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SOME *NEUTRAL* REASONS WHY THE KALAM *PHILOSOPHICAL*  
COSMOLOGICAL ARGUMENT FAILS FOR THEISTS AND NONTHEISTS

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1. This paper shows why the Kalam Philosophical Cosmological Argument (hereafter, KPCA) fails for theists and nontheists—especially those sharing a *first philosophy*. Which is to say that it concerns the Kalam Cosmological Argument (hereafter, KCA) insofar as it is based upon the philosophical ground that any infinite temporal series is metaphysically impossible. Accordingly, this paper challenges the bold claim by William Lane Craig that the KPCA constitutes "the cosmological argument which is most likely to be a sound and persuasive proof for the existence of God."

2. The KPCA can be conveniently stated as follows: (a) That temporal series which constitutes or includes the history of this physical universe had a beginning in the sense of having a first event or moment. (b) Whatever begins to exist must have a cause. (c) Therefore, the beginning of the temporal series (pertaining only to natural entities) which constitutes or includes the history of this physical universe had a cause. Further analysis and argumentation are required to show that the temporal series in question is of finite duration; and that the beginning of this temporal series was causally due to the agency of a spiritual personal being of great power and intelligence, who sans creation is changeless and in some sense timeless. The KPCA presupposes an A (or dynamic) theory of time. According to this theory events or moments, besides being before, simultaneous, or after other events or moments, are past, present, and future, and thus temporal becoming is a

real and objective feature of the physical universe. Premise (b). that whatever begins to exist must have a cause, is not to be understood as entailing ontic or epistemic determinism. As Craig succinctly puts it: "To be uncaused in the relevant sense of an absolute beginning, an existent must lack any non-logical necessary or sufficient conditions whatsoever." I accept as true the A theory of time, and also hold that premise (b) states a metaphysically necessary truth with respect to whatever temporal series the beginning of which is understood to be its first event or moment in the series which constitutes or includes the history of this universe—assuming to there have been such a first event or moment. Those who disagree with these propositions are invited to assume their truth for the sake of argument.

3. Craig himself distinguishes between metaphysical (real or absolute time as others might call it) and physical (or empirical or clock time). The former is characterized by the succession of events or moments which can have relations of absolute simultaneity. The latter is the time pertaining to the physical world as measured by those processes which constitute natural clocks. According to him, it is metaphysically possible for this physical universe (assuming it have had a temporal beginning) to have been preceded by a temporal series of events encased in moments of nonzero duration that are not defined in terms of spatial relations or dynamic processes. Such events (e.g., mental events in the lives of angelic beings or nonmental events pertaining to other physical universes (if any)) are "mapped" onto metaphysical time. Since every moment in metaphysical time has been (or can only be) immediately succeeded by a next moment, metaphysical time consists of denumerably infinite moments (i.e., infinitely many moments in one-to-one correspondence with the set of all natural numbers) provided that metaphysical time is an

actual infinite. The paper proceeds with the assumption made for argument's sake (but which assumption I also hold as true) that metaphysical time obtains.

4. The first premise (i.e., premise (a) of the KPCA) is purportedly justified upon two philosophical grounds. First, that any infinite set of concrete entities or events in the real world (hereafter, real infinite) is metaphysically impossible and that a purported infinite temporal series is such a set. Second, that even were an infinite set of coexisting concrete entities metaphysically possible, an infinite temporal series would nevertheless be metaphysically impossible. But to assert that an real infinite set or an infinite temporal series is metaphysically impossible is not to assert that either is logically impossible. The logical possibility of both is accepted by Craig, just as he accepts the logical but not metaphysical possibility of the uncaused beginning of this universe.

5. As presented by Craig, the first philosophical subargument (i.e., that any real infinite is metaphysically impossible) in support of the KPCA involves a showing that the attempted application of Cantorian set theory to the real world of concrete entities and events generates counterintuitive absurdities.

