Chapter 8: Arguments and Conditionals

Logic may be defined as the…science that evaluates arguments…An argument is a group of statements, one or more of which (the premises) are claimed to provide support for, or reasons to believe, one of the others (the conclusion).[1] Arguments do the job of expressing *inferences*, processes of reasoning which may also be expressed by *conditionals*. In this chapter we will consider conditionals and arguments—how they are related and how they differ—as we segue into the study of formal logic.

1 Conditionals

A conditional is an IF-THEN statement. In a conditional the 'if' clause is called the *antecedent*; the other clause is called the *consequent*.[2] So, consider the conditional

(1) If Ducati is a dog then Ducati is a mammal

In (1), the antecedent is 'Ducati is a dog'; the consequent is 'Ducati is a mammal'. The if…then is a *sentential connective* or *sentence forming operator*, that is, a device that forms more complicated sentences from simpler ones.

Conditionals state *necessary* and *sufficient* conditions: in any conditional, the ***antecedent is sufficient for the consequent*** and the ***consequent is necessary for the antecedent***. So (1) says that Ducati's being a dog is a *sufficient* condition on Ducati's being a mammal and that Ducati's being a mammal is a *necessary* condition on his being a dog.

To say that some condition A is *necessary* for some other condition B is to say that A is *a requirement* for B.[3] This means that the falsity of A guarantees the falsity of B: if you don't have A, you don't have B.

Suppose, for example, you need to be a college graduate in order to be considered for a job. In that case, having a college degree is a *necessary condition* for getting the job, that is to say, a requirement. If you don't have that piece of paper you won't get the job.

Of course having the piece of paper doesn't guarantee you will get the job because there are 183 other applicants with the same credentials you have. So even though having that degree

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[1] Patrick Hurley, *A Concise Introduction to Logic*. P. 1

[2] Sometimes you hear other terminology, e.g. instead of antecedent and consequent, hypothesis and conclusion.

[3] Don't confuse this business of necessary conditions with the notion of necessity we talked about in discussing necessary and contingent propositions!

is a *necessary condition* for getting the job it is not a *sufficient condition* for getting hired. Not all necessary conditions are sufficient.

'Sufficient' means exactly what it sounds like it means, viz. 'enough'. A condition A is said to be *sufficient* for a condition B when A guarantees the truth of B. For example, being caught cheating is a *sufficient condition* on failing this course, that is, it guarantees an F. As we can see from this case however not all sufficient conditions are necessary. There are other ways to fail a course with which most students are familiar. So while cheating is sufficient for getting an 'F,' it is not necessary.

Now even though necessary and sufficient aren't the same thing, there are cases where condition A is both necessary and sufficient for condition B. Most of these sound strange and trivial because they're essentially definitions. So, for example, being a father is both necessary and sufficient for being a male parent. Or again, a day's being neither Saturday nor Sunday is both necessary and sufficient for its being a weekday. Less trivially, x's being taller than y is both necessary and sufficient for y's being shorter than x. We call such two-way conditionals, where each condition is both necessary and sufficient for the other, *biconditionals* and often express them using the phrase 'if and only if'—sometimes abbreviated as 'iff'.

Here are some questions to see if you understand necessary and sufficient conditions. In each case determine whether the condition in question is necessary, sufficient, both necessary and sufficient or neither necessary nor sufficient. The answers are at the end of the chapter

1.1 Exercise

1 Is sunlight a necessary or sufficient condition for the roses to bloom?

2 Is earning a final grade of C a necessary or sufficient condition for passing the course?

3 Is being a male a necessary or sufficient condition for being a father?

4 Is having the flu a necessary or sufficient condition for being sick?

5 Is being 20 years old a necessary or sufficient condition for being a college student?

6 Is completing all the requirements of your degree program a necessary or sufficient condition for earning your degree?

2 ARGUMENTS

An Argument is a group of statements, one or more of which (the *premises*) are claimed to provide evidential reasons to believe one of the others (the *conclusion*). Every argument, in effect, makes a *factual claim*, viz. that the premises are true. This doesn't mean that the premises really *are* true: it just means that this is what the arguer is claiming, at least for the sake of the argument. In an argument, the premises, and the conclusion, are *asserted*, that is, put forth as true.

But arguments do more. Every argument also makes an *inferential claim*, namely that the premises provide *evidential reasons* for believing the conclusion. That is what makes that group of statements the arguer asserts an *argument* rather than a mere collection of factual claims. Not everything is an argument.

