CHAPTER 10: PREDICATE LOGIC

1 WHAT PROPOSITIONAL LOGIC CAN'T DO

There are many arguments whose validity cannot be shown using just the resources of propositional logic. Consider, for example, the following argument:

The Socrates Argument
1 All men are mortal.
2 Socrates is a man.
3 Therefore, Socrates is mortal.

Propositional logic cannot 'see' inside the statements that constitute this argument: from the perspective of Propositional Logic neither the premises nor the conclusion of this argument have any internal structure. The only structure that propositional logic can 'see' is the structure contributed by sentential connectives, so from the perspective of propositional logic the best we can do to translate the Socrates Argument is:

1 H
2 S
3 M

where 'H' translates 'All men (i.e. humans) are mortal', 'S' translates 'Socrates is a man' and 'M' translates 'Socrates is mortal'. Translated in this way, we have three sentences that appear to be completely unrelated.

Intuitively this argument is formally valid—but it is valid in virtue of features of its form that propositional logic cannot see! To show the validity of arguments like this we need a logic that can see inside sentences like the premises and conclusion of this argument—and that is predicate logic, which adds to the language of propositional logic new grammatical categories that allow us to display the structure of these sentences and so the explain the validity of the Socrates Argument.

THE LANGUAGE OF PREDICATE LOGIC

Predicate Logic Vocabulary

Connectives: ~ , ∨ , · , ⊃ , ≡

Individual Constants: lower case letters of the alphabet (a, b, c, … , w)

Predicates: upper case letters (A, B, C, … , Z)

Variables: x, y, z

Quantifiers: ∀ (universal quantifier)*, ∃ (existential quantifier)

*NOTE: sometimes the universal quantifier is just understood—so instead of writing '(∀x)' to say 'for all x' we just write '(x)'. This is the way it appears in Hurley.

The language of predicate logic retains all the vocabulary of propositional logic, including the connectives but adds predicates and individual constants, variables, which act as place-holders for individual constants, and quantifiers, which interact with variables.

Individual constants name individuals: persons, places or 'things'—including abstract objects like numbers, times and places.

Predicates ascribe properties and relations to individuals. One-place ('monadic') predicates assign properties to single individuals. Examples: being human, being mortal, being Greek. Two-place ('dyadic') predicates assign relations to pairs of individuals. Examples: being the teacher of, being to the north of. Three-place ('triadic') predicates assign relations to triples of individuals, and so on. According to our grammatical convention, in assigning properties or relations to individuals we put the predicate first and follow it by the name or names of the individuals to which it ascribes a property or relation. Hence:

Hs	Socrates is human.
Ms	Socrates is mortal.
Tsp	Socrates is the teacher of Plato.
Nas	Athens is to the north of Sparta.
Bcas	Corinth is between Athens and Sparta.

Statements like these, which assign properties or relations to named individuals are *singular statements*. We can translate compound singular statements by using the connectives, which carry over from propositional logic.

Gs · Ps	Socrates was a Greek philosopher. (Socrates was Greek and Socrates was a philosopher)
Hs ⊃ Ms	If Socrates is human then Socrates is mortal.

Predicates and individual constants provide the resources for translating the second premise and conclusion of the Socrates Argument in such a way that we can see their structure: each of these statements assigns a property to Socrates. So far, however, we do not have the resources to translate the first premise since it does not ascribe a property to any named individual—it says rather that every individual who is human is also mortal, viz. that if something has the property of being human then it has the property of being mortal.

Now you might think that we could get around this by considering 'men' as a singular term naming the collection of all humans. Singular terms, after all, can name gappy or diffuse things like flocks of sheep, suits and suites of living room furniture. But when we say that all men are mortal we are *not* saying that this is a property of humans collectively but rather that mortality is a property that every human being individually. We can certainly speak of humans collectively if we wish. We can say, for example, that humans have been around since before the last ice age. That is true of the human species but not of any individual human now alive. We can also say that the human species will one day go extinct—and so, to that extent, that the human species is mortal. But that is not what the first premise of the Socrates Argument says. It says that every human individual, including Socrates, will die. To make sense of it and other statements like it therefore we need a way of saying things about individuals generally without picking out any given individual or individuals. And that is where quantifiers and variables come into play.

Informally, you can think of the variables of predicate logic as pronouns. So, we might say:

Hx	He is human.
Nxs	It is north of Sparta.

Taken in isolation, these English sentences are incomplete. Without further information, without context, they don't tell us what is being talked about: *who* is human? *what* is north of Sparta? By themselves these English sentences have no truth value. And so it is with 'Hx' and 'Nxs'. Something in missing. To turn them into complete true-or-false sentences we add quantifiers that interact with the variables or, as we say, to *bind* them. In particular, we use the existential quantifier to say that some individual or other has a property or stands in a relation and the universal quantifier to say that every individual has a property or stands in a relation. So, for example, we use the existential quantifier in the following sentence to say that there is some individual or other that is human:

(∃x)Hx	There are humans.