6. Two finite sets are *equipollent* if and only if they are in one-to-one correspondence. In the domain of finite arithmetic two sets each of which is equipollent to a third set are necessarily equipollent to each other. Thus the equipollence-relation is transitive. A proper subset of a finite set is not equipollent to the latter; they have two different cardinal numbers. (A proper subset of a finite set is one in which the master set has at least one member more than the subset in question.) A so-called potential infinite is actually a finite set to which new members can be indefinitely added. The cardinal number of any finite set is a natural number.

7. Next we turn to the domain of pure mathematics insofar as transfinite arithmetic is involved. An actual infinite is a set the cardinality of which is greater than any natural number. Its cardinal number is the same or greater than that of the set of all natural numbers, which is itself an actual infinite. So is any proper infinite subset of  $N$ , e.g., the set of all even numbers. This is so because there is a function-equation (i.e.,  $o = 2n$ ) which is a rule ordering each natural number such that there is another natural number which is twice its numerical value. So strange but true: the set of all natural numbers is equipollent to its proper infinite subset-- the set of all odd numbers. It suffices for our purposes to stipulate that the term *Cantorian set theory* is used in this paper to refer to any set theory pertaining to transfinite arithmetic that includes three important propositions concerning pure transfinite arithmetic: (1) that any two infinite sets have the same cardinality if and only if they are equipollent to each other; (2) that an infinite set and any of its infinite proper subsets are equipollent to each other and thus have the same cardinality; and (3) that two mathematical infinities each equipollent to a third mathematical infinite are equipollent to each other. What is the cardinality of the set of all natural numbers? It is aleph-zero (or aleph-null--( $\aleph_0$ ), the lowest transfinite number. The set of natural numbers and any other mathematical infinite that is equipollent to it is termed denumerable or denumerably infinite. There are transfinite numbers greater than aleph-zero, but this paper does not concern them as it suffices for our purposes to presuppose that any real infinite of concrete entities is denumerably infinite.

8. Craig acknowledges (or at least assumes for argument's sake) that modern set theory, a branch of pure mathematics, is logically consistent and coherent as well as being fruitful in that domain. But let us now turn to the real world, the world consisting of substantive

concrete entities whether natural or supernatural (as distinguished from the world of abstract entities such as numbers, points, and the like), their properties, and events pertaining to them. According to Craig, the attempted application of relevant Cantorian set theory to the real world generates counterintuitive absurdities. Craig describes several scenarios in which these counterintuitive absurdities purportedly obtain, such as Hilbert's Hotel or the library consisting of infinitely many books. Permit me to set forth another scenario which perhaps even more usefully constitutes an ideal paradigm for our inquiry.

9. This physical universe (let us call it World-A) includes physical objects that are spatially located such that each is spatially distant from another. But we can conceive of another physical universe (nWorld-B), any physical object of which is not spatially distant from any physical object in World-A. Now let us assume that there is a denumerably infinite set of physical worlds similar to World-A and World-B, none of which is spatially related to any other. Suppose further that each such world has finitely many humans with each human having only two hands (a right hand and a left hand). So there are infinitely many worlds, infinitely many humans, infinitely many human hands, infinitely many left hands, infinitely many right hands, and infinitely many pairs of human hands. The scenario described above, although to be sure antecedently improbable, is at least logically coherent and does not exhibit anything otherwise weird (prescinding from issues solely based upon Cantorian set theory).

10. Now Craig's argument involves consideration of what can be called the *standard version* of the attempted application of the relevant Cantorian set theory to the real world. According to this version, each real infinite is deemed to be a set within the meaning of transfinite arithmetic. This is a bridging (or correspondence) rule which as such is not

