THE REAL NUMBERS

Work with a group of 2 or 3 students discussing the following problems. (Some of these problems appear on the homework this week.)

Some problems from today

(1) Find the number whose infinite repeating decimal is $5.14444444\ldots$.
(2) Same question for $0.666666\ldots$ (this one maybe you can just recognize and do in your head)
(3) Add the two rational numbers from parts 1 and 2 above and then find the decimal expansion for that fraction.

Remember now that we have only defined the real numbers as a set. We haven't said how to multiply or add two real numbers. I want you to talk about the difficulties involved in adding and multiplying infinite decimals. Example, what's the 1000th decimal in the real number $\sqrt{2} \cdot \pi$?

(4) Discuss how one might go about adding infinite decimals? How would you take care of all the carrying? In theory do you think you could write down a precise algorithm if the decimals were at least repeating?
(5) What about non-repeating decimals (i.e. irrational numbers?) (It's hard!) What about multiplying them together? (Even harder!)
(6) Our goal as a class today is to come up with a description of how to multiply two real numbers together - what do you think? Tell me how I could write down all the decimals in $\pi \cdot \sqrt{2}$ if I know the decimals for π and $\sqrt{2}$.