $\mathbb{Z}/2\mathbb{Z}[x]$ which of the following is equal to $(x^2 + x + 1)^2$?

(a) $x^4 + x^2 + 1$
(b) $x^4 + x + 1$
(c) $x^4 + x^3 + x^2 + 1$
(d) $x^4 + x^2 + 1$

5.

(1) (4 points) Long divide the polynomial $g(x) = x^4 + 2x^2 - 3$ by the polynomial $x - 2$ in $\mathbb{Q}[x]$. Clearly indicate your remainder.

(2) (4 points) Now in general, suppose that $f(x)$ is a nonzero polynomial. If you divide $f(x)$ by the polynomial $x - 2$ explain why you know the remainder will always be a constant polynomial c.

VI.

(1) (3 points) Why is $\mathbb{Z}/4\mathbb{Z}$ not a field?

(2) (4 points) Find the prime factorization of $x^3 + x^2 + x$ in $F_3[x]$ where F_3 is the field $\mathbb{Z}/3\mathbb{Z}$.

(3) (3 points) Factor 10 into prime factors in the Gaussian integers $\mathbb{Z}[i]$? (Recall $\mathbb{Z}[i] = \{a + bi,\ a, b \in \mathbb{Z}\}$, and things factor here more than they might in just the usual $\mathbb{Z}$.)

Extra Credit. (3 points) Suppose I tell you that I am working over a field $F = \mathbb{Z}/p\mathbb{Z}$ where p is some prime number. Further, suppose that I tell you that the prime factorization of $x^4 - x^3 + 3x^2 + 6$ is $(x + 2)^2(x^2 + 2x + 5)$. What is p?