part of Cantorian set theory within the domain of pure mathematics. Bear also in mind, we are dealing only with infinite sets the cardinality of each is aleph-zero, i.e., each such set in question is equipollent to the set of all natural numbers. Given the foregoing, it follows that each real infinite described in the immediately preceding paragraph is equipollent to every other similarly described set. Thus the set of infinitely many hands is equipollent (i.e., in one-to-one correspondence) with the set of infinitely many humans, and the set of infinitely many hands and the set of infinitely many humans are each equipollent with the set of infinitely many left hands or the set of infinitely many pairs of hands. So we have a scenario as to which it was initially stipulated that every human has only two hands and as to which any finite subset of humans is not equipollent to the finite subset of human hands which belong to the humans in question; but as to which the infinite set of humans is equipollent to the set of all human hands. Surely Craig is right when he contends that the application (but via the standard version) of Cantorian set theory to the real world generates counterintuitive absurdities—such absurdities constituting the sufficient ground for concluding that real infinities are metaphysically impossible.

11. Although Craig is quite right in holding that the standard version generates counterintuitive absurdities, his conclusion that any real infinite is metaphysically impossible only follows if there is no other but yet plausible way by which Cantorian set theory may be applied to the real world. And there is. It is called the *alternative version*, the application of which to the real world precludes the generation of the counterintuitive absurdities.

12. According to the alternative version, there is no bridging (or correspondence) rule according to which a real infinite is deemed to be a set within the meaning of transfinite arithmetic. Rather, the nexus of real infinities with transfinite arithmetic is accomplished simply by positing that every real infinite is equipollent to the set of all natural numbers and thus has aleph-zero (the lowest transfinite number) as its cardinal number. This being the case, every real infinite is also equipollent with every denumerable mathematical infinite. But although, according to the alternative version, every two real infinities are each equipollent to the set of natural numbers, they are not necessarily equipollent to each other. On the other hand, two real infinities each equipollent to a third real infinite are equipollent to each other. Finally, a real infinite is not equipollent to any of its proper infinite subsets. Since, according to the alternative version, equipollence is not necessarily transitive, I avoid using the term *equivalent* as a synonym for *equipollent* when I speak of elements of two real infinities being in one-to-one correspondence. The principal difference between the standard and alternative versions is as follows. The standard version holds that two real infinities have the same cardinality if and only if they are equipollent. However, according to the alternative version, it is a sufficient but not necessary for equipollence to obtain between two real infinities in order for them to have the same cardinality.

13. So let us return to the scenario of the denumerably infinite set of physical worlds, each such world with finitely many humans, and each human with only two hands. Application of the alternative version to this scenario does not generate any counterintuitive absurdity. The set of infinitely many humans and the set of infinitely many hands are in one-to-two correspondence—as they should be. The set of infinitely

many humans and the set of infinitely many pairs of human hands are in one-to-one correspondence—as they should be. The set of all human hands and that of all left hands are in one-to-two correspondence—and they should be. And so on.

14. What is to be remembered is that any equipollence obtaining between any two mathematical infinities does so by virtue of their mathematical properties as intuitively discerned, and as further embodied in naïve or logicist set theory, or some particular axiomatic set theory. Such equipollences obtain by virtue of function-equations, rules which order the sets in question into relations of equipollence (e.g., every natural number has an even number which is twice its numerical value). Such equipollences do not obtain by virtue of factually contingent or definitional matter pertaining to a real world consisting as it does of concrete substantive entities, their properties and states, and events pertaining to them. On the other hand, whether or not two real infinities are equipollent does indeed depend upon factually contingent or definitional matter. Because of a function-relation ( $z = -n$ ), there is a complementary negative number for every natural number. Because of a factually contingent law of nature (hypothetical, to be sure), every human (of the set of infinitely many humans) has a pair of hands. Because of a definitional matter, there is a one-to-twelve correspondence between the set of feet and that of inches. Were space Euclidean, would there be infinitely many feet along a geometric straight line beginning at any given point on the Earth's surface? There would, according to the standard version, be infinitely many feet with necessarily a one-to-one correspondence between feet and inches. But, according to the alternative version, there would be a one-to-twelve correspondence between feet and inches along this line. For

Craig, the only alternative is to flatly deny the metaphysical possibility of (an infinitely extended) Euclidean space.