We pronounce this sentence as 'there exists an x such that x is human'. It says that there is at least one individual that has the property of being human or, 'there are humans'—which is a little misleading since it doesn't commit us to saying that there is *more* than one human but just that there is at least one. So, in general, if we want to say that this is at least one individual of a certain kind, we use the existential quantifier. We can also use it, together with negation to say that there are no individuals of a certain kind. There are no unicorns, for example, and we can say that:

~ (∃x)Ux	There are no unicorns.

As you might expect, we pronounce this sentence, barbarically, as 'it is not the case that there exists an x such that x is a unicorn'. It says that there does not exist even a single individual that is a unicorn. That is to say, if you scour the universe, if you check out every single individual there is, there is not a one that is a unicorn.

Suppose however we want to say something about every individual. Then we use the universal quantifier. A materialist, for example, claims that everything is material, that is:

$(\forall x)Mx$ Everything is material

In our usual barbaric manner, we pronounce this as 'for all x, x is material'. Those of us who are not materialists, including people who believe in abstracta like numbers, in God, or in other things that are not material deny this. We say

$\sim (\forall x)Mx$ Not everything is material

EXERCISES

Translate the following sentences.

1 Socrates is mortal.

2 Socrates is mortal but Apollo is immortal.

3 If Socrates dies then his wife will be widowed.

4 There are philosophers.

5 There are no (pagan) gods.

6 Everything is dubitable (i.e. can be doubted).

7 Nothing is certain.

8 If there are numbers then not everything is material.

9 Either everything is material or there are some things that are not material.

10 There are neither nymphs nor satyrs.

2 CHANGE OF QUANTIFIER RULES: THE PUSH-THROUGH EQUIVALENCES

Now at this point you may notice that sentences using the universal quantifier can be translated into sentences using the existential quantifier and vice versa. So, to say that not everything is material is just to say that there's at least one thing that is not material:

$(\exists x) \sim Mx$ There is something that is not material

Or, 'there exists an x such that x is not material', that is, there is something immaterial. Saying that there's something that's not material is saying the same thing as saying that not everything is material, so $(\exists x) \sim Mx$ says the same thing as $\sim (\forall x)Mx$. By the same reasoning, saying that there isn't a single thing that has a certain property is the same as saying that for anything there is, it doesn't have that property: to say that no one is perfect

$$\sim (\exists x)\ Px$$

Is to say that everyone is imperfect, that is, that for anyone you consider, that individual is not perfect.

$$(\forall x) \sim Px$$

We can summarize this by the following equivalences:

$$(\exists x) \sim Fx :: \sim (\forall x)\ Fx$$

$$(\forall x) \sim Fx :: \sim (\exists x)\ Fx$$

The idea, from the mindless mechanical point of view, is that when you push the negation sign through a quantifier, it flips the quantifier from existential to universal and vice versa. And when you think of examples this makes sense: there's something that doesn't F if and only if not everything Fs and everything doesn't F if and only if there isn't a single thing that Fs. That is what the equivalences above say.

Exercises

Prove the following equivalences

1 $\sim (\exists x) \sim Fx \equiv (\forall x)Fx$

2 $\sim (\forall x) \sim Fx \equiv (\exists x)Fx$

3 $(\forall x)(Fx \supset \sim Gx) \equiv \sim (\exists x)(Fx \bullet Gx)$

4 $(\exists x)(Fx \bullet Gx) \equiv \sim (\forall x)(Fx \supset \sim Gx)$

5 $\sim (\exists x)(Fx \bullet \sim Gx) \equiv (\forall x)(Fx \supset Gx)$

Categorical propositions

So far we've considered translations of very simple statements asserting the existence and non-existence of things of various kinds. We can also use predicate logic to translate more complicated statements, for example, sentences of the four 'categorical proposition' forms identified by Aristotle. These are sentences of the following forms:

All S are P

No S are P

Some S are P

Some S are not P

'S' and 'P' are place-holders for terms that designate kinds or 'categories' of objects. Each of these statement forms describes a relation between such 'categories.' So, 'All men are mortal', the first premise of the Socrates, which is of the first form, says that everything of the human kind is of the mortal kind. That is to say: *if* anything is of the human kind then it is of the mortal kind. We therefore translate it and other sentences of that form as conditionals:

$(\forall x)(Hx \supset Mx)$	All humans are mortals.

Taken literally, we pronounce this as 'for all x, if x is human then x is mortal'. And that is just what the first premise of the argument says. We translate sentences of the other four forms accordingly. Here are some examples:

$(\forall x)\ (Hx \supset \sim Mx)$	No humans are Martians
$(\exists x)\ (Hx \bullet Mx)$	Some horses are mares.
$(\exists x)\ (Hx \bullet \sim Mx)$	Some horses are not mares.

So now, returning to the Socrates Argument, have the resources to explain its validity. Translating the premises and conclusion into the language of predicate logic we have:

The Socrates Argument

1 $(\forall x)(Hx \supset Mx)$	All men are mortal
2 Hs	Socrates is a man
3 Ms	Socrates is mortal

Now we can see why the premises of the Socrates Argument necessitate the conclusion. The first premise says that *if* anything belongs to the human kind it belongs to the mortal kind. The second premise says that there is indeed something that belongs to the human kind, viz. Socrates. Since Socrates belongs to the human kind, it follows therefore that, he belongs to the mortal kind. Case closed.

