CHAPTER 6: KNOWLEDGE AND TRUTH

In doing formal logic we will make some idealizing assumptions about *truth value* (that is, truth-or-falsity) that seem at best naïve. We assume, for example, that there are just two truth values, viz. true and false: nothing else and nothing in between. That means we treat all propositions as either true or false, and that's it—even though this black-and-white picture doesn't seem to capture the shades of gray that figure in the real world.

In this chapter we will argue that this and other assumptions we make about truth value for the sake of doing classical logic aren't as dumb as they seem, providing we are careful to distinguish truth value from other notions with which is sometimes confused—like belief, justification and knowledge. In the course of our discussion we will touch on some philosophical issues and discuss some logical maneuvers. These include the 'justified true belief account of knowledge that comes to us from Plato, the use of counterexamples and mathematical induction in arguments and proofs, and the dread Sorites Paradox.

6 FACT AND OPINION

'When debating ethics and other controversial topics, one frequently hears the claim "That's just your opinion." 'It is a pernicious claim', writes John Corvino, 'devoid of clear meaning, and it should be consigned to the flames – or so I shall argue here'.[1]

6.1 WHAT'S THE DIFFERENCE BETWEEN FACT AND OPINION?

[W]hen we ask, 'What is the difference between facts and opinions?' what we're really asking is 'What is the difference between *statements of fact* and *statements of opinion*?' This seems like it should be an easy question, but it actually tends to stump most people on the street. Mind you, they have no trouble in offering examples of either, or in categorizing others' examples. So for instance, given

(1a) There is beer in my refrigerator.	(1b) Wine tastes better than beer.
(2a) The earth revolves around the sun.	(2b) The earth was created by an omnipotent God.
(3a) Thousands were killed in Darfur.	(3b) Genocide is wrong.
(4a) The current US president is a Democrat.	(4b) A Democrat will win the presidency in 2016.

[1] 'The Fact/Opinion Distinction'. John Corvino. *The Philosophers Magazine*. March 4, 2015
The following is excerpted from Corvino's article. For the complete text click the link.

they'll say that the A statements are facts and the B statements are opinions. When asked to explain the *principle of distinction* between the two, however – the rule that tells us how to assign statements to one category or the other – they often get tongue-tied…

If everyday observers are confused about the distinction, 'experts' fare little better. Curious as to the standard explanation, I googled 'facts vs. opinions'. (This may not be the best way to conduct serious philosophical research, but it can be a useful way of gauging common thoughts on a subject.) Here's the first result I received, from a 'Critical Thinking Across the Curriculum Project' website:

6.1.1 Direct observability

> 'Fact: statement of actuality or occurrence. A fact is based on direct evidence, actual experience, or observation.
>
> 'Opinion: statement of belief or feeling. It shows one's feelings about a subject. Solid opinions, while based on facts, are someone's views on a subject and not facts themselves.'

This way of drawing the distinction makes 'The earth revolves around the sun an opinion – or at least, not a fact – since no one directly observes it happening (not even astronauts!). It also jumbles together occurrences (what we earlier called 'states of affairs'), statements about occurrences, and the evidence for those statements. Perhaps more confusing is its labeling opinions as 'statement(s) of belief'…[*A*]*ll* statements express beliefs: our task is to determine which of them express factual beliefs and which express opinions.

6.1.2 Provability

So I looked further. Here are the second and third results from my quick internet search, from an 'Education Oasis' and 'Enchanted Learning' website, respectively:

> 'A fact is a statement that can be proven true.
>
> 'An opinion expresses someone's belief, feeling, view, idea, or judgment about something or someone.'

and

> 'Facts are statements that can be shown to be true or can be proved, or something that really happened. You can look up facts in an encyclopedia or other reference, or see them for yourself. For example, it is a fact that broccoli is good for you (you can look this up in books about healthy diets).'
>
> 'Opinions express how a person feels about something – opinions do not have to be based upon logical reasoning. For example, it is an opinion that broccoli tastes good (or bad).'

Both of these connect fact with provability. But in common parlance, 'provability' seems audience-relative as well: While one person might find Anselm's ontological argument to be a sufficient proof for God's existence (thus rendering 'God exists' a fact *for that person*); others may not.

The Education Oasis site announces that 'An opinion expresses someone's belief ... about something'. So if I believe that there's beer in my refrigerator, is that just an opinion? The Enchanted Learning site muddies the waters even further by claiming that you can look up facts in an encyclopaedia (always? but then were there no facts before books?), and by including an evaluative notion ('good for you') among examples of facts.

If this is 'Critical Thinking', I'd hate to see what Sloppy Thinking looks like.

6.2 The Real Distinctions

Let me offer a conjecture: the fact/opinion distinction is ambiguous, and in trying to explain it, people typically conflate it with other distinctions in the neighborhood. Let's consider three of those other distinctions.

6.2.1 Belief/Reality

Take, first, the familiar philosophical distinction between *belief and reality*. In common understanding, there's a world (reality), and then there are our representations of that world (beliefs: sometimes true, sometimes not). I might believe that there's beer in the refrigerator, whether or not there's any there. I might believe that God created the earth, whether or not God did – indeed, whether or not there is a God. Generally, we strive to make our beliefs as accurate as possible in representing reality, but that doesn't remove the gap (some would say 'gulf') between the two.

The problem, obviously, is that attempts to bridge that gap always proceed via our own fallible cognitive capacities. Beliefs about reality are still beliefs, and some of them, despite our best efforts, turn out to be false. That's true whether we're talking about beliefs that usually show up in the 'fact' column ('There's beer in the refrigerator') or in the 'opinion column ('God created the earth')…

6.2.2 Subjective/Objective

Second, consider the *subjective/objective* distinction. Something is subjective insofar as it is mind-dependent, objective insofar as it is mind-independent. Given this definition, all beliefs (qua beliefs) are subjective, because beliefs depend on minds. And since we've been treating both facts and opinions as *statements* of belief, facts and opinions are similarly 'subjective': In other words, we can always ask 'Whose belief?' or 'Whose statement?'