Traditionally, intro logic books give lists of 'premise indicators' and 'conclusion indicators' that are supposed to clue you into when a passage contains an argument—though many arguments do not include these indicator words.

Premise Indicators	**Conclusion Indicators**
since	therefore
as indicated by	wherefore
because	thus
for	consequently
in that	we may infer
may be inferred from	accordingly
as	we may conclude
given that	it must be that
seeing that	for this reason
for the reason that	so
inasmuch as	entails that
owing to	hence
	it follows that
	implies that
	as a result

Premise indicators typically introduce (that is, precede), the premises, and conclusion indicators typically introduce the conclusion—as they do in the following argument:

> ***Since*** the good, according to Plato, is that which furthers a person's real interests, ***it follows that*** in any given case when the good is known, men will seek it.

But premise and conclusion indicators are just clues, and sometimes even misleading ones. They may give you some hints but it would be counterproductive to get hung up on them. What makes a list of statements an argument is not the presence of indicator words or any other purely linguistic marker but what those statements are supposed to do. An argument is supposed to persuade: the arguer's aim is to convince his hearer of the conclusion, so the easiest way to identify an argument is to find the conclusion. To determine which of the statements is the conclusion we ask: 'what claim is the writer or speaker trying to get me to accept?'

An arguer who is doing her job properly starts with premises she assumes her hearer or reader *already* believes and attempts to convince him that believing those premises should lead to his believing the conclusion—which he doesn't *yet* believe—as well. Understood in this way we can see that one way to sort out the premises and conclusion of an argument is to recognize that typically (not always, but usually) the conclusion is less obvious and more controversial than the premises. Consider the following argument:

> Since private property helps people define themselves, since it frees people from mundane cares of daily subsistence, and since it is finite, no individual should accumulate so much property that others are prevented from accumulating the necessities of life.[4]

I'm not sure what it means to say that private property helps people 'define' themselves so let's leave that. The other two premises are pretty obvious. Property frees people from the mundane cares of daily subsistence: it means that they don't have to go out every day scratching for the day's food, wood for the campfire and materials to make a shelter for the night. Houses, refrigerators, and bank accounts us to do more interesting things. The third premise is also pretty obvious. Property is finite. There's only so much stuff to go around. Even if we increase the amount of stuff there's always a limit.

What is far from obvious is the conclusion, that 'no invidual should accumulate so much property that others are prevented from accumulating the necessities of life'. This is a very radical proposal and one that is highly controversial. Even in the absence of the premise indicators, that should tip you off that it is the conclusion. Another tip-off: the premises are *factual* claims—statements that assert that something *is* the case; the conclusion is a *normative* claim—a statement making a claim about what *should* be the case. Typically, normative claims, e.g. moral judgements, aesthetic assessments, and other value judgments, are less obvious and more controversial than factual claims.

[4] Leon P. Baradat, *Political Ideologies, Their Origins and Impact* cited from Hurley, *A Concise Introduction to Logic*

2.1 EXERCISE

Identify the conclusions of the following arguments:

I. His [God's] power, we allow, is infinite; whatever he wills is executed; but neither man nor any other animal is happy; therefore, he does not will their happiness.

—David Hume

2. It is right that men should value the soul rather than the body; for perfection of soul corrects the inferiority of the body, but physical strength without intelligence does nothing to improve the mind.

—Democritus

3. The table, which we see, seems to diminish, as we remove farther from it: but the real table, which exists independent of us, suffers no alteration: it was, therefore, nothing but its image, which was present to the mind.

—David Hume

4. There cannot be any emptiness; for what is empty is nothing, and what is nothing cannot be.

—Melissus

5. We see that things which lack intelligence, such as natural bodies, act for an end, and this is evident from their acting always or nearly always, in the same way, so as to obtain the best result .... Now whatever lacks intelligence cannot move towards an end, unless it be directed by some being endowed with knowledge and intelligence; as the arrow is shot to its mark by the archer. Therefore, some intelligence exists by whom all natural things are directed to their end; and this being we call God.

—Thomas Aquinas

6. About the gods, I am not able to know whether they exist or do not exist, nor what they are like in form; for the factors preventing my knowledge are many: the obscurity of the subject, and the shortness of human life.

— Protagoras

7. It is by no means established that the brain of a woman is smaller than that of a man. If it is inferred merely because a woman's bodily frame generally is of less dimensions than a man's, this criterion would lead to strange consequences. A tall and large-boned man must on this showing be wonderfully superior in intelligence to a small man, and an elephant or a whale must prodigiously excel mankind.