EXERCISES

1 All dogs are mammals.

2 No fish are warm-blooded.

3 Some mammals fly.

4 Some birds don't fly.

5 If 2 is prime then some primes are even.

3 SETS AND VENN DIAGRAMS: WHAT IT REALLY MEANS

At this point we stop to ask: what does it all *really* mean? Recall that in the discussion of propositional logic, after giving the conventional English translations of the connectives, we noted that what they *really* meant was given by their characteristic truth tables. By the same token, now that we've considered the conventional English translations of some sentences in predicate logic we can come clean about their real meaning: what truth tables are to propositional logic, sets and operations on sets are to predicate logic!

A set is a collection of distinct objects. The objects that make up a set (also known as the elements or members of a set) can be anything: numbers, people, letters of the alphabet, other sets, and so on. Sets are conventionally denoted with capital letters. Sets *A* and *B* are equal if and only if they have precisely the same elements. If a is a member of B, this is denoted $a \in B$, while if c is not a member of B then $c \notin B$. For example, with respect to the sets $A = \{1,2,3,4\}$, $B = \{\text{blue, white, red}\}$, and $F = \{n^2 - 4 : n \text{ is an integer; and } 0 \leq n \leq 19\}$ defined above,

$4 \in A$ and $12 \in F$; but

$9 \notin F$ and green $\notin B$.

If every member of set *A* is also a member of set *B*, then *A* is said to be a *subset* of *B*, written $A \subseteq B$ (also pronounced *A is contained in B*). Equivalently, we can write $B \supseteq A$, read as *B is a superset of A*, *B includes A*, or *B contains A*. The relationship between sets established by $\subseteq$ is called *inclusion* or *containment*. Warning: do not confuse membership with inclusion.

We understand singular sentences that ascribe properties to individuals in terms of set membership. We say that the *extension* of a singular term, that is, what it picks out or designates, is an individual, and that the extension of a predicate is a set. So to ascribe a property to an individual is to say that that individual belongs to the set designated by a predicate.

Examples:

Socrates is mortal	Socrates is a member of {mortals}
1 is odd	1 is a member of {1, 3, 5, …}

Things get a little more complicated when it comes to sentences that ascribe relations to individuals. However if we complicate the story a little further we can still understand such sentences in terms of set membership. According to this more complicated story 2-place predicates designate sets of ordered pairs, 3-place predicates sets of ordered triples and, in general, n-place predicates designate sets of ordered n-tuples. In ascribing an n-place relation to individuals, we say that an n-tuple is a member of a set of n-tuples. Ordered n-tuples are sets of a special kind. When it comes to ordinary sets, order doesn't matter: sets are identical just in case they have exactly the same members. So, e.g. {1, 2, 3} = {2, 3, 1} Order is, however, crucial for the identity of *ordered* sets. We distinguish ordered sets from the ordinary, unordered kind by using pointy-brackets instead of squiggly ones. So ordered triples which have the same members may not be identical: <1, 2, 3> ≠ <2, 3, 1>. In ascribing relations to individuals we are saying that some ordered n-tuples are members of sets of ordered n-tuples.

Examples:

Socrates is married to Xanthippe	<Socrates, Xanthippe> is a member of {individuals x, y, such that x is married to y}
2 > 1	<2,1> is a member of {<3,2>, <4,3>…}

BTW it is now time to notice that we can designate sets in different ways. We can refer to them by describing what their members are like or by enumerating their members. We can, e.g. pick out the set of odd numbers as {x: x is odd}—pronounced as 'the set of x such that x is odd} or as the set of 1, 3, 5, yada-yada-yada: {1, 3, 5, …}.

In the Venn Diagram convention circles represent sets. To say that a set is empty we shade out the circle; to say that it is non-empty, i.e. that it has at least one member, we put an X in it.

Horses **Unicorns**

There are horses. There are no unicorns.

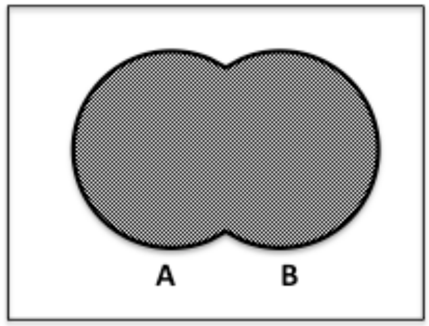
There are several fundamental operations on sets. The *union* of *A* and *B*, denoted by $A \cup B$, is the set of all things that are members of either *A* or *B*.

Examples:

{1, 2} ∪ {1, 2} = {1, 2}

{1, 2} ∪ {2, 3} = {1, 2, 3}

The shaded area represents the union of sets A and B:



A ∪ B: things that are either A or B

The *intersection* of *A* and *B*, denoted by *A* ∩ *B*, is the set of all things that are members of both *A* and *B*. If *A* ∩ *B* = ∅, then *A* and *B* are said to be *disjoint*. The **intersection** of *A* and *B*, denoted *A* ∩ *B*.

