IS THERE A FIELD WITH 4 ELEMENTS?

(1) Check that $\mathbb{Z}/5\mathbb{Z}$ is a field. Remember all the rules like associativity and commutativity and distributivity we get for free, so all you really need to check is that there is $0, 1$ additive and multiplicative inverses.

(2) Write down the 4 elements of $\mathbb{Z}/4\mathbb{Z}$. Explain why $\mathbb{Z}/4\mathbb{Z}$ is NOT a field. (I'll ask this question on the exam - make sure you have a good answer)

FIGURE 1. A (lavender) field with 4 elements

(3) Just because this set with its 4 elements with clock arithmetic doesn't form a field, doesn't mean that there isn't some other way to make a field. Your group is going to figure out if there is or isn't one.

I want to leave you fairly free to come up with your own ideas on this - but here are a few guidelines. You'll need to come up with a set with four elements. You could call them $\{a, b, c, d\}$ if you want. But aren't there two distinguished elements that EVERY field has? Maybe you could call two of your elements that? Now think about what the addition and multiplication rules must be like. Drawing a multiplication table might be helpful. Our goal before the end of class is to come up with a set, and a way of multiplying and adding so that all the rules of a field are satisfied. Let's do this!