—John Stuart Mill

8. In the beginning man was born from creatures of a different kind; because other creatures are soon self-supporting, but man alone needs prolonged nursing. For this reason he would not have survived if this had been his original form.

— Anaximencs

9. If the teacher happens to be a man of sense, it must be an unpleasant thing to him to be conscious, while he is lecturing his students, that he is either speaking or reading nonsense, or what is little better than nonsense. It must, too, be unpleasant to him to observe that the greater part of his students desert his lectures, or perhaps attend upon them with plain enough marks of neglect, contempt, and derision. If he is obliged, therefore, to give a certain number of lectures, these motives alone, without any other interest, might dispose him to take some pains to give tolerably good ones.

—Adam Smith

10. [S]ome whales have been captured far north in the Pacific, in whose bodies have been found the barbs of harpoons darted in the Greenland seas. Nor is it to be gainsaid, that in some of these instances it has been declared that the interval of time between the two assaults could not have exceeded very many days. Hence, by inference, it has been believed by some whalemen, that the Nor'-West Passage, so long a problem to man, was never a problem to the whale.

—Herman Melville

2.2 Deductive and Inductive Arguments

Every argument makes an inferential claim to the effect that the premises 'support' the conclusion. But the degree of support the premises are intended to provide differs between those arguments we call *deductive* and those we call *inductive*. In a *deductive argument* the premises are supposed to force (necessitate, guarantee) the conclusion. In other words, the intention is that the premises provide the strongest possible support for the conclusion: *it is supposed to be logically impossible that the premises be true and the conclusion false.* That is what is meant by saying that the premises are supposed to 'necessitate' the conclusion.

How does this necessitation magic happen when a deductive argument succeeds? It happens because the conclusion isn't really saying anything new. In a *valid* deductive argument, that is a deductive argument were the premises *really do* necessitate the conclusion, there is no information in the conclusion that isn't already in the premises. The premises are said to 'contain' the conclusion. If they really do then the conclusion is *of course* guaranteed since it was there all along in the premises!

In an inductive argument by contrast the premises are just supposed to make the conclusion more probable. Even in a successful inductive argument the premises do not necessitate the conclusion because there is information in the conclusion that is not in the premises. The conclusion, in this sense, goes beyond the premises so the premises don't guarantee that we have the conclusion.

Note, though, that inductive arguments aren't in any sense inferior to deductive ones: the intent of an inductive argument is just to make the conclusion plausible. If the arguer succeeds in doing that on the basis of the premises, the argument is a success. Note also that inductiveness and deductiveness are about what is *supposed* to happen. Some inductive arguments and some deductive arguments fail: their premises don't provide the *intended* degree of support for their conclusions. Finally, note that a failed deductive argument does not thereby become an inductive argument: failing at one task doesn't mean succeeding at another! Deductiveness and inductiveness are a matter of the arguer's intention—what the arugment is *supposed* to do.

How do we know what the arguer intends? Sometimes we don't but most of the time we can get a fairly good idea of whether an arguer intends his argument to be deductive or inductive. There are characteristically inductive argument forms and characteristically deductive forms we can identify. In logic we're mainly interested in deductive arguments. But to appreciate their special features it's useful to compare them to some inductive arguments.

2.3 Some Inductive Arguments

The most familiar kind of inductive argument is inductive generalization. Inductive generalization proceeds from information about a selected sample to a claim about the whole group. Apart from the census, all surveys proceed in this way. We want to learn something about a whole population—the human population of the United States, the elephant population of Kenya or the 'population' of stones on the beach. So we start with a sample, assumed to be representative of the whole population, and conclude that the target of our investigation, the whole population, is like the sample.

When we do a survey, we try to select a sample that fairly represents the population we're studying. So, for example, if we're interested in the US population we'll assemble a sample that includes the same proportion of people in various demographic categories as the total US population. A 'fair sample' will match the total population when it comes to the percentage of men and women, of college graduates, of people of various ages and so on. If the sample is sufficiently large and fair then we can extrapolate from information we get about the sample. So, for example, according to a 2005 Gallup Poll surveying beliefs concerning paranormal phenomena, about one fourth of the sample of Americans surveyed thought there was something to astrology. By inductive generalization we argue:

An Inductive Argument

Premise: One fourth of the Gallup sample believe there's something to astrology

Conclusion: approximately one fourth of the American public believe there's something to astrology

If the Gallup sample was indeed fair and the survey was done properly then the premise gives us good reason to believe the conclusion *but it does not necessitate the conclusion*. It is still *logically possible* that many more or far fewer Americans give credence to astrology. The premise does not guarantee the conclusion because the conclusion goes beyond the premise: we're arguing from the character of a small group to a conclusion about the character of a larger population of which it is a part. There is information in the conclusion that is not contained in the premise.