Of course, there are different kinds of beliefs and statements. Some are about objective matters, such as whether there is beer in the refrigerator. Others are about subjective matters,

such as whether one would enjoy a Guinness more than a Corona. Perhaps the fact/opinion distinction tracks the distinction between statements with objective content (facts?) and those with subjective content (opinions?). But if so, we would need to revise what usually gets put in each column. In particular, the statement that 'God created the earth' will need to move over to the 'fact' column, since whether God created the earth is an objective matter – whether that happened or not is independent of whether we believe it happened. The same is true for 'God exists' – not an opinion, on this schema, but a factual claim (maybe true, maybe false).

It is also by no means obvious that 'Genocide is wrong' should remain in the 'opinion' column. While some philosophers hold that moral beliefs are subjective, many do not. Moreover, there is a strong commonsense intuition that genocide would be wrong whether or not anyone believes that it is wrong, suggesting that the claim is objective, not subjective. So while the subjective/objective distinction might be useful in explaining the fact/opinion distinction, adopting this approach would require us to revise our common thinking about facts and opinions. That is not necessarily a bad thing, since – as we have seen – our common thinking about facts and opinions appears rather confused.

6.2.3 Descriptive/Normative

Finally, consider the *descriptive/normative* distinction. Descriptive statements describe or represent the world; normative statements evaluate it. For example: the statement that thousands were killed in Darfur is descriptive; the statement that such killing was wrong is normative.

The descriptive/normative distinction is sometimes called the fact/value distinction, which might lead it to be confused with the fact/opinion distinction. But it's controversial whether all normative claims are matters of opinion. Moreover, many of the standard 'opinion' examples are not normative: consider 'God exists' or 'A Democrat will win the presidency in 2016'. If the fact/opinion distinction were identical to the fact/value distinction, then once again we would need to revise our common thinking about facts and opinions.

6.3 Statements of Fact and Statements of Opinion

Having teased apart these various distinctions, and looking back over the several attempts to explain the difference between fact and opinion, we might propose the following definitions:

- A statement of fact is one that has objective content and is well-supported by the available evidence.

- A statement of opinion is one whose content is either subjective or else not well supported by the available evidence.

These definitions have several advantages. First, they capture some of the concerns that lead people to insist on the fact/opinion distinction in the first place – in particular, the concern

that claims not be accepted without good evidence. Second, they explain why some objective matters – in particular, controversial matters such God's existence or predictions about the future – get placed in the category of opinion, despite their objective content. And third, they avoid the sloppiness of some of the earlier proposals. That said, they are still somewhat revisionist: They do not fully capture everyday usage (since everyday usage is messy and confused), but instead serve to refine that usage.

Why worry about the fact/opinion distinction? One reason is that precise thinking is valuable for its own sake. But there's another, more pragmatic reason. Despite its unclear *meaning*, the claim 'That's just your opinion' has a clear *use*: It is a conversation-stopper. It's a way of diminishing a claim, reducing it to a mere matter of taste, which lies beyond dispute. (*De gustibus non est disputandum*: there's no disputing taste.)

Indeed, the 'opinion' label is used not only to belittle others' stances, but also to deflate one's own. In recognizing that a personal belief differs sharply from that of other individuals and cultures, one may conclude, 'I guess that's just *my* opinion – no better than anyone else's.' This conclusion may stem from an admirable humility. On the other hand, it can have pernicious effects: it leads to a kind of wishy-washiness, wherein one refrains from standing up for one's convictions for fear of imposing 'mere opinions'. Such reticence conflicts with common sense: surely some opinions are more thoughtful, more informed, more coherent, and more *important* than others.

This diminishment is especially troubling in moral debates. Moral debates are *practical* – they're debates about what to do – and they concern our *values*: things that matter to us. Either we send troops to Syria or we don't. Either we allow same-sex couples to marry or we don't. Either we lie to our parents about what happened to the car or we don't. Categorizing these issues as 'matters of opinion' doesn't make them any less urgent or vital.

I therefore propose that we abandon the ambiguous fact/opinion distinction, and especially the dismissive retort 'That's just your opinion'. We should focus instead on whether people can offer *good reasons* for the claims they make – reasons that might compel us to share their views. That's my opinion, anyway. If you think yours is better, don't merely say so: Say *why*.

John Corvino is Chair of the Philosophy Department at Wayne State University

6.4 The Fact/Opinion Distinction as a Noble Lie

A 'noble lie' is a myth or untruth (often but not always of a religious nature) told to maintain social harmony or to advance an agenda. So Seneca argued that religious belief was a noble lie—a fraud perpetrated on the public to sustain the social order: 'Religion', he said, 'is regarded by the common people as true, by the wise as false, and by magistrates as useful'. The distinction between fact and opinion, now part of the 'common core' curriculum in American schools, is a noble lie promulgated by educators to teach a number of important truths and to

promote good behavior. Their aim is to encourage critical thinking and tolerance. Though fact/opinion distinction is a lie, it is a noble one: it was invented in order to promote of a range of good principles and practices that we will argue are supported on other, more cogent grounds, including the following:

6.4.1 Don't be so sure.

Reflect on your own beliefs and commitments—subject them to scrutiny. You may be wrong. Think critically and don't be dogmatic: be open to new information and to arguments that may undermine your most firmly held convictions. There are a great many claims that are uncontroversial: 2+2=4, the earth goes around the sun rather than vice versa, Baltimore is south of Philadelphia, there is an Apple Store in Fashion Valley. These claims are easily established and uncontroversial. Other claims—typically the more interesting ones—are controversial because they cannot be easily established. Who will be the next US president? What is the most effective way to promote economic growth? Does God exist? What makes an action right or wrong? These are hard questions: the answers to them are controversial, and smart, educated people who've done all their homework disagree. We don't know now who the next president will be: it is, to that extent, 'a matter of opinion'. But we'll soon find out! We don't know whether there is a God or not and cannot find out in this life. But the proposition 'God exists' is either true or false: we just don't know which (yet?) and shouldn't jump to conclusions!