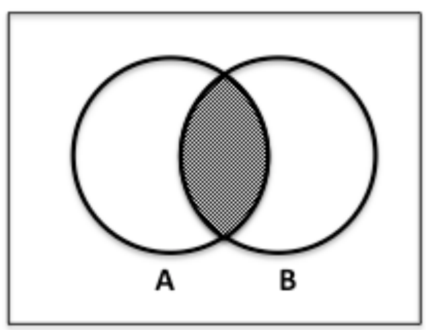
Examples:

{1, 2} ∩ {1, 2} = {1, 2}

{1, 2} ∩ {2, 3} = {2}

{1, 2} ∩ {3, 4} = ∅

The shaded area represents the intersection of sets A and B:



A ∩ B: things that are both A and B

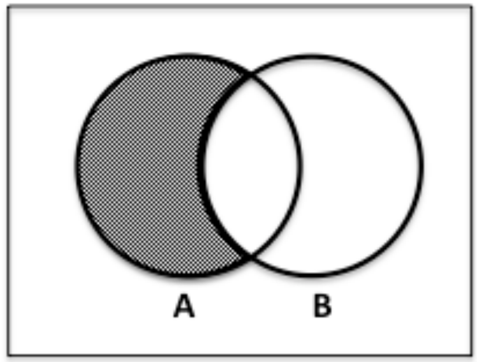
The *relative complement* of *B* in *A* (also called the *set-theoretic difference* of *A* and *B*), denoted by *A* \ *B* (or *A* − *B*), is the set of all elements that are members of *A* but not members of *B*.

Examples:

{1, 2} \ {1, 2} = ∅.

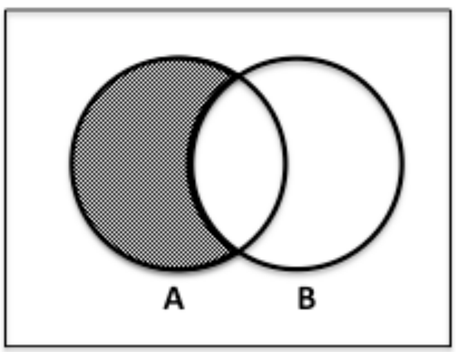
{1, 2, 3, 4} \ {1, 3} = {2, 4}

The shaded area represents the relative complement of B in A:

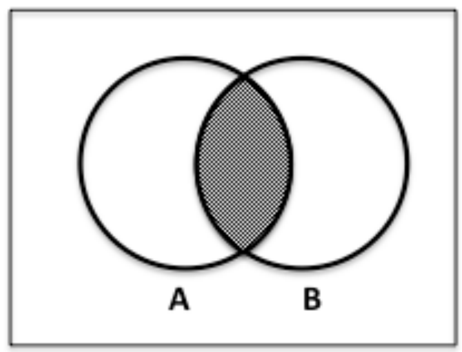


A \ B: things that are A but not B

Sentences of the four categorical proposition forms assert the emptiness or non-emptiness of sets. 'All A are B' says that the relative complement of B in A is empty and 'No A are B' says that the intersection of A and B is empty.

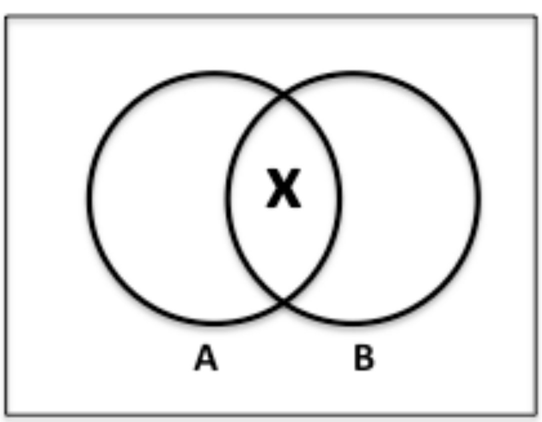


All A are B: (∀x)(Ax ⊃ Bx)

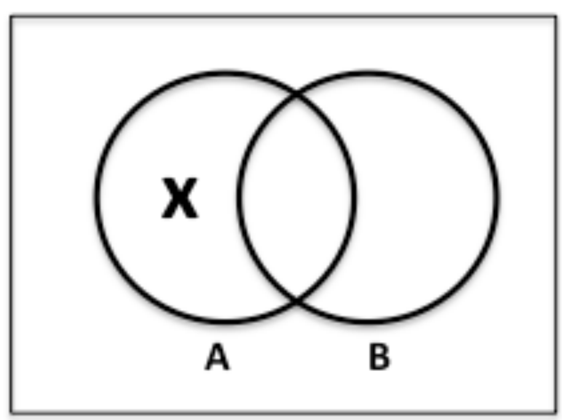


No A are B: (∀x)(Ax ⊃ ~ Bx)

'Some A are B' and 'Some A are not B' are translated using the existential quantifier so, as we might expect they assert the existence of objects, that is, the *non-emptiness* of sets.



