Aann Alkhori

5.7 Trig. Substitution Questions

1. Use the Sub $x = 3\sin\theta$, $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and The Identity $\cot^2\theta = \csc^2\theta - 1$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

2. Evaluate the integral

$$\int \frac{x^3}{\sqrt{x^2+9}}\,dx$$

Hann Alkhori
5.7 Trig Substitution
Solutions

Subs you Should Know:

1. $\int \sqrt{a^2 - x^2}\, dx \qquad x = a\sin\theta$

2. $\int \sqrt{a^2 + x^2}\, dx \qquad x = a\tan\theta$

3. $\int \sqrt{x^2 - a^2}\, dx \qquad x = a\sec\theta$

1. Use the Substitution $x = 3\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, and the Identity $\cot^2\theta = \csc^2\theta - 1$

$$\int \frac{\sqrt{9 - x^2}}{x^2}\, dx$$

$x = 3\sin\theta$
$a^2 = 9$
$a = 3$
$dx = 3\cos\theta\, d\theta$

$$= \int \frac{\sqrt{9 - (3\sin\theta)^2}}{(3\sin\theta)^2}\, 3\cos\theta\, d\theta$$

$$= \int \frac{\sqrt{9 - 9\sin^2\theta}}{\cancel{9}\sin^2\theta}\, \cancel{3}\cos\theta\, d\theta$$

$$= \int \frac{\sqrt{9(1 - \sin^2\theta)}}{3\sin^2\theta}\, \cos\theta\, d\theta$$

$$= \int \frac{3\sqrt{\cos^2\theta}}{3\sin^2\theta}\, \cos\theta\, d\theta$$

$$= \int \frac{\cos\theta}{\sin^2\theta}\cos\theta\, d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \int \cot^2\theta\, d\theta$$

$$= \int (\csc^2\theta - 1)\, d\theta = -\cot\theta - \theta + C$$

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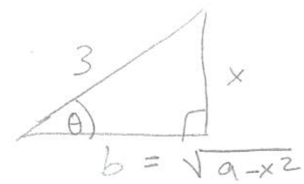
Cont. of. #1

$-\cot\theta - \theta + C$

$$\boxed{-\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C}$$

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$



$3^2 = x^2 + b^2$

$\sqrt{9-x^2} = \sqrt{b^2}$

$\tan\theta = \frac{x}{\sqrt{9-x^2}}$

$\cot\theta = \frac{\sqrt{9-x^2}}{x}$

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$

$\sin^{-1}\left(\frac{x}{3}\right) = \cancel{\sin^{-1}}(\sin\theta)$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

Aann Alkhori

5.7 Trig. Substitution Solutions

Subs you should know:

1. $\sqrt{a^2-x^2}\,dx$ $\quad x = a\sin\theta$

2. $\sqrt{a^2+x^2}\,dx$ $\quad x = a\tan\theta$

3. $\sqrt{x^2-a^2}\,dx$ $\quad x = a\sec\theta$

4. $\tan^2\theta + 1 = \sec^2\theta$

2. Evaluate the integral

$$\int \frac{x^3}{\sqrt{x^2+9}}\,dx$$

$x = 3\tan\theta$

$a^2 = 9$

$a = 3$

$dx = 3\sec^2\theta\, d\theta$

$$= \int \frac{(3\tan\theta)^3}{\sqrt{(3\tan\theta)^2+9}}\,3\sec^2\theta\, d\theta$$

$$= \int \frac{27\tan^3\theta}{\sqrt{9\tan^2\theta+9}}\,3\sec^2\theta\, d\theta$$

$$= \int \frac{27\tan^3\theta}{\sqrt{9(\tan^2\theta+1)}}\,3\sec^2\theta\, d\theta$$

$$= \int \frac{27\tan^3\theta}{\cancel{3}\sqrt{\sec^2\theta}}\cdot \cancel{3}\sec^2\theta\, d\theta$$

$$= \int \frac{27\tan^3\theta}{\cancel{\sec\theta}}\cdot \sec^{\cancel{2}}\theta\, d\theta$$

$$= \int 27\tan^3\theta\cdot\sec\theta\, d\theta$$

$$= 27\int \tan^2\theta\cdot\tan\theta\cdot\sec\theta\, d\theta$$

Perform U-sub at this step:

$u: \sec\theta$

$du: \sec\theta\tan\theta\, d\theta$

$$= 27\int (\sec^2-1)\tan\theta\sec\theta\, d\theta$$

$$= 27\int (u^2-1)\,du = \frac{u^3}{3} - u = 27\left(\frac{\sec^3\theta}{3} - \sec\theta\right) + C$$

$$= 9\sec^3\theta - 27\sec\theta + C$$

Cont. on other paper

cont. of #2

$$9\sec^3\theta - 27\sec\theta + C$$

$$= 9\sec\theta\,(\sec^2\theta - 3) + C$$

$$= \cancel{9}\left(\frac{\sqrt{9+x^2}}{\cancel{3}}\right)\left[\left(\frac{\sqrt{9+x^2}}{3}\right)^2 - 3\right] + C$$