6.4.2 Don't mistake your feelings for facts or values.

Feeling strongly about something doesn't mean that it's true. Feeling disgusted by a practice doesn't mean it's morally wrong. Finding a thesis counterintuitive doesn't mean it's false. Don't go with your gut!

6.4.3 Don't assume that people who disagree with you are ignorant, intellectually lazy, naïve or dishonest.

There are a great many issues about intelligent, informed people disagree. These include a ethical principles, theological doctrines and political policies, and predictions about the course of world events, the weather and the behavior of the stock market. These issues pose hard questions. When people get the answers to *easy* questions wrong it reflects adversely on them. But when it comes to the hard questions getting wrong answers is no shame. Smart, educated, reflective people, who've made every effort to arrive at the truth, may get it wrong when it comes to these matters.

Holding false beliefs does not always reflect adversely on a person. And this, if anything, is the theme of the current chapter: '*It's alright to be wrong!*' A person may by *justified* in holding a belief that is in fact false. To make this out we need to distinguish between truth, belief and justification.

7 TRUTH

In classical logic, there are just two truth values: true and false. A proposition is true if it corresponds to reality; otherwise it is false. We make, and will defend, the following controversial claims about truth value:

(1) Every proposition is either true or false;

(2) Truth (and falsity) do not admit of degree.

(3) Truth (and falsity) are 'absolute' in the sense that they are not relative to persons, times, places, cultures or circumstances.

How do we know that this is the way things are? We don't: we *stipulate* that this is the way in which we will understand truth value for the purposes of doing logic!

This account represents a simplification or *idealization* of the way in which notions like truth and falsity figure in our ordinary ways of thinking, however we shall argue that, for most purposes, it does not represent a significant loss if we are careful to distinguish *semantic* notions, concepts which concern the way in which language connects to the world like truth, falsity and meaning, from *epistemic* notions, like belief and justification, concepts which pertain to people's knowledge about the world.

7.1 IDEALIZATION

Idealization is the process by which scientific models assume facts about the phenomenon being modeled that are not strictly true. Often these assumptions are used to make models easier to understand or solve.

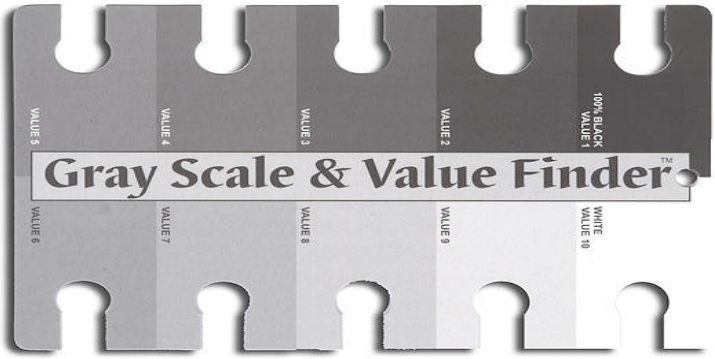
> [An] example of the use of idealization in physics is in Boyle's Gas Law: *Given any x and any y, if all the molecules in y are perfectly elastic and spherical, possess equal masses and volumes, have negligible size, and exert no forces on one another except during collisions, then if x is a gas and y is a given mass of x which is trapped in a vessel of variable size and the temperature of y is kept constant, then any decrease of the volume of y increases the pressure of y proportionally, and vice versa.*

In physics, people will often solve for Newtonian systems without friction. While, we know that friction is present in actual systems, solving the model without friction can provide insights to the behavior of actual systems where the force of friction is negligible. Another discipline, geometry, arises by the process of idealization because it, at its core, is a universe of ideal entities, forms and figures. Perfect circles, spheres, straight lines and angles are the essential elements of this discipline, all which would be near impossible without idealization…Similarly, in economic models individuals are assumed to be maximally rational choosers with perfect

information. This assumption, although known to be violated by actual humans, can often lead to insights about the behavior of human populations.[2]

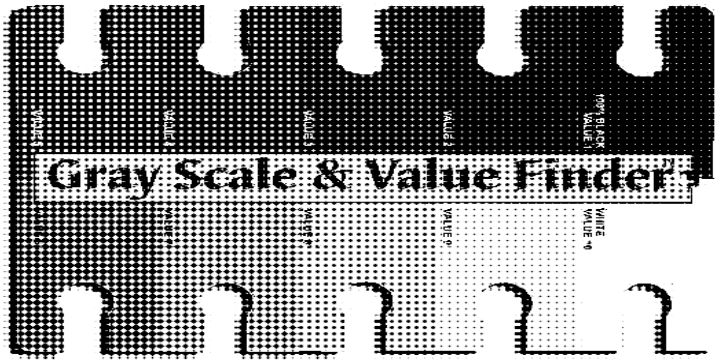
Maps of idealize—they selectively include and omit details in order to serve the purposes of users. Some maps, like those that mark public transportation routes distort in order to provide the information riders need in the most convenient form. If a map accurately represented the territory in every detail it would be useless!

For the purposes of doing formal logic we will idealize truth value. We stipulate that there are just two truth values, true and false, which do not admit of degree. What does this mean? Many properties 'admit of degree' in the sense that an object can have the property in question to a greater or lesser degree. Tallness and shortness admit of degree: someone who is 6 feet tall is tall; someone who is 6 foot 4 is taller; someone who is 7 feet tall has the tallness property to an even higher degree. Other properties, like the oddness and evenness of numbers do not admit of degree. A number is either even or odd. No number is more or less odd, or more or less even than any other. That's the way we understand truth value: when it comes to truth and falsity there are no shades of gray.