Inductive generalization is only one inductive argument form. There are many others. What they all have in common is that they are all intended to have information in their conclusions that is not contained in their premises. There is, for example, *inference to the best explanation*. Arguably, this kind of argument provides the most compelling reason to reject solipsism. Recall that in our discussion of skepticism we worried that we might be brains in vats or that the world might have come into being 5 minutes ago. We suggested that even though these were logical possibilities, the external world hypothesis was the best explanation of the character of our experience, even though the conclusion, that there exists a world that goes beyond the data of immediate experience, goes beyond what we asserted in the premises about the character of our experiencer. That was an argument to the best explanation—an inductive argument—and I think (hope!) a pretty good one. But, even given the truth of the premises about the character of our experience it is still *logically possible* that the conclusion be false!

Then there is *argument from analogy,* which depends on the existence of an analogy, or similarity between two things or states of affairs. Because of the existence of such similarities, a condition that affects the better-known thing or situation is concluded to affect the similar, lesser-known thing. In the philosophical literature, the most famous argument from analogy is the argument for the existence of other minds.

Suppose we grant that the world hasn't come into existence 5 minutes ago, that we are not brains in vats and that we are not dreaming. Suppose we grant that there is a world of material things independent of ourselves. Do we have any good reason to believe that some of those material objects, those that roughly resemble us and behave in similar ways, are conscious? This is the philosophical Problem of Other Minds. The only mental states to which we have direct access are our own. We cannot think other peoples thoughts, experience their emotions, or feel their sensations: the only thoughts, emotions and sensations of which we are directly aware are our own. So what reason do we have to believe that those other beings that look and behave more or less like us have thoughts, emotions, sensations or anything else going on inside? How do we know they aren't just automata? Or zombies?

Intuitively, we rely on an argument from analogy. We note the similarities between ourselves and those other beings that look and act very much as we do. We know that we are conscious—that we think and feel. So we infer, by analogy, that they do too. This is a good argument and provides us with a very good reason to believe in Other Minds. But it is still logically possible that we are wrong because what we conclude, viz. that those other material beings that look and behave like us are conscious like us, goes beyond the premises.

2.4 Deductive Arguments

In a deductive argument, by contrast, the premises are supposed to *guarantee* the conclusion. The argument is *valid* if the premises do what they're supposed to do—so that *if* the premises are true it is logically impossible for the conclusion to be false. Validity is the big deal in logic. It is an 'internal' feature of arguments: it concerns the connection between premises and conclusion *whether they're true or not*. Note the Big If: in a valid argument, IF the premises are true then the conclusion must be true. So, beware: 'validity' is a term of art in logic and doesn't mean what it means in ordinary English. In logic, we understand *validity* as follows:

An argument is valid if…

1. The premises necessitate ('force', 'guarantee') the conclusion

2. It is not logically possible for the premises to be true and the conclusion false (that is, there is no possible world at which the premises are true and the conclusion is false)

3. It is *truth-preserving*: IF the premises are true then the conclusion must be true

4. There is no information in the conclusion that's not in the premises ('The conclusion is 'contained' in the premises')

5. It is not possible to represent the premises without representing the conclusion in a logic diagram

Every valid argument has all of these properties, which are related to one another. How are we to understand (1)? The claim that the premises 'necessitate' the conclusion just means (2), that it is not logically possible for the premises to be true and the conclusion false. Another way of putting that is to say that IF the premises are true then the conclusion must be true, which is to say (3) the argument is truth-preserving. Now we ask further why this happens and how. The answer is (4): since the conclusion doesn't really say anything more than what the premises say, the premises of course guarantee the conclusion. (5) concerns a method of testing arguments for validity using logic diagrams. We can represent or *picture* the information contained in the premises and conclusions of an argument. The claim is that when an argument is valid it is not possible to draw the picture of the information in the premises without automatically displaying the conclusion.