15. According to either the standard or the alternative version, the addition or subtraction of finitely many members from or to a real infinite does not change the cardinality of that set. However, contrary to the standard version, the set before the addition or subtraction is not equipollent with the set after the addition or subtraction, as the case may be. So, in one sense, the set before the addition or subtraction of members can be said to have as many members as the set after the addition or subtraction because the cardinality remains the same. But, in another sense, the set before the addition or subtraction can be said not to have as many members as the set after addition or subtraction because the set before and the set after the addition or subtraction are not equipollent to each other.

16. Here we may pause to consider what Craig proposes as a block to any attempt to apply Cantorian set theory to the real world. He argues that any such attempt is blocked because Cantorian set theory precludes the inverse arithmetical operations of subtraction and division. True, in the domain of pure Cantorian set theory, inverse arithmetical operations of subtraction and division pertaining to transfinite numbers are precluded. This is the case because the nonarithmetical operations of subtracting or removing different infinities from others have disparate arithmetical results. For example, in one case the removal of the set of all natural numbers greater than 4 from the set of all natural numbers leaves a remaining finite set consisting of the numbers 1, 2, 3 and 4, with 4 being the cardinal number of this finite set. In another case, the removal of the set of all odd natural numbers from the set of all natural numbers results in a remaining set of all even natural numbers with, of course, aleph-zero as its cardinal number. Craig claims that

a contradiction obtains because the subtraction of equal quantities from equal quantities generates different answers. Thus, with respect to the two scenarios just given, the subtraction of the identical number of numbers (i.e., aleph-zero) from the identical number of numbers (i.e., aleph-zero) does not yield identical results (i.e., 4 and aleph-zero). But, au contraire, there is no contradiction because the rule forbidding subtraction with respect to transfinite *numbers* does not apply to *members of sets*.

17. Any apprehension, based upon Craig's contention, is even less warranted when we turn our attention to the real world. For in the real world, we can speak in one sense of equal quantities with respect to two real infinities inasmuch as they have the same cardinal number, i.e., aleph-zero. But, in another sense, we can speak of unequal quantities with respect to two real infinities whenever they are not equipollent to each other. So we do not have a simple case of subtraction of equal quantities from equal quantities arriving at different answers. The term "equal quantities" as used by Craig thus turns out to be harmlessly ambiguous.

18. Craig has urged that the metaphysical impossibility of any real infinity is exacerbated in the case of infinite temporal series. He maintains, for example, that the infinite temporal series of years ending in any particular year has the same number of years as any of its infinite proper subsets ending in any given earlier year. In so doing, he would be quite right since, according to the alternative version as well as the standard, any infinite temporal series is equipollent to the set of all natural numbers and thus has aleph-zero as its cardinal number. However, his fundamental point is that (given the standard version) the infinite temporal series ending in a particular year is equipollent to any of its infinite proper subsets. Thus he contends that a counterintuitive absurdity obtains because

if an event  $x$  occurs in a given year, there is no reason why it should not have occurred in to any of its infinite subsets. But as the astute reader easily discerns, this difficulty is obviated with the alternative version since the two sets are not equipollent. So the happening of  $x$  in a given year in an infinite temporal series cannot possibly be a warranted basis for raising the issue of why it did not happen in some preceding year based upon the ground asserted by Craig.

19. At this point we pass on to the second philosophical subargument in KPCA, i.e., that an infinite temporal series is metaphysically impossible, even were we to assume for argument's sake the metaphysical possibility of some real infinities of coexisting concrete entities. The specific grounds for this position are: (1) a temporal series is a collection formed by successive addition; and (2) a collection formed by successive addition cannot be an actual infinite. The conclusion is that no temporal series can be an actual infinite.

