# POLYNOMIALS

(1) Consider the following polynomials in $\mathbb{R}[x]$:

$$f(x) = 2x^3 - 8, \quad g(x) = x^4 + 1, \quad h(x) = x^2 + x + 1$$

- Compute $f + g$, $fg$, and $gh$.
- What is the degree of $f$? of $g$? of $h$?
- Can you write down a formula for the degree of the product of two polynomials in $\mathbb{R}[x]$?
- Can you write down a formula for the degree of the sum of two polynomials? (Warning: this is tricky)

(2) Now consider the polynomials in $\mathbb{Z}/10\mathbb{Z}[x]$: (Here I will just write $c$ to mean $[c]$. Remember this means that the coefficients are all taken mod 10.)

$$f(x) = 2x^3 - 8, \quad g(x) = 2x^4 - 2, \quad h(x) = 5x^2 + x + 1$$

- Compute $f + g$, $fg$, and $gh$.
- Can you write down a formula for the degree of the product of two polynomials in $\mathbb{Z}/10\mathbb{Z}[x]$?
- Why did you get a different answer for $\mathbb{Z}/10\mathbb{Z}$ and for $\mathbb{R}$?

(3) In a ring, we say that an element $a$ is a **zero divisor** if $a \neq 0$ and there some $b \neq 0$ such that $ab = 0$. Find all the zero divisors in $\mathbb{Z}/12\mathbb{Z}$.

(4) Prove that in a field there are no zero divisors. (Hint: The defining property of a field is that EVERY nonzero element has a multiplicitive inverse. In other words if $a$ is nonzero, we can divide an equation by $a$)

(5) Consider the following statements. Decide whether you think they are going to be true?

- Let $F$ be a field. If $f(x)$ and $g(x)$ are in $F[x]$ and are not zero then $\deg(fg) = \deg f + \deg g$.
- What do you think the degree of the zero polynomial should be?
- Does the polynomial $x$ have a multiplicative inverse in $\mathbb{R}[x]$?
- What about the polynomial $x + 1$? What do you think the polynomials that DO have multiplicative inverses are?
- In general (maybe over $\mathbb{Z}/10\mathbb{Z}$ if $f(x)g(x) = f(x)h(x)$ can we conclude that $g(x) = h(x)$?