But surely you say this is terrible! How can a logic like this make sense of a world where just about everything is in shades of gray. Well, we digitize, so to speak…

[2] http://en.wikipedia.org/wiki/Idealization Links above are to Wikipedia.



Using just black and white dots arranged in different ways at different densities we can display black, white, and every visible shade of gray in between. Digitizing sharpens the picture and allows us to display it using fewer, simpler resources. Instead of a zillion different shades of paint we just need black ink and white paper.

This is in effect what we're doing when we opt for a *bivalent* (that is, 'two-valued') logic. Could we have made a different decision? Yes. There are a variety of logics that have been developed differently, e.g. logics with three truth values and logics with continuum many truth values between 0 (absolutely false) and 1 (absolutely true). But these logics are *hard*. You don't want to deal with them—believe me! And, as we'll argue, you can get everything you want for most practical purposes from good old bivalent classical logic. So that's what we're going to do around here.

However, this being logic, we don't dogmatize. We argue. So in this chapter I will at least sketch responses to some of the worries people have about the claims we make about truth value in logic, including the shades-of-gray problem.

8 COUNTEREXAMPLE

Some worries come in the form of apparent *counterexamples*. Given the claim that truth value doesn't admit of degree people come up with cases where they think a proposition is 'partly true or 'not entirely' false. If there really were such cases they would be counterexamples to the claim that truth value does not admit of degree.

In general, a *counterexample* is an exception to a proposed general rule or proposition—a case that shows a general claim to be false. Suppose you say that no women are good scientists and I say, 'Um, well there was Madam Curie who won two Nobel Prizes'. I am offering a counterexample to your claim. Suppose you say, 'All Polish people are dumb' and I say, 'Um, 'scuse me: what about Copernicus? And Pope John Paul II? And Chopin? (and also Madam Curie)'. These are counterexamples to your claim, which shows it to be false. Counterexamples are great! To show that a general claim is false all you have to do is produce one

counterexample! This means that refuting such claims is easier than proving them! Suppose someone says, 'All primes are odd'. Ha! No problem with this one: 2 is prime but isn't odd. This is an easy win.

Nevertheless, not everything that looks like a counterexample really is one. We need to be careful. Suppose you want to produce a counterexample to the general claim that all monkeys have tails. You note that chimpanzees don't have tails thinking you've got a counterexample. But you don't because chimps aren't monkeys—they're apes!

Now we're interested in producing *putative* counterexamples to our claims about truth value—claims that *seem* to be counterexamples—and responding to them. Are there any counterexamples to the claim that truth value is not a matter of degree? What are they? Are they really counterexamples? That is what we are going to do here. We have an idealized account of truth value which, we claim, is close enough to our ordinary understanding of things to be serviceable. Is this account defensible? Let's see…

9 BIVALENCE: APPARENT COUNTEREXAMPLES AND RESPONSES

9.1 CONJUNCTIONS

Lots of times you hear people say, 'There's some truth in all the Great World Religions' or 'what you're saying isn't *entirely* false'. What is going on here? I think in most cases like this we're dealing with conjunctions.

Suppose I try to unload my used car, with the following ad on Craigslist:

For Sale: 1996, 4-door Nissan Sentra. New clutch, low mileage.

Well that is 'partly true' in the sense that *some* of the claims I'm making in this ad are true. Yes, the car is for sale. Yes, it is a 4-door Nissan Sentra. Yes, it has a new clutch—and I can produce the papers from Becka Motors to prove it. But it has 209,173 miles on the clock and that isn't, by anyone's standard, low mileage.

I made a bunch of claims, some of which were true and so to that extent we can say, colloquially, that what I said was partly true. But when we consider each claim individually, each is either true or false—not partly true or partly false or anything in between. So we can in effect digitize the picture. The whole ad is, so to speak, gray, but when we consider each claim individually it's either black or white—either true or false.

At this point you may object that there could be cases where *the individual claims are themselves* neither black nor white. Suppose my car had had 50,000 miles on it. Is that low mileage? By my standards it is, but are those community standards? What counts as 'low mileage'? You might even get fussy about how recently a clutch has to have been installed to

count as 'new'. This brings us to a more troubling range of apparent counterexamples to bivalence: those that involve *vagueness*.

9.2 VAGUENESS

When it comes to vagueness there are the easy cases and the hard case. Here is an easy one:

Stealing is wrong.

The problem with this sentence is that it does not express a complete thought. Is the claim that stealing is *always* wrong? Is it that stealing is *sometimes* wrong—and, if so, under what conditions? In logic we cannot tolerate vagueness so we consider sentences like this junk and refuse to deal with them as they stand. We ask for clarification: '*Always* wrong? *Sometimes* wrong? If so, when? What exactly do you mean?'

Unfortunately sometimes what we mean is itself inherently vague. There are many properties that *admit of degree*—like being hot, being tall and being bald. These cause trouble because when we assign these properties to objects what we mean is something vague so pressing for further clarification is futile. You say this guy is bald. But what do you mean? That he has no hair whatsoever on his head? No. That he has fewer than 531 hairs on his head? Fewer than 1013 hairs? No. No. You don't mean anything that definite: what you mean is irremediably vague. 'Bald' is inherently vague. And it is the presence of inherently vague words like this in natural languages that beget the dread *Sorites Paradox*.

9.3 THE SORITES PARADOX[3]: THE HARD CASE (STATED BUT NOT SOLVED!)

The 'sorites paradox' is the name given to a class of paradoxical arguments, also known as little-by-little arguments, which arise as a result of the indeterminacy surrounding limits of application of the predicates involved. For example, the concept of a heap appears to lack sharp boundaries and, as a consequence of the subsequent indeterminacy surrounding the extension of the predicate 'is a heap', no one grain of wheat can be identified as making the difference between being a heap and not being a heap. Given then that one grain of wheat does not make a heap, it would seem to follow that two do not, thus three do not, and so on. In the end it would appear that no amount of wheat can make a heap. We are faced with paradox since from apparently true premises by seemingly uncontroversial reasoning we arrive at an apparently false conclusion…

The name 'sorites' derives from the Greek word *soros* (meaning 'heap') and originally referred, not to a paradox, but rather to a puzzle known as *The Heap*: Would you describe a

[3] http://plato.stanford.edu/entries/sorites-paradox/ This is excerpted from the Stanford Encyclopedia of Philosophy article on the Sorites Paradox. For further discussion and some proposed solutions, hit the link and go to the article!

single grain of wheat as a heap? No. Would you describe two grains of wheat as a heap? No…You must admit the presence of a heap sooner or later, so where do you draw the line?