What is claimed is that just because every finite segment of a temporal series is a collection formed by successive addition one cannot legitimately conclude that any temporal series can be a collection formed by successive addition. But I cannot but consider this argument to be other than fatally defective because his first premise is gratuitous. For a temporal series, rather than being essentially characterized as a collection formed by successive addition, should instead be so characterized as being a collection that is instantiated sequentially or successively in time (each event or moment following one after another). Thus being a collection formed by successive addition is not constitutive or definitive of any temporal series; although it is such for any finite temporal series. For a specific temporal series totally formed by successive addition is one that must necessarily be numerically finite. We must begin at sometime if something

is formed by successive addition. Hence it is true that no actual infinite is formed by successive addition. But an infinite temporal series is not formed at all; although finite segments of the series have been so formed. So Craig's first premise is a nonstarter.

20. One can neither count to or from infinity because infinity (i.e., aleph-zero in the case of a denumerable infinite) has no immediate predecessor or successor. Counting, as a temporal process, in order to determine the cardinality of a set of concrete entities applies only to finite sets and requires a starting number. Of course, every infinite temporal series is equipollent with the set of all natural numbers. But then, according to the alternative version, every infinite temporal series is not equipollent with any of its infinite proper subsets. However, what would be something but not actually like counting or an enumeration of an infinite temporal series of moments (here understood to include future moments) is a series which is yet to terminate at some designated target moment, when some important event is to take place, where at all times it is already known just how finitely many moments are yet to be gone through to the future target moment from any specified moment in the series. God, assumed here to have the requisite knowledge, simultaneously apprehends at all times the one-to-one correspondence of the set of natural numbers with the present, past, and future members of the infinite temporal series terminating in the designated moment. He, assumed here to have the requisite power, could have programmed a temporally beginningless counter or meter to announce upon the arrival of any moment just how finitely many moments yet remain before the arrival of the target moment. But this countdown scenario is quite unlike the one in which someone claims to have just finished at the present hour writing down all the negative numbers ending in -1 at, say, the rate of one negative number per one hour

where neither the counter nor his supernatural programmer knows at all times just how many finitely many hours remain from any given past hour to the present hour when the negative numbers run out.

21. Nevertheless, it might well be urged that a infinite temporal series, just as is the case with any real infinite, cannot be traversed—crossed, passed, or gone through one by one. But bear in mind that Craig himself has denied that the notion of a traversal entails a beginning point. For Craig, "[t]o traverse a series means just to cross it or pass through it one member at a time." I readily agree that any real infinite yet to be traversed cannot be traversed. Thus the metaphysical impossibility of a traversal of a real infinite pertains only to a traversal which begins with an initial moment or event from which to start the traversal. But an infinite temporal series is not such a series because at any time (i.e, the happening of an event or temporal interval of nonzero duration) it was never the case that there was an infinite temporal series yet to be traversed. So therefore that an infinite temporal series has been traversed does not entail that it was at anytime an infinite to be traversed from some starting point. Although all past events or moments intervals in any temporal series are each finitely distant from the present, this does not entail that every such set has a natural number as its cardinality. Moreover, the assertion that all infinitely past events or temporal intervals moments are each (that is individually considered) finitely distant from the present does not entail that all such events or temporal intervals (collectively considered) are finitely distant from the present since there is no event or temporal interval moment in such a series that is furthest away from the present. As far as I can see whatever other plausible grounds are offered as defeaters

for their position are ultimately based upon considerations provided by the standard version of the application of Cantorian set theory to the real world.

22. Given the failure of the KPCA, the metaphysical possibility of an infinite temporal series of events is more plausible than not given (that it is at least presumptively more probable than not) that every event was temporally preceded by another event to which it was causally related. Suppose that the temporal series of moments (pertaining only to natural entities) which constitutes or includes the history of this universe is denumerably infinite and of infinite duration. In that case this infinite temporal series, some naturalists (such as I) would maintain, is not not only metaphysically possible but necessary as well. On the other hand, it remains open for the theist to argue that the temporal series constituting the history of this universe, whether of finite or infinite duration, was causally due to supernatural agency without, however, relying upon the untenable ground that an infinite temporal series is metaphysically impossible.

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