Some A are B: (∃x)(Ax • Bx)



Some A are not B: (∃x)(Ax • ~ Bx)

Exercises

Consider the following sets and determine which of the sentences that follow are true and which are false:

A = {1, 2, 3}; B = {1, {2, 3}} ; C = {<1, 2>, <2, 3>}; D = {1, 2}; E = {2, 3}; F = {<2, 3>, <1, 2>}

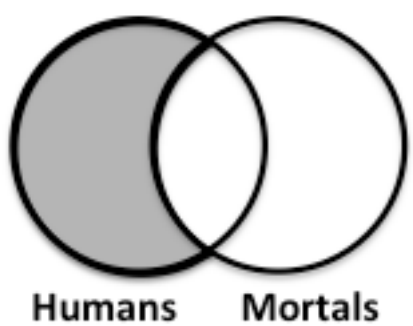
1	A = B	6	C = F
2	$2 \in A$	7	$D \in A$
3	$E \in A$	8	$D \subseteq A$
4	$E \in B$	9	$D \cup E$ = {1, 2, 3}
5	$E \subseteq A$	10	$D \cap E$ = {1}

4 Using logic diagrams to show validity

Now we can show the validity of arguments *visually* using logic diagrams.

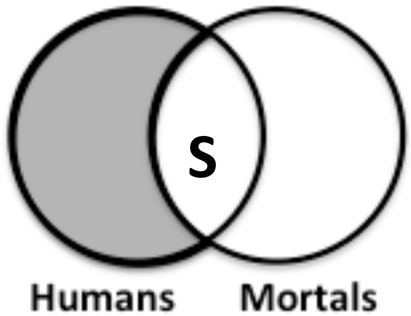
Remember how we understand validity: an argument is valid if it is not logically possible for the premises to be true and the conclusion false. Now we can represent statements ascribing properties to individuals and the emptiness or non-emptiness of sets using diagrams. Given our understanding of validity, therefore, an argument is valid if it is not logically possible to represent its premises without automatically representing its conclusion.

Here, for example, is 'All men are mortal', the first premise of the Socrates Argument:



It says that there are no immortal humans: the set of humans is included in the set of mortals, that is, there is no member of the set of humans outside of the set of mortals.

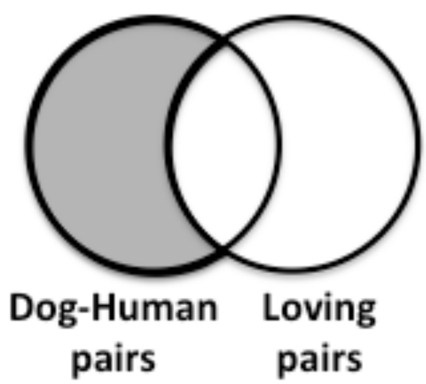
The second premises, 'Socrates of a man', says that Socrates is a member of the set of humans. To represent that, we put a name of Socrates, 's', inside the humans circle. But the first premise blocks out the part of that circle that's outside the mortal's circle: the only place for the 's' to go is in the part of the circle that's not shaded. This then is the second premise:



So far so good. In this argument we're ascribing a property to an individual: the circles represent sets of individuals. However now that we know about ordered pairs, triples, etc.—'ordered n-tuples'—we can do the same for arguments in which relations are ascribed to individuals. There is, for example:

The Dog Argument
1 All dogs love their humans.
2 Bo is Obama's dog.
3 Therefore, Bo loves Obama.

Here the sets we're talking about are sets of ordered pairs of individuals, viz. the set of pairs consisting of humans and their companion animals and the set of individuals such that the first loves the second. (And, to make life simpler, let's assume that the universe consists of dogs and humans). So the first premise says that all pairs of humans and their companion animals are loving pairs—more precisely, pairs such that the first member of the pair loves the second:



Exercises

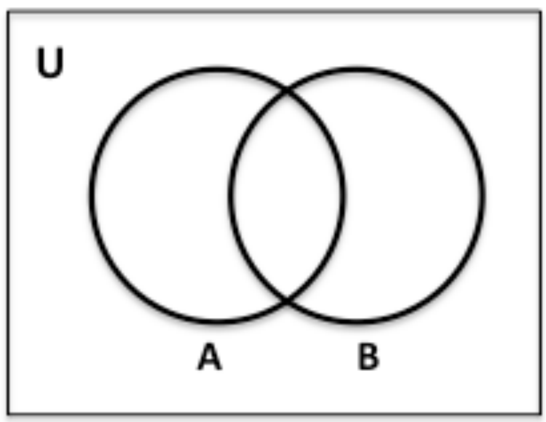
Show that the following arguments are valid using Venn diagrams

No philosophers are crooks. Nixon was a crook. Therefore Nixon was not a philosopher.

All married couples are entitled to file joint tax returns. Bill and Hillary are married. Therefore Bill and Hillary are entitled to file a joint tax return

There is an interesting analogy between the logical negation, conjunction and disjunction and the set-theoretical operations of complement, intersection and union respectively. Things that are <u>not</u> A are members of the complement of A in U (the universe); things that are A <u>and</u> B are members of A $\cap$ B; things that are A <u>or</u> B are members of A $\cup$ B. So De Morgan's Laws hold for set theory as well as logic!