It was one of a series of puzzles attributed to the Megarian logician Eubulides of Miletus. Also included were:

***The Liar*:** A man says that he is lying. Is what he says true or false?

***The Hooded Man*:** You say that you know your brother. Yet that man who just came in with his head covered is your brother and you did not know him.

The Bald Man: Would you describe a man with one hair on his head as bald? Yes. Would you describe a man with two hairs on his head as bald? Yes. … You must refrain from describing a man with ten thousand hairs on his head as bald, so where do you draw the line?

This last puzzle…was seen to have the same form as the Heap and all such puzzles became collectively known as sorites puzzles…

The following argument form of the sorites was common:

1 grain of wheat does not make a heap.
If 1 grain of wheat does not make a heap then 2 grains of wheat do not.
If 2 grains of wheat do not make a heap then 3 grains do not.
…
If 9,999 grains of wheat do not make a heap then 10,000 do not.

10,000 grains of wheat do not make a heap.

The argument certainly seems to be valid, employing only *modus ponens* and *cut* (enabling the chaining together of each sub-argument which results from a single application of *modus ponens*). These rules of inference are endorsed by both Stoic logic and modern classical logic, amongst others…Yet the conclusion seems false…

As presented, the paradox of the Heap and the Bald Man proceed by addition (of grains of wheat and hairs on the head respectively). Alternatively though, one might proceed in reverse, by subtraction. If one is prepared to admit that ten thousand grains of sand make a heap then one can argue that one grain of sand does since the removal of any one grain of sand cannot make the difference. Similarly, if one is prepared to admit a man with ten thousand hairs on his head is not bald, then one can argue that even with one hair on his head he is not bald since the removal of any one hair from the originally hirsute scalp cannot make the relevant difference. It was thus recognized, even in antiquity, that sorites arguments come in pairs, using: ‘non-heap’ and ‘heap’; ‘bald’ and ‘hirsute’; ‘poor’ and ‘rich’; ‘few’ and ‘many’; ‘small’ and ‘large’; and so on.

For every argument that proceeds by addition there is another reverse argument which proceeds by subtraction.

Curiously, the paradox seemed to attract little subsequent interest until the late nineteenth century when formal logic once again assumed a central role in philosophy…

A common form of the sorites paradox presented for discussion in the literature is the form discussed above. Let '*F*' represent the soritical predicate (e.g., 'is bald', or 'does not make a heap') and let the expression 'a_n' (where n is a natural number) represent a subject expression in the series with regard to which '*F*' is soritical (e.g., 'a man with n hair(s) on his head' or 'n grain(s) of wheat'). Then the sorites proceeds by way of a series of conditionals and can be schematically represented as follows (and whether the argument is taken to proceed by addition or subtraction will depend on how one views the series):

Conditional Sorites

Fa_1
If Fa_1 then Fa_2
If Fa_2 then Fa_3
…
If Fa_1 then Fa_i

Fa_i (where *i* can be arbitrarily large)

In English, we read this as saying the following. Start with the first case, in which an object a_1 has the property *F*, where *F* is the kind of vague property—like being bald, being a heap, etc.—that produces sorites problems. Properties of this kind are sometimes called 'inductive' because when an object in a series has a property of this kind it 'induces' the next object in the series to have the property. So, if a_1 has the *F* property it 'induces' a_2, the next member to the series, to have the *F* property. And, the argument continues, if a_2 has the *F* property then a_3 has it, and so on down the line to any further member of the series a_i.

A 'soritical predicate' is a predicate that designates one of those vague properties that produce sorites problems. The key feature of soritical predicates, which drives the paradox…[is] 'tolerance' and is thought to arise as a result of the vagueness of the predicate involved. Predicates such as 'is a heap' or 'is bald' appear tolerant of sufficiently small changes in the relevant respects—namely number of grains or number of hairs. The degree of change between adjacent members of the series relative to which 'F' is soritical would seem too small to make any difference to the application of the predicate 'F'. Yet large changes in relevant respects will make a difference, even though large changes are the accumulation of small ones, which don't seem to make a difference. This is the very heart of the conundrum, which has delighted and perplexed so many for so long.

Any resolution of the paradoxes is further complicated by the fact that they can be presented in a variety of forms and the problem they present can only be considered solved when all forms have been defused.

One variant replaces the set of conditional premises with a universally quantified premises. Let 'n' be a variable ranging over the natural numbers and let '∀n(…n…)' assert that every number n satisfies the condition …n… . Further, let us represent the claim of the form '∀n(if Fa_n then Fa_{n+1})' as follows:

$$\forall n\ (Fa_n \rightarrow Fa_{n+1})$$

In English this says: for all n, if an object a_n, an object at the *n* position in a series, has the *F* property, then the next object in the series, a_{n+1} has the *F* property. Then the sorites is now seen as proceeding by the inference pattern known as mathematical induction:

Mathematical Induction Sorites

Fa_1
$\forall n\ (Fa_n \rightarrow Fa_n+1)$

$\forall n\ Fa_n$

So, for example, it is argued that since a man with 1 hair on his head is bald and since the addition of one hair cannot make the difference between being bald and not bald (for any number n, if a man with n hairs is bald then so is a man with n+1 hairs), then no matter what number n you choose, a man with n hairs on his head is bald. Speaking of mathematical induction…

9.4 Mathematical Induction

Mathematical induction is a method of mathematical proof typically used to establish that a given statement is true of all natural numbers. It is done by proving that the **first** statement in the infinite sequence of statements is true, and then proving that if **any one** statement in the infinite sequence of statements is true, then so is the **next** one…Mathematical induction should not be misconstrued as a form of inductive reasoning, which is considered non-rigorous in mathematics! In fact, mathematical induction is a form of rigorous deductive reasoning…

The simplest and most common form of mathematical induction proves that a statement involving a natural number *n* holds for all values of *n*. The proof consists of two steps:

The **basis (base case)**: showing that the statement holds when *n* is equal to the **lowest** value that *n* is given in the question. Usually, *n* = 0 or *n* = 1.