$$= 3\sqrt{9+x^2}\left[\frac{9+x^2}{9} - 3\right] + C$$

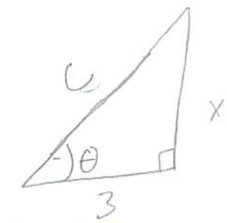
$$= 3\sqrt{9+x^2}\left[\frac{9+x^2}{9} - \frac{27}{9}\right] + C$$

$$= \cancel{3}\sqrt{9+x^2}\left[\frac{9+x^2-27}{\cancel{9}}\right]$$

$$= \boxed{\frac{1}{3}\sqrt{9+x^2}\,(x^2-18) + C}$$

$x = 3\tan\theta$

$\frac{x}{3} = \tan\theta$



$\sqrt{c^2} = \sqrt{3^2 + x^2}$

$c = \sqrt{9+x^2}$

$\cos\theta = \frac{3}{\sqrt{9+x^2}}$

$\sec\theta = \frac{\sqrt{9+x^2}}{3}$

5.4 Fundamental Theorem of Calculus

Let $f(x) = \int_1^x \sin t \, dt$. Find $f'(x)$.

$f'(x) =$

$h(x) = \int_1^{e^x} \ln t \, dt$ Find $h'(x)$

$h'(x) =$

Joey Petersen

$$F(x) = \int_1^x \sin t \, dt$$

$$f'(x) = \sin(x) \frac{dx}{dt}$$

$$= \sin(x) \cdot 1 = \sin x$$

$$h(x) = \int_1^{e^x} \ln t \, dt \qquad u(x) = e^x$$

$$h'(x) = \ln(u(x)) \frac{du}{dx}$$

$$= \ln(u(u(x)) \cdot e^x = \ln(e^x) e^x$$

$$= x e^x$$

Kyle Wijayatilleke
Dr. Parker
MATH151-02
May 9, 2023

5.6 Integration by Parts

1. Evaluate the integral using integration by parts with the indicated choices for u and dv.

$$\int xe^{2x}\,dx \qquad u = x \quad dv = e^{2x}\,dx$$

---

44. First make a substitution (change variables) and then use integration by parts to evaluate each integral.

$$\int_0^{\pi} e^{\cos t} \sin 2t \, dt$$

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5.6 Integration by Parts

Evaluate the integral using integration by parts with the indicated choices for u and dv.

1

$$\int xe^{2x}\,dx \qquad u = x \quad dv = e^{2x}\,dx$$

Solution

$$\int u\,dv = uv - \int v\,du$$

$u = x$ $\quad du = dx$

$dv = e^{2x}dx \quad v = \frac{1}{2}e^{2x}$

$$\int e^{2x}dx = \frac{1}{2}e^{2x}$$

$u = 2x$

$$\int \frac{1}{2}e^{u}du$$

$$u \cdot v - \int v\,du$$

$$= x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}dx$$

$$= x\left(\frac{1}{2}e^{2x}\right) - \frac{1}{4}e^{2x} + C = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

$$\boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C}$$

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5.6 Integration by Parts

First make a substitution (change variables) and then use integration by parts to evaluate each integral.

44 $\displaystyle\int_0^{\pi} e^{\cos t} \sin 2t \, dt$

Solution

$$\int_0^{\pi} e^{\cos t} \sin 2t \, dt = \int_0^{\pi} e^{\cos t} \cdot 2 \sin t \cos t \, dt$$

$$= \int_0^{\pi} e^{\cos t} \cdot 2 \cos t \underbrace{\sin t \, dt}$$

$$\left[\begin{array}{c} u = \cos t \\ du = -\sin t \, dt \\ -du = \sin t \, dt \end{array}\right]$$

$$= \int e^u \cdot 2 \cdot u \, (-du) = -2 \int e^u \cdot u \, du \qquad = tv - \int v \, dt$$

$$= -2\left[ue^u - \int e^u \, du\right] = -2ue^u + 2e^u$$

$$\left[\begin{array}{ll} \text{By Parts} & \\ t = u & v = e^u \\ dt = du & dv = e^u \, du \end{array}\right]$$

$$= \left[-2\cos t \cdot e^{\cos t} + 2e^{\cos t}\right]_0^{\pi}$$

$$= \left(-2\cos \pi e^{\cos \pi} + 2e^{\cos \pi}\right) - \left(-2\cos 0 e^{\cos 0} + 2e^{\cos 0}\right)$$

$$= \left(-2(-1)e^{-1} + 2e^{-1}\right) - \left(-2 \cdot 1 \cdot e^1 + 2e^1\right)$$

$$= 2e^{-1} + 2e^{-1} + \cancel{2e} - \cancel{2e} = 4e^{-1} = \frac{4}{e}$$

$$\boxed{\frac{4}{e}}$$

Name : Varada Rohokale

Section 5-10 - Comparison Test

Q1) $\displaystyle\int_{1}^{\infty} \frac{1+\sin^2 x}{\sqrt{x}}\,dx$

Q2) $\displaystyle\int_{0}^{