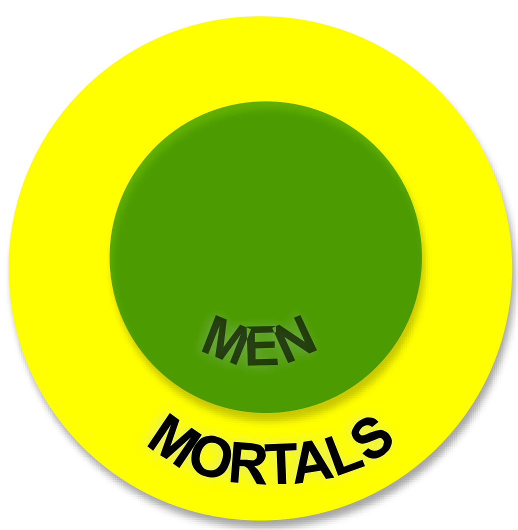
To see how this works, let us consider the most famous argument of all, henceforth to be called 'The Socrates Argument':

The Socrates Argument

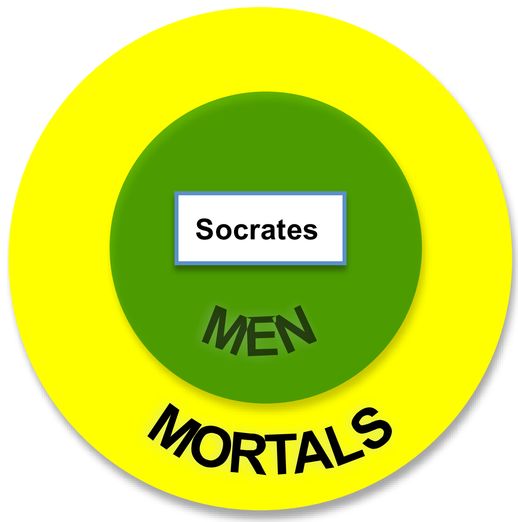
1. All men are mortal.
2. Socrates is a man.
3. Therefore, Socrates is mortal

Premise 1 asserts set inclusion: it says that every member of the set of men is a member of the set of mortals. We can represent set inclusion by spatial inclusion. We can represent sets as disks. So we can represent the set of men and the set of mortals like this:

We can represent set inclusion as the spatial inclusion of one disk in another. So we can picture the first premise, which asserts that all men are mortal, by showing the set of men as contained in the set of mortals, like this:



The second premise says that Socrates is a member of the set of men. We can represent that by putting Socrates into the MEN disk:



What d'ya know: putting Socrates into the MEN disk automatically puts him into the MORTALS disk! So we automatically get the conclusion that Socrates is mortal! It couldn't be otherwise given the premises. We cannot—logically cannot—put MEN into MORTALS and Socrates into MEN without thereby putting Socrates into MORTALS: the conclusion is contained in the premises. And that shows that the Socrates argument is valid. It is easy to see why this is so. Premise 1 says that the set of men is *included* in the set of mortals, that is, that it is a *subset* of the set of mortals. This means that every member of the set of men is a member of the set of mortals. So, given Premise 2, which says that Socrates is a member of the set of men, it follows that he, along with all other members of that set, is a member of the set of mortals.

Now please remember that validity just concerns the internal structure of arguments. It doesn't say anything about whether the premises or conclusion accurately represent the way the world is. It doesn't say that the premises are true. It doesn't say that the conclusion is true. Validity just says that **IF** the premises are true then the conclusion must be true—and that is a mighty big

IF

What this means is that really stupid arguments, which have idiotic premises and ridiculous conclusions can still be valid—providing that the stupid premises and conclusions hang together in the right way. Validity just says that **IF** the premises are true then the conclusion must be true. So here's a valid argument for you:

A Valid but Unsound Argument

1. All numbers greater than 17 are even
2. 3 is greater than 17
3. Therefore 3 is even

Is this argument valid? You betcha! (1), (2) and (3) are all false. The premises are garbage and the conclusion is garbage and, as we used to say: garbage in—garbage out. But this argument is garbage that hangs together: IF the premises were true then the conclusion WOULD BE true. And that is what makes for validity.

What this means is that for practical purposes validity isn't good enough—unless we're willing to settle for garbage. We want arguments that guarantee the truth of their conclusions. We want *soundness*. Soundness is just validity-plus: an argument is *sound* if (1) it is valid and (2) has all true premises. The argument above is valid but not sound since its premises are false. The Socrates Argument is sound: it is valid and has all true premises. Given the definition of soundness, and assuming the definition of validity, it follows that if an argument is sound its conclusion must be true. Validity says that IF the premises of an argument are true then the conclusion must be true. Soundness says that the argument is valid and its premises are true. So it follows that the conclusion of a sound argument must be true.