State set-theoretical De Morgan's laws. And using the diagram below show why these equivalences hold.



5 Relational predicates and overlapping quantifiers

So far we've only considered sentences that involve, at most, one quantifier. However, as noted, in addition to one-place predicates, which ascribe properties to individuals, we also have predicates that assign relations to individuals in pairs, triples, and so on. Where *singular statements* in which such predicates figure, those that don't involve quantifiers, are no problem:

Socrates was the teacher of Plato	Tsp
Anthony loved Cleopatra	Lac
Thomas preferred Aristotle to Plato	Ptap

Even when just one quantifier is involved, translation is relatively unproblematic.

Plato had a teacher	(∃x)Txp	There exists an x, such that x was the teacher of Plato
Anthony loved someone	(∃x)Lax	There exists an x, such that Anthony loved x
Thomas preferred someone to Plato	(∃x)Ptxp	There exists an x, such that Thomas preferred x to Plato

Some such sentences involve the universal quantifier:

Aristotle knew everything	(∀x)Kax	For all x, Aristotle knew x
Everyone loved Cleopatra	(∀x)Lxc	For all x, x loved Cleopatra
Everyone prefers Russell to Hegel	(∀x)Pxrh	For all x, x prefers Russell to Hegel

Notice that in these sentences the order in which we string variables and constants after the predicate matters. There's a difference between loving and being loved—and sometimes loving is not mutual. There's a difference between saying that everyone loved Cleopatra and saying that:

Cleopatra loved everyone	(∀x)Lcx	For all x, Cleopatra loved x

So long as we remember to put the variables and constants in the right order everything is ok. When more than one quantifier is involved however things get stickier. There are two kinds of problems when more quantifiers are involved: problems of scope and overlapping quantifiers.

SCOPE

As it is with the connectives, so it is with quantifiers: we use parentheses to indicate scope. Recall that there was an important difference between the following sentences:

You can't both have your cake and eat it	~ (H • E)
You can't have your cake and you can't eat it	~ H • ~ E

In the first sentence the whole conjunction, H • E, was what was negated: the whole conjunction was in the scope of the negation sign. In the second, only H was within the scope of the first negation. And, as we know, tacking a negation on each of the sentence letters individually isn't the same as sticking it on to the whole conjunction.

The same thing goes for quantifiers:

Some numbers are even and prime	(∃x)(Ex • Px)

Some numbers are even and some are odd	(∃x)Ex • (∃x)Ox

Both these sentences are true. Assuming a universe of numbers, the first says there exists a number that is both even and prime. And there is: 2 has both of those properties. The second however does not say that there is a number that has both the property of being even and the property of being odd. It says, truly, that some numbers are even and some are odd, leaving it open whether there is any number that is both even and odd. And, of course, there is no number that has both of those properties so the following sentence is false:

Some numbers are both even and odd	(∃x)(Ex • Ox)

So long as we're dealing with one-place predicates and quantifiers of the same kind—all existential or all universal—scope issues are relatively easy to deal with. But when 'overlapping quantifiers' of different kinds are involved, translation gets hairy.

OVERLAPPING QUANTIFIERS

Remember that variables are like pronouns in that their meaning is incomplete. Suppose M is the predicate '__is material'. Mx just says 'it's material' and that doesn't tell you anything—or have truth value. *What*'s material? Something? Everything? Variables get their meaning by working in tandem with quantifiers, which 'bind' them:

Everything is material	(∀x)Mx
Some things are material	(∃x)Mx

Quantifiers bind only those variables that are within their scope. And when two or more quantifiers have the same scope we have to be careful about which quantifier binds which variable. The stock cases are:

Everybody loves somebody or other.	(x)(∃y)Lxy:	for all x, there exists a y such that x loves y
Everybody is loved by somebody or other.	(x)(∃y)Lyx:	for all x, there exists a y such that y loves x
There's somebody who loves everybody.	(∃x)(y)Lxy:	there exists an x such that for all y, x loves y
There's somebody who everybody loves.	(∃x)(y)Lxy:	there exists an x such that for all y, x loves y

Hurley provides further examples:

Any heavyweight can defeat any lightweight. $(\forall x)[Hx \supset (y)(Ly \supset Dxy)]$

Some heavyweights can defeat any lightweight. $(\exists x)[Hx \bullet (y)(Ly \supset Dxy)]$

No heavyweight can defeat every lightweight. $(\forall x)[Hx \supset (\exists y)(Ly \bullet \sim Dxy)]$
or $\sim(\exists x)[Hx \bullet (\forall y)(Ly \supset Dxy)]$

EXERCISES

Translate the following sentences

1 Every number has a successor.

2 There is no greatest prime.

3 Everybody loves somebody sometime. (three-place relation here!)

4 There are some experiences that are as good as it gets.

5 Whatever you've thought of, someone has thought of it first. (as in 3, some of the individuals involved are times)

6 IDENTITY AND DEFINITE DESCRIPTIONS

(Text from Hurley 8.7)

Many arguments in ordinary language involve a special relation called *identity,* and translating this relation requires special treatment. Consider, for example, the follow- ing argument:

The only friend I have is Elizabeth. Elizabeth is not Nancy. Nancy is a Canadian. Therefore, there is a Canadian who is not my friend.

