# UNIQUE FACTORIZATION

The goal with today's worksheet is to come up with (and prove) the right statement about unique factorization in $\mathbb{Z}$ and $F[x]$. The key is that both of these rings are Euclidean domains.

(1) In $\mathbb{Z}$, write down some factorizations of the number 12 into primes. You should find at least 3 different ways!

(2) Do the same in $\mathbb{Q}[x]$ with the polynomial $x^2 + 3x + 2$. There are infinitely many ways to factor this. Write down 4 different ways.

(3) What relationships do you see between the different factorizations? (What are some commonalities and some differences?)

(4) Talk about why you think the following statement is true.

If $p$ and $p'$ are both primes and $p$ divides $p'$ then $p$ and $p'$ are in fact associated.

(5) Now try to state the fundamental theorem of arithmetic: If $a$ is an element in a Euclidean domain, then "$a$ factors uniquely into a product of primes" - try to come up with a precise version of this statement and prove it!