The **inductive step**: showing that ***if*** the statement holds for some *n*, ***then*** the statement also holds when *n* + 1 is substituted for *n*.

The assumption in the inductive step that the statement holds for some *n* is called the **induction hypothesis** (or **inductive hypothesis**). To perform the inductive step, one assumes the induction hypothesis and then uses this assumption to prove the statement for *n* + 1.

The choice between n = 0 and n = 1 in the base case is specific to the context of the proof: If 0 is considered a natural number, as is common in the fields of combinatorics and mathematical logic, then n = 0. If, on the other hand, 1 is taken as the first natural number, then the base case is given by n = 1.

This method works by first proving the statement is true for a starting value, and then proving that the process used to go from one value to the next is valid. If these are both proven, then any value can be obtained by performing the process repeatedly. It may be helpful to think of the domino effect; if one is presented with a long row of dominoes standing on end, one can be sure that:

The first domino will fall

Whenever a domino falls, its next neighbor will also fall,

so it is concluded that *all* of the dominoes will fall, and that this fact is inevitable....

Example: Mathematical induction can be used to prove that the following statement

$$0 + 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

holds for all natural numbers n. It gives a formula for the sum of the natural numbers less than or equal to number n. The proof that the statement is true for all natural numbers n proceeds as follows.

Call this statement *P*(*n*).

Basis: Show that the statement holds for *n* = 0.
P(0) amounts to the statement:

$$0 = \frac{0 \cdot (0+1)}{2}\,.$$

In the left-hand side of the equation, the only term is 0, and so the left-hand side is simply equal to 0. In the right-hand side of the equation, 0·(0 + 1)/2 = 0. The two sides are equal, so the statement is true for *n* = 0. Thus it has been shown that *P*(0) holds.

Inductive step: Show that *if P*(*n*) holds, then also *P*(*n* + 1) holds. This can be done as follows.

Assume *P*(*n*) holds (for some unspecified value of *n*). It must then be shown that *P*(*n* + 1) holds, that is:

$$(0+1+2+\cdots+n)+(n+1)=\frac{(n+1)((n+1)+1)}{2}$$

Using the induction hypothesis that *P*(*n*) holds, the left-hand side can be rewritten to:

$$\frac{n(n+1)}{2}+(n+1)\,.$$

Algebraically:

$$\begin{aligned}\frac{n(n+1)}{2}+(n+1) &= \frac{n(n+1)+2(n+1)}{2}\\ &= \frac{(n+1)(n+2)}{2}\\ &= \frac{(n+1)((n+1)+1)}{2}.\end{aligned}$$

thereby showing that indeed P(n + 1) holds.

Since both the basis and the inductive step have been proved, it has now been proved by mathematical induction that P(n) holds for all natural n. Q.E.D.

9.5 We're gonna go with bivalence anyway!

So, after this long excursion into the intricacies of mathematical induction and the horrors of the Sorites Paradox we adopt what is known as *strategic withdrawal*. That is, we resolve that from this day forward we will ignore the problem of vagueness in this course. We will assume that the propositions with which we are concerned are not vague and are, therefore, unproblematically either true or false.

So why did we go through all this anyway? First, because it's only fair to tell you about the problems with the system we're going to be studying, even if it's beyond the scope of this course to discuss solutions. Secondly, because this is a great excuse to introduce you to the idea of

mathematical induction, which is something you should know about, and to the Sorites Paradox which is an intriguing philosophical puzzle.

We will now consider some easier problems…

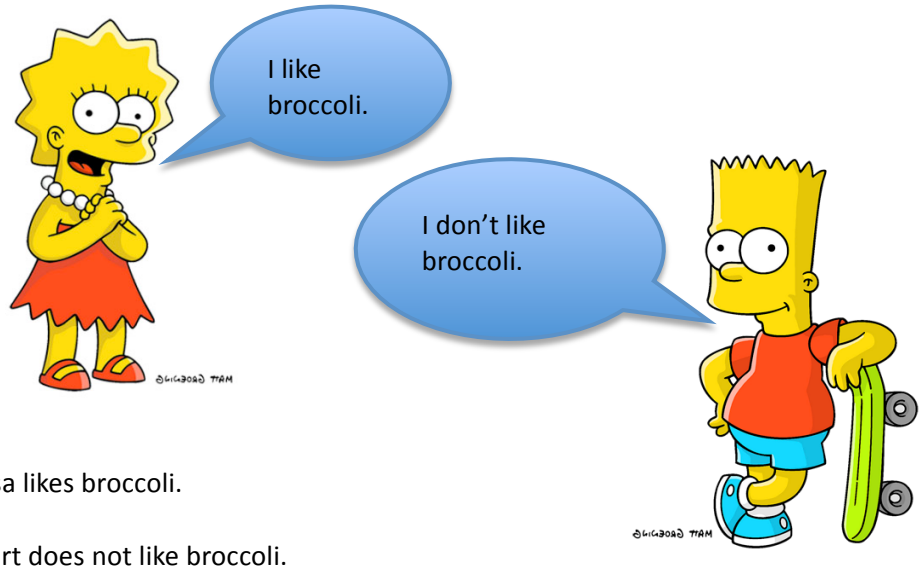
10 Truth is not relative: apparent counterexamples and responses

We claim that truth is not relative to persons, places, times, cultures or circumstances. In this section we respond to some apparent counterexamples to this claim.

