

The Truth Tree Test for Validity

List the premises and the *negation* of the conclusion in a vertical column.

- This is to assume invalidity.
- This column, which consists of the *initial sentences* on which the tree was grown, forms the "trunk" of the tree.

Take any compound statement in the trunk, check it off, and draw its truth-conditions at the bottom of the trunk, following the truth tree rules for decomposition.

- If compound statements remain on any branch, check them off and break them down, listing their truth conditions (by the branching rules) at the bottom of every open branch below them.
- Repeat until each compound statement in each branch has been checked off and broken down (decomposed).
- Tip: To save labor and paper, check off and break down all non-branching compounds before any branching compounds.

If a branch contains contradictory information anywhere along it, including the trunk, then *close* that branch with an X at its bottom.

When these steps are complete, then either:

All branches are closed.

- This means that the initial assumption of invalidity leads to contradiction; hence the assumption is false. *The argument is valid.*

or

At least one branch is open.

- This means that the assumption of invalidity is not contradictory; there is at least one assignment of truth values which makes all the premises true and the conclusion false. *The argument is invalid.*
- Each open branch represents an interpretation in which all premises are true and the conclusion false--a vertical, or jagged, representation of an invalidating row of a truth table.

Truth Trees and Truth Tables: Open branches on a completed tree represent assignments of truth values to sentence letters that occur on the truth table in rows where all initial sentences are true. Where a sentence letter occurs on an open branch, the corresponding row of the truth table assigns true to the sentence letter; where the negation of a sentence letter occurs on an open branch, the corresponding row of the truth table assigns false to the sentence letter on that row of the truth table. So, in addition to telling us whether an argument is valid or invalid, in the case of invalid arguments, the completed truth tree tells us which assignments of truth values to the constituent sentence letters makes all the premises true and the conclusion false.

Validity and Consistency: The truth tree test for validity is an *indirect proof* method. It tests arguments for validity "indirectly" by testing the initial sentences i.e. the premises + the negation of the conclusion, for consistency. If the tree is open this shows that the initial sentences are consistent, hence that the argument is invalid; if the tree is closed this shows that the initial sentences are inconsistent, hence that the argument is valid.

Truth Tree Rules

