# THE NATURAL NUMBERS

Work with a group of 2 or 3 students discussing the following problems. (Some of these problems appear on the homework this week.)

**Today's Tools**

- Definition of the natural numbers
- Induction
- Definition of addition
- Subsets

## Some problems from today

- Get to know your group! Chat for a few minutes. Would you like to work on this homework assignment together?
- How do you think you would define multiplication? For instance, if $m$ is a natural number, what would a good **definition** of $2 \cdot m$ be? What about $3 \cdot m$? Can you come up with an inductive definition for $m \cdot n$ for any $n$?
- Give a definition of the factorial function $f(n) = n!$ by induction. Make sure you specify the domain and codomain for this function and check that your definition will define $n!$ for every $n$ in your range.
- I define $X_n$ to be the set $\{1, 2, \ldots, n\}$. Write down $X_4$ and all of its subsets. (Don't forget the empty set!) Prove by induction that there are in general $2^n$ subsets of $X_n$. Hint: exactly half of the subsets of $X_{n+1}$ contain $n + 1$.

## Looking Forward

- **Warmup for tomorrow** So far we have talked about adding and subtracting natural numbers. What happens if you try to subtract or divide? On Wednesday we'll focus on division (with remainder) of natural numbers. To get some practice - try to come up with a statement about when we can divide two numbers $a$ and $b$ with a quotient $q$ and remainder $r$. Be brave - if you don't know what I mean, just ask. For example, what happens if we divide 23 by 5?
- Before Wednesday's class, please read Aaron Bertram's notes on the natural numbers. (The first section of the notes) Pay particular attention to the proof of the principle of math induction.