2.5 Validity

Soundness is the gold standard for arguments. But in logic we aren't really that interested in it because we aren't interested in facts about the world. We're interested in whether, and why, arguments hang together internally—that is, we're interested in validity.

So far, while we've given an account of what validity is we haven't even given any hints about how to recognize it. How do we determine whether an argument is valid or invalid? There are a number of ways of doing this, which we'll be considering. Right now we want to consider a way of determining that an argument is *invalid*, namely the method of counterexample. The idea is this. We assume that validity is a matter of form, so that all arguments of the same form are the same as regards validity or invalidity. So if there is an argument whose validity is in question, to determine whether it's valid or invalid we look for another argument of the same form whose validity/invalidity is known. If we determine that that argument is valid then we know that the argument whose validity is in question is also valid; if we determine that the argument is invalid we know that the argument whose validity is in question is also invalid.

The method of counterexample, however, can only be used to show invalidity: *it cannot show validity.* The reason for this is that we have a way of knowing for sure when an argument is invalid but—until we have some system of formal logic for testing validity—no way of determining whether an argument is valid. Remember the definition of validity? An argument is valid if it is logically impossible that the premises be true and the conclusion false. This means that if we have an argument with all true premises and a false conclusion we know that it must be invalid. No other combination of truth values for premises and conclusion tells us anything about validity. An argument with all true premises and a true conclusion could be valid—or it could be invalid. How could such an argument be invalid? Well the premises and conclusion could be true individually but just not hang together. Here is an example of an invalid argument with all true premises and a true conclusion:

The Cat Argument

1. All cats are vertebrates.
2. All mammals are vertebrates.
3. Therefore, all cats are mammals.

The premises and conclusion of this argument at all true. But the argument doesn't hang together in the right way: the premises do not guarantee the conclusion. This is an argument with all true premises and a true conclusion, which is nevertheless invalid.

You can also have a valid argument with false premises and a false conclusion. Crazy as it may seem, the following argument is valid. 'Valid' for us is a term of art: for the purposes of logic, we aren't using this term in the way it's used in ordinary English.

The Even-Odd-Prime Argument

1. All even numbers are odd

2. All odd numbers are prime
3. Therefore, all even numbers are prime

This argument is totally stupid and ridiculous: both of its premises and its conclusion are manifestly false. But it's valid. IF the premises were true then the conclusion WOULD BE true: that's validity. So validity is a virtue—but it isn't a complete virtue. To summarize, given our definition of validity, a valid argument can have any combination of truth values for premises and conclusions *except* all true premises and false conclusion:

	Valid	**Invalid**
True premises/True conclusion	All men are mortal. Socrates is a man. Therefore, Socrates is mortal.	All cats are vertebrates. All mammals are vertebrates. Therefore, all cats are mammals.
True premises/False conclusion	**NO CAN DO! BY DEFINITION, A VALID ARGUMENT CAN'T HAVE ALL TRUE PREMISES AND A FALSE CONCLUSION**	All cats are mammals. All dogs are mammals. Therefore all dogs are cats.
False premises/True conclusion	All dogs are reptiles. All reptiles are warm-blooded. Therefore, all dogs are warm-blooded.	All dogs are cold-blooded. All mammals are cold-blooded. Therefore all dogs are mammals.
False premises/False conclusion	All even numbers are odd. All odd numbers are prime. Therefore, all even numbers are prime.	The Department of Defense Building in Washington, DC has the shape of a hexagon. Therefore, it has seven sides

The definition of validity rules out a case in which an argument has all true premises and a false conclusion but is nevertheless valid. Anything else goes, as the table shows, so no other combination of truth values tells you anything. Suppose you know that the premises and conclusion of an argument are true. Does this tell you that the argument is valid? Nope. It could be that although the premises and conclusion are individually true they don't hang together in such a way that the premises necessitate the conclusion, that is, so that the argument is valid. Consider, for example, The Cat Argument.

We could give examples of arguments that fit into each of the boxes in the table above except the one that is ruled out by definition. There cannot be a valid argument with all true premises and a false conclusion because according to the definition of validity an argument is valid if it is *not logically possible that the premises be true and the conclusion false*. So end of story.