10.1 Context Dependence

As we saw earlier, some sentences are *context dependent*: what they say depends upon where, when, by whom or in what circumstances they are uttered. So, if I say, 'My favorite ice cream is spumoni' and President Obama says 'My favorite ice cream is spumoni' we are, in an important sense, not saying the same thing. It could be that the things we're saying are both true: maybe Obama likes spumoni as much as I do. But we're still not saying the same thing because he's talking about *his* ice cream preference while I'm talking about *my* ice cream preference.

Remember that context dependent sentences can be translated into context independent ones by replacing the indexicals with names, dates, descriptions and the like. So each of the following sentences, as uttered by the individuals indicated, can be translated into a sentence that is not context dependent:



(1) Lisa likes broccoli.

(2) Bart does not like broccoli.

In logic we don't like context-dependent sentences! The language of classical logic doesn't have any indexicals. But that's not a loss because we can translate utterances of these sentences into context independent sentences like (1) and (2). So we resolve not to worry about context dependence. When we say that truth value is not relative to persons, places, times or circumstances we're talking about the truth value of *context-independent* sentences. **Nothing in this account of truth value, according to which it is not relative persons, places, times or circumstances is inconsistent with the fact that different people have different preferences or different beliefs.**

10.2 'TRUE-FOR': TRUTH AND BELIEF

Sometimes we get confused because in idiomatic English we use the idiom 'true for' to mean 'believed by' and so say things like

(3) For the ancient Greeks, the earth was the center of the universe.

We sometimes follow up on this by noting that Ptolemeic astronomy, the theory according to which the sun, moon, stars and planets all orbited around the earth, was 'true for' the ancient Greeks. But this is clearly **idiomatic**. What we mean is:

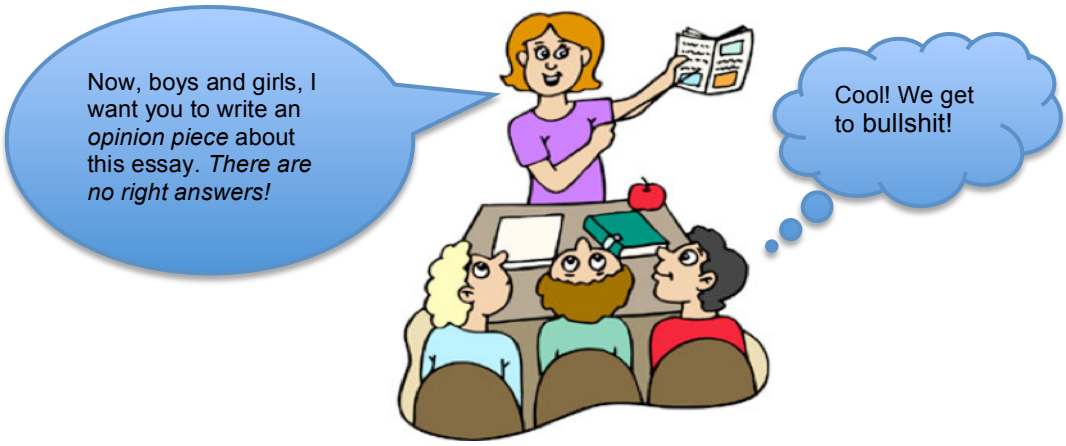
(3′) The ancient Greeks *believed* that the earth was the center of the universe.

However, even though we can see that 'true for' is just an idiom in cases like this, many people are still reluctant to let go of the idea that truth is relative to persons or cultures when it comes to controversial claims—in particular, disputed questions of ethics and theology. When it comes to these hard questions there are good arguments on all sides and smart, educated, informed people disagree: no one *knows* the answers. Does God exist? There are arguments pro and con, as we'll note in the next chapter, and strictly speaking, no one *knows*. It's in cases like this that many people are inclined to say it's just a matter of opinion—that irremediably controversial claims like 'There is a God' are neither true nor false. After all, the folk explain, 'who's to say?'

We note, however, that 'who's to say?' is a very different question from 'true or false'. The first is a question about *epistemic authority*, that is about who, if anyone, is in a position to know the answer. In the case of disputed questions no one is. The second is a question about how things are. It may be that no one *knows* how things are but that doesn't mean that there isn't a way that things are—or aren't. Until recently, no one knew there were craters on the moon, or was even in a position to make an educated guess. But they were there even before people knew about them. No one now knows, or ever will know, whether Lucy, a primitive hominid who lived over 3 million years ago had exactly four children. But there is a fact of the matter: she either did or she didn't.

People get confused about truth and knowledge when it comes to disputed questions because in school teachers assigning an opinion pieces on a controversial topics tell students:

‘there are no right answers’. That’s not exactly right. In cases like this there *are* right answers—it’s just no one knows what they are. What teachers mean is that students will be graded on how well they write and how cogently they argue—not on the truth or falsity of their conclusions, which neither she nor anyone else knows.



11 Knowledge

In any case it’s worth making some brief comments about what knowledge is. As a working definition we will adopt Plato’s account of knowledge as *justified true belief*. This, for obvious reasons, is called ‘The JTB account’. The JTB account of knowledge of course raises further questions, viz. What is truth? What is belief? And what is justification? We shall consider these very briefly in turn.

11.1 What is truth?

This is a very old question. See John 18:38.[4] We will adopt the conventional view that truth is correspondence with reality—with ‘the world’, the facts of the matter, the way things are. We leave open the question of what correspondence comes to. You can think of it as representation or, metaphorically, as picturing.

What sorts of things do that representing? We’ll say, for convenience, that propositions are the things that correspond, or fail to correspond, to reality and so that these are the things that are true or false. The propositions that are true or false are those that can be expressed by context-independent sentences. Such propositions are true if they correspond to (adequately represent, accurately picture) reality; otherwise they are false.

