MATH 4030 – MIDTERM #3

Your Name

- You have 80 minutes to do this exam.
- No calculators!
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
EC	3	
Total	60	

This page contains helpful information about prime polynomials in $\mathbb{Z}_2[x]$ and $\mathbb{Z}_3[x]$. You may refer to this during the exam.

- In $\mathbb{Z}_2[x]$ the following are the polynomials that are prime with degree less than or equal to 4:

$$x,\ x+1$$

$$x^2+x+1$$

$$x^3+x+1,\ x^3+x^2+1$$

$$x^4+x+1,\ x^4+x^3+1,\ x^4+x^3+x^2+x+1$$

- In $\mathbb{Z}_3[x]$ the following are the polynomials **WITH LEADING COEFFICIENT 1** that are prime with degree less than or equal to 3:

$$x,\ x+1,\ x-1$$

$$x^2+1,\ x^2+x-1,\ x^2-x-1$$

$$x^3-x+1,\ x^3-x-1,\ x^3+x^2-1,\ x^3+x^2+x-1$$

$$x^3+x^2-x+1,\ x^3-x^2+1,\ x^3-x^2+x+1,\ x^3-x^2-x-1$$

Number 1.

(1) (3 points) According to the rational roots test, what are the possible rational roots of the polynomial $3x^4 - 5x^3 + 12x + 6$? (Just list them, you don't need to check whether they are roots.)

(2) (3 points) How many elements are in the field $\mathbb{Z}_3[x]_{f(x)}$ if $f(x) = x^{25} + 4x^2 + 6$?

(3) (4 points) What is the multiplicative inverse of 17 in $\mathbb{Z}_{31}$?

Numero 2.

(1) (3 points) Calculate the characteristic polynomial of the complex number $\alpha = 2i + 3$.

(2) (3 points) Write down two different complex numbers that have the same characteristic polynomial.

(3) (4 points) Calculate the characteristic polynomial of the complex number $\alpha = (1; 2\pi/6)$.

 # 3. In this problem, say (with explanation) whether the given polynomial is prime in $\mathbb{Q}[x]$. You may use things we've done in class:

(1) (3 points) $f(x) = x^{25} + 9x^7 - 27x^5 + 3x^2 + 12x + 3$

(2) (3 points) $g(x) = 105x^4 + 151x^3 + 25$

(3) (4 points) $h(x) = x^4 + 1$

No. 4.

(1) Consider the polynomial $g(x) = x^2 + 4x + n$.

- (3 points) For what values of n will $g(x)$ have only real roots? (Your answer should be a equation or inequality involving n)
- (3 points) For what values of n will $g(x)$ have only one root?

(2) (4 points) Find all real and complex roots of the polynomial $x^3 + 2x^2 + 6x + 5$. (Hint: It has a nice rational root)

5.

(1) (3 points) Define the term *algebraic number*.

(2) (3 points) True or False: If $f(x)$ is prime in $\mathbb{Q}[x]$ then so is $f(x^2)$. If true, give an explanation. If false, provide an example of a polynomial $f(x)$ that is prime, but where $f(x^2)$ is not.

(3) (4 points) If α is a root of $x^{10} - 5x^8 - 10x^5 + 15$ then is α constructible with ruler and compass? (Hint: Your explanation should use the word "prime" somewhere)

VI.

(1) True/False (no justification needed) (2 points each)

- There a field with 6 elements.

- There a field with 4 elements.

- $\mathbb{Z}_4$ is a field.

(2) (4 points) The following polynomials are prime in $\mathbb{Z}_7[x]$:

$$x^2 - 2x - 2, \quad x - 1, \quad x^3 - 2x^2 - 2x - 1, \quad x^2 - 2x + 3.$$

Use this information to write down a field with 49 elements. (Your answer should be in the form $F[x]_{f(x)}$.)

Extra Credit. (3 points) Factor the polynomial $x^4 + 1$ over the real numbers. Hint: it factors as the product of two quadratic polynomials