But this is not bad news. In fact it is very good news because it means that for at least some arguments we can answer the validity question just by knowing the truth values of their premises and conclusion. Knowing that an argument has all true premise and a true conclusion doesn't tell us anything about validity: you can have an argument with all true premises and a true conclusion that isn't valid, like the Cat Argument. Knowing that the premises and conclusion of an argument are all false doesn't tell you anything. BUT…knowing that an argument has all true premises and a false conclusion does give us an answer to the validity question: it tells us that the argument is definitely invalid.

So, for the special case or arguments where the premises are true and the conclusions false, we can answer the validity question. But it gets even better. If we assume that validity is a matter of form, we can determine that arguments with different combinations of truth value for premises and conclusion are invalid if they are of the same form as arguments that have all true premises and false conclusions. That is how the method of counterexample works.

2.6 The Method of Counterexample

We know that any argument that has all true premises and a false conclusion is invalid. But what about other arguments? The method of counterexample allows us to determine when arguments that do not have the telltale true premises/false conclusion are invalid—providing that we are willing to assume (or, let's be honest, *stipulate*) that *validity is a matter of form*.

What does this mean? The idea is that every sentence and more grandly every argument has a certain 'logical form' which we can recognize. So, for example, intuitively the following two sentences have the same form:

(1) If you lie down with dogs, then you get up with fleas.

(2) If it rains, then it pours.

Both (1) and (2) are conditionals, sentences of the form

(3) if *p* then *q*

In (3) we are using *p* and *q* as *variables*, that is, place-holders. (3) is not a sentence. It does not express any proposition and it is neither true nor false. It is a sentence *form* that can be turned into a sentence that has truth value by substituting sentences for the variables. So, (1) and (2) are said to be *substitution instances* of (3) insofar as they are the result of plugging in sentences for the variables. (1) substitutes "you lie down with dogs" for the *p* variable and "you get up with fleas" for the *q* variable; (2) substitutes "it rains" for the *p* variable and "it pours" for the *q* variable.

Now arguments are groups of sentences, so we can also talk about their logical form too. Consider the following two arguments:

The Even-Odd-Prime Argument	**The Dog Argument**
1. All even numbers are odd 2. All odd numbers are prime **3.** Therefore, all even numbers are prime	1. All dogs are mammals 2. All mammal are vertebrates 3. Therefore, all dogs are vertebrates

In each of these arguments we can distinguish what we may call *logical expressions* from *non-logical* expressions. *Logical expressions* are the colorless, structural words that occur in the discussion of all different subject matters and, intuitively, constitute the *form* of the arguments. These include words like 'all', 'some', 'no', 'and', 'or', 'if-then' and so on. We call these 'logical expressions' not because they're somehow 'more logical' than other words but because it's these structural words, which constitute the form of arguments, that we study in formal logic. The logical expressions in the arguments above are 'all' and 'are'.

These arguments are of the same logical form because they meet two conditions:

(i) They have the same framework of logical expressions, i.e.

All…are…

All…are…

Therefore, all…are

But this isn't good enough. There is also a second condition

(ii) They have the same pattern of same non-logical expressions plugged in.

To see this, compare the Dog Argument to the Cat argument:

The Dog Argument	**The Cat Argument**
1. All dogs are mammals 2. All mammals are vertebrates 3. Therefore, all dogs are vertebrates	1. All cats are vertebrates 2. All mammals are vertebrates 3. Therefore, all cats are mammals

They meet condition (i) but not condition (ii) since the pattern of same non-logical expressions that plug into the blanks is different. That's why we need variables rather than just blanks! Variables tell us when you have to plug in the same expression. If variables are the same, the expressions you substitute for them must be the same.[5] So, using the variables X, Y and Z we can see that these two arguments are different in form:

The Dog Argument Form	**The Cat Argument Form**
1. All X are Y 2. All Y are Z 3. Therefore, all X are Z	1. All X are Y 2. All Z are Y 3. Therefore, all X are Z

We can see from this that the pattern is different, and so that the Dog and Cat Arguments are not of the same logical form. Moreover, as it happens, the Dog argument is valid but the Cat Argument, even though it has all true premises and a true conclusion, is not valid. But how could we determine that, given that it doesn't have the telltale combination of truth values, viz. all true premises and false conclusion?