The peculiar feature of this argument is that it involves special statements about indi- viduals. To translate such statements, we adopt a symbol from arithmetic, the equal sign (=), to represent the identity relation. We can use this symbol to translate a large variety of statements, including simple identity statements, existential assertions about individuals, statements involving "only," "the only," "no . . . except," and "all except," and statements involving superlatives, numerical claims, and definite descriptions. After seeing how the identity relation is used to translate such statements, we will see how natural deduction is used to derive the conclusions of arguments involving identity…

A *definite description* is a group of words of the form "the such-and-such" that identifies an indi- vidual person, place, or thing. Here are some examples:

The author of *Evangeline*

The capital of Nebraska

The mother of John F. Kennedy

The first designates Henry Wadsworth Longfellow, the second the city of Lincoln, and the third Rose Fitzgerald Kennedy. Definite descriptions are like names in that they identify only one thing, but unlike names they do so by describing a situation or relationship that only that one thing satisfies.

Statements incorporating definite descriptions have given rise to disputes in logic, because alternate interpretations of such statements can lead to conflicts in truth value...[M]ost logicians today accept an interpretation of definite descriptions originally proposed by Bertrand Russell. According to this interpretation, a statement that incorporates a definite description asserts three things: an item of a certain sort exists, there is only one such item, and that item has the attribute assigned to it by the statement. If we accept this interpretation, the statement about the queen of the United States is false, because no such person exists.

EXERCISES

Translate the following sentences.

1 The cat is on the mat.

2 There is just one even prime.

3 Everything is identical to itself.

4 Ponce de Leon searched for the fountain of youth.

5 There are two quantifiers.

EPILOGUE: A PHILOSOPHICAL PROBLEM

The deepest and perhaps most intractable problems in philosophy are problems of *ontology*: questions about what there is—about what, in the most fundamental sense, exists. We touched briefly on some of these questions. Does God exist? Maybe. Does the 'external world' exist? I hope so. What about possible worlds, Platonic universals, or numbers? What about unicorns and other mythological beasts, or fictional characters like Sherlock Holmes?

Russell worried about definite descriptions because they pose an ontological problem. They *seem* like names. We can refer to a certain American author as 'Samuel Clemens' or by his pen name, 'Mark Twain'. Alternatively we can refer to him by the definite description, 'the author of *Huckleberry Finn*'. On the surface it seems like 'the author of *Huckleberry Finn*', like 'Mark Twain' is just another name for that author and that the following sentences are of the same form—each a singular statement ascribing a property to an individual:

Mark Twain was born in Hannibal, Missouri

The author of *Huckleberry Finn* was born in Hannibal, Missouri

However treating definite descriptions as names causes trouble. Intuitively, the meaning of a whole sentence is determined by the meaning of its parts: if you have a dictionary to look up the individual words, and a grammar of the language to explain how they function within the sentence, you can figure out what the sentence means. But proper names have no dictionary entries so there's a sense in which, strangely, they contribute to the meaning of sentences in which they occur without themselves having any meaning. This is what led Russell to suggest that *the individuals themselves* 'enter into' the meanings of singular sentences. On this account, Mark Twain himself contributes to the meaning of the sentence 'Mark Twain was born in Hannibal, Missouri'.

It may be tempting to suggest that he contributes to the meaning of the second sentence in the same way. After all, 'the author of *Huckleberry Finn'* picks him out. However, we can't adopt this account for all sentences whose grammatical subjects are definite descriptions because in some cases there *are* no individuals picked out by their grammatical subjects. Russell's famous example was:

The present King of France is bald.

Writing at a time when France was a republic, there was no such king and yet, Russell noted, the sentence was perfectly meaningful.

Other philosophers had the same worry. Sentences of this grammatical form, it seemed, ascribed properties to individuals. *If* that's what they really did then there must exist individuals to which they ascribed those properties—perhaps merely possible individuals, or even impossible individuals—to answer to definite descriptions like 'the present King of France' or 'the bachelor's wife'. But do we really want to say that things like this exist—especially if they just seem conjured up to explain how sentences like 'the present King of France is bald' could be meaningful?

Russell's response was that such sentences didn't really ascribe properties to individuals—that their surface grammar was deceptive. Definite descriptions were not to be understood as names and sentences in which they occurred as surface-grammatical subjects were not singular statements ascribing properties to individuals: they were really existentially quantified sentences that didn't involve any names at all. So 'the present King of France is bald' was to be understood as

$$(\exists x)((Fx \bullet (\forall y)Fy \supset y=x) \bullet Bx)$$

Semi-colloquially: 'There is an x who's King of France and for any y, if that y is King of France then he is none other but x himself and x (a.k.a. y) is bald.