[4] That’s in the Bible—from the bit where Jesus is interrogated by Pilate: ‘Pilate saith unto him, What is truth? And when he had said this, he went out again unto the Jews, and saith unto them, I find in him no fault at all’.

What are propositions—really? On this we remain non-committal. Some regard them as thoughts—subjective, mind-dependent objects; others as abstract, mind-independent objects. We may regard them as sets of sentences—those that group together when we 'count by proposition'. For the purpose of this course we are agnostics when it comes to this philosophically vexed matter. We note only that truth value occupies the space between propositions (whatever they are) and 'the world'.

11.2 What is belief?

Belief is a *propositional attitude*. Propositional attitudes are ways in which people are related to propositions. Take a proposition like the one expressed by the sentence 'Seabiscuit won the race'. I can *wonder whether* Seabiscuit won the race—that's a propositional attitude and there are many more. I can *hope* that Seabiscuit won the race (because I've placed a substantial bet on his nose). I can *be afraid* that Seabiscuit won the race. Or I can simply *believe* that Seabiscuit won the race. Wondering whether, hoping, fearing and believing are propositional attitudes.

The propositional attitude in which we're most interested though is belief. Like all propositional attitudes, belief, so to speak, occupies the space between people and propositions. This means that truth value and belief are two separate and independent things. Truth value sits between propositions and the world: it doesn't have anything to do with people or their beliefs.[5] Belief sits in the space between people and propositions: it doesn't have to do with whether those propositions correspond with reality or not, that is with whether they're true or false.

However, we call those beliefs that connect people to true propositions *true beliefs* and those that connect people to false propositions *false beliefs*. What makes such beliefs true or false however isn't that they are believed or why they are believed but their being beliefs concerning propositions that are true or false respectively. Put simply: believing doesn't make something so; disbelieving doesn't make anything not so.

11.3 What is justification?

On our working definition of knowledge true belief in itself isn't good enough. For true belief to count as *knowledge* it must be *justified*, that is, the believer must have good enough *evidential reasons* for holding his belief.

Now asking why a person holds a belief, asking for reasons, is ambiguous. We may be asking for *causal* reasons, that is, requesting information about what *caused* a person to hold the belief in question. Some answers to this kind of why question might be: 'he had it drilled into his head as a kid'; 'it's the result of post-hypnotic suggestion'; 'a mad neurosurgeon monkeyed with his brain and inserted this belief into his head'. Reasons like this may explain why a person came

[5] Unless, of course, it's a belief about people or their beliefs.

to have a belief but don't *justify* him in holding it, that is, they don't provide any reason to think that the proposition he believes is true.

The kind of reasons we're looking for are those that provide evidence for the truth of a belief. It is these *evidential reasons* that go toward *justifying* a belief. Why do I believe that there is a table in front of me? Because I see it: sense perception provides good evidence for our beliefs. Why do I believe that I had chicken noodle soup for lunch? Because I remember it: memory also provides good evidence for our beliefs. Why do I believe that Constantinople fell in 1453? Because I read it in Steven Runciman's history *The Fall of Constantinople*. Expert testimony provides good evidential reason for our beliefs.

We are justified in holding a belief when we have good enough evidential reasons for that belief. How good does it have to be to be good enough? There is no general answer to this question. But what we can say is that good enough stops short of certainty. Sense perception, memory, and expert testimony, introspection, that is, reflecting on our own mental states and 'reason', the business of doing calculations, proofs and otherwise figuring things out—are reliable sources of knowledge. They lead to truth most of the time. But they are not infallible: they sometimes go wrong. Requiring certainty for knowledge would set the bar too high. We would have to say that we didn't know much of anything. We want an account of knowledge that squares with our commonsense understanding of knowledge. So we will say that knowledge doesn't require certainty.

11.4 Knowledge and Truth

Knowledge on our account is justified true belief. There are true beliefs that aren't justified, like lucky guesses. These don't count as knowledge. There are also false beliefs that aren't justified, like unlucky guesses. Are there cases of false belief that are nevertheless justified? Unfortunately there are. Even reliable means of acquiring evidence for our beliefs are not infallible. Sometimes even when we do everything in our power to arrive at the truth and have what would generally be regarded as the best evidence, we still go wrong.

What this means is that **IT'S ALRIGHT TO BE WRONG!** Getting it wrong doesn't mean that you haven't done what you should do to arrive at the truth. And disagreeing with people isn't attacking them or putting them down.

12 Is it ever rational to believe without good evidential reasons?

If you have good evidential reasons for holding a belief, then it's rational to hold that belief. But what if you don't? Is it ever rational to hold a belief for which we do not have sufficient evidence? According to W. K. Clifford, a 19th century British mathematician writing on the ethics

of belief 'it is wrong always, everywhere, and for anyone to believe anything on insufficient evidence'.[6] Clifford writes:

> Every time we let ourselves believe for unworthy reasons, we weaken our powers of self-control, of doubting, of judicially and fairly weighing evidence. We all suffer severely enough from the maintenance and support of false beliefs and the fatally wrong actions which they lead to…But a greater and wider evil arises when the credulous character is maintained and supported, when a habit of believing for unworthy reasons is fostered and made permanent.[7]

The American philosopher and psychologist William James disagreed. In his 1897 lecture, 'The Will to Believe', James defends the right to violate Clifford's *evidentialist* principle. According to James, there are some cases where, even in the absence of compelling evidential reasons for a belief we may nevertheless have *pragmatic* reasons for adopting it, that is, holding the belief may be good for us. In the next chapter on religious belief we will address the question of whether it is ever rational or right to hold a belief, theological or otherwise, on insufficient evidence.

[6] W. K. Clifford, "The Ethics of Belief," in *Philosophy and Contemporary Issues*, ed. J. Burr and M. Goldinger (New York: Macmillan, 1984), p. 142.

[7] Ibid.