The method of counterexample gives us a way. If we assume that validity is a matter of form then if any two arguments are of the same form we know that either both are valid or both are invalid. What this means is that if we can find an argument *of the same form* as the cat argument that is known to be invalid we thereby establish that the Cat Argument is invalid. We can identify the form of an argument by inspection. But how do we know when an argument is invalid? Well, as we've seen, if an argument has all true premises and a false conclusion we know it's invalid, given the definition of validity. So to show that the Cat Argument is invalid all we have to do is find another argument that is (1) of the same form and (2) has all true premises and a false conclusion. Such an argument is called a *counterexample* to the original argument. We show invalidity by the *method of counterexample* by contriving counterexamples to arguments to be tested.

In general, the method of counterexample banks on two assumptions:

Factoid #1: Validity is a matter of form—i.e. arguments of the same form are the same as regards validity/invalidity[6]

Factoid #2: If an argument has all true premises and a false conclusion then it must be invalid.

[5] However, as we'll see later, if variables are different you don't have to plug in different expressions: you could plug in either same or different expressions.

[6] What does this have to do with counterexamples in the more general sense? Well, the idea is that when we say an argument is valid, on this account, we are claiming that the argument *form* of which it's a substitution instance is valid, i.e. that all substitution instances of that form are valid. Producing an *invalid* argument of that form is coming up with a counterexample to that general claim.

#2 is unproblematic. It just falls out of the definition of validity, as we have seen. #1 is basically what Plato called a 'noble lie' or, to put a better spin on it, a stipulation. A great many arguments are valid even though their validity doesn't come about in virtue of their logical form. For example:

An Argument that is not Formally Valid

George is a bachelor

Therefore, George is unmarried

The validity of this argument comes about in virtue of the meanings of the non-logical expressions, viz. the content expressions 'bachelor' and 'unmarried'. But this argument is nevertheless valid: the premise guarantees the conclusion. Since the validity of arguments like this comes about because of the meanings of content words rather than the logical expressions, which constitute their form, we say that such arguments are *valid, but not formally valid*.

In logic, we are interested in validity insofar as it comes about in virtue of logical form. From henceforth, we stipulate that, unless otherwise indicated, when we say 'valid' we mean 'formally valid'. So we have made #1 true by stipulating that we will restrict our attention to those arguments whose validity or invalidity is a matter of form.

Now given these two assumptions we can see how the method of counterexample works to show invalidity.

An argument, C, is a *counterexample* to an argument, A, if the following conditions are met:

1. A and C are substitution instances of the same logical form and
2. C has all true premises and a false conclusion

If these two conditions are met, then Argument C is a counterexample to Argument A and argument A is thereby shown to be invalid.

3 ON TO FORMAL LOGIC!

Through the use of logic diagrams and the method of counterexample we have ways to test arguments for validity and invalidity. In our discussion of formal logic which follows we will tighten up all the stuff we've considered so far and consider three further methods for testing arguments: truth tables, truth trees and the method of 'natural deduction' as a way of doing proofs in propositional logic.

Answers to Exercise 1.1

1 Sunlight is a necessary condition for the roses to bloom, since without sunlight it would be impossible for the roses to bloom. It is not a sufficient condition, though, because sunlight alone does not guarantee that the roses will bloom.

2 Earning a final grade of C is a sufficient condition for passing this course because earning a C guarantees passing it. It is not a necessary condition because there are other ways to pass the course other than earning final grade of C.

3 Being a male is a necessary condition for being a father since it is impossible to be a father without being a male. Being a male is not a sufficient condition, however, since being a male does not guarantee that a male will be a father.

4 Having the flu is sufficient for being sick, but not necessary since there are other ways to be sick besides having a flu virus.

5 Being 20 years old is neither necessary nor sufficient for being a college student. One can be a college student without being 20 years old, and there are other ways to be a college student than being 20 years old, i.e you don't deed to be 20 to be a college student.

6 Completing all of your requirements is both a necessary and sufficient condition for earning your degree. Without completing all requirements, it is impossible to earn a degree, and completing all requirements guarantees earning a degree.

Answers to Exercise 2.1

1 God does not will the happiness of either men or other animals.

2 It is right that men should value the soul rather than the body.

3 The table was nothing but its image, which was present to the mind.

4 There cannot be any emptiness.

5 Some intelligence exists by whom all natural things are directed to their end; and this being we call God.

or

God exists

6 I cannot know whether the god exist or not

7 It is by no means established that the brain of a woman is smaller than that of a man.

8 In the beginning man was born from creatures of a different kind.

9 If a man of sense is obliged to give a certain number of lectures then the desire to avoid nonsense is enough to dispose him to take some pains to give tolerably good lectures.

10 The Nor'West Passage, was never a problem to the whale.