We use the language of identity here to say that there is just one King of France since 'the' implies uniqueness. The first conjunct just says that there's *at least* one King of France, leaving open the possibility that there are more. The second however excludes that possibility: it says that there is *at most* one King of France. The two together say that there is exactly one King of France.

This account of definite descriptions comes from Russell's 1905 paper 'On Denoting', which was perhaps the most significant and influential philosophical essay of the 20th century though by no means uncontroversial. And the discussion continues.

ANSWERS TO EXERCISES

EASY TRANSLATIONS

1 Socrates is mortal.
Ms

2 Socrates is mortal but Apollo is immortal.
Ms • ~ Ma

3 If Socrates dies then his wife will be widowed.
Ds ⊃ Ww

4 There are philosophers.
(∃x)Px

5 There are no (pagan) gods.
~ (∃x)Gx

6 Everything is dubitable (i.e. can be doubted).
(∀x)Dx

7 Nothing is certain.
~ (∃x)Cx

8 If there are numbers then not everything is material.
(∃x)Nx ⊃ ~ (∀x)Mx

9 Either everything is material or there are some things that are not material.
(∀x)Mx ∨ (∃x) ~ Mx

10 There are neither nymphs nor satyrs.
~ ((∃x)Nx ∨ (∃x)Sx)

PROVING EQUIVALENCES

Prove the following equivalences

1 ~ (∃x) ~ Fx ≡ (∀x)Fx

2 ~ (∀x) ~ Fx ≡ (∃x)Fx

3 (∀x)(Fx ⊃ ~ Gx) ≡ ~ (∃x)(Fx • Gx)

4 $(\exists x)(Fx \bullet Gx) \equiv \sim (\forall x)(Fx \supset \sim Gx)$

5 $\sim (\exists x)(Fx \bullet \sim Gx) \equiv (\forall x)(Fx \supset Gx)$

TRANSLATING 'CATEGORICAL PROPOSITIONS'

1 All dogs are mammals.
$(\forall x)(Dx \supset Mx)$

2 No fish are warm-blooded.
$(\forall x)(Fx \supset \sim Wx)$

3 Some mammals fly.
$(\exists x)(Mx \bullet Fx)$

4 Some birds don't fly.
$(\exists x)(Bx \bullet \sim Fx)$

5 If 2 is prime then some primes are even.
$Pt \supset (\exists x)(Px \bullet Ex)$

OVERLAPPING QUANTIFIERS

1 Every number has a successor.
$(\forall x)(\exists y)Sxy$

2 There is no greatest prime.
$(\forall x)(Px \supset (\exists y)(Py \bullet Gyx)$
(For any prime number, x, there's a prime, y, that's greater then x)

3 Everybody loves somebody sometime. (three-place relation here!)
$(\forall x)(\exists y)(\exists t)Lxyt$
That is, for everybody, every single x one of them, there exists somebody, y, that they love at some time, t. But, don't worry, I'm not going to put something this complicated on the exam.

4 There are some experiences that are as good as it gets.
$(\exists x)(Ex \bullet \sim (\exists y)(Ey \bullet Byx)$
There's an x that's an experience, and there isn't any experience, y, that's better than x

5 Whatever you've thought of, someone has thought of it first. (as in 3, some of the individuals involved are times)
$(\forall x)((\exists y)(\exists t)(Tyxt \supset (\exists z)(\exists t')(Tzxt' \bullet Et't)$

DEFINITE DESCRIPTIONS

(see the last section of my Hurley re-write and the epilogue. Definite descriptions, expressions of the form 'the so-and-so', e.g. 'the cat on the mat' and 'the fountain of youth' express uniqueness, and to express uniqueness, or number, we need '='—the identity predicate.

1 The cat is on the mat.
$(\exists x)(Cx \bullet (\exists y)(Cy \supset y = x \bullet Mx$
There's a thing, x, which is a cat and, if there's a y that's a cat then she's none other but x herself! (That is, we're talking about exactly one cat). And that cat, x, is on the mat.

2 There is just one even prime.
$(\exists x)((Ex \bullet Px) \bullet (\forall y)((Ex \bullet Px) \supset y = x$
There's this number, x, which is even and prime, and for any number, y, that's even and prime it's none other than x itself!

3 Everything is identical to itself.
$(\forall x)x = x$
For anything, x, it'

4 Ponce de Leon searched for the fountain of youth.
$(\exists x)(Fx \bullet (\forall y)(Fy \supset y = x) \bullet Spx$
There is this thing, x, that's a fountain of youth. And there's no other fountain of youth. And Ponce de Leon searched for it. (There's a philosophical problem here. We want to say that this sentence is true—even though one of the conjuncts, viz. '$(\exists x)Fx$', is false. But let's not worry about that for now.

5 There are two quantifiers.
$(\exists x)(\exists y)(Qx \bullet Qy \bullet x \neq y \bullet (\forall z)[Qz \supset z = x \vee z = y))$
There exist two different things, x and y,, both of which are quantifiers. And they're not identical, each of which is a quantifier (i.e. there are at least 2 quantifiers). And for any z, if it's a quantifier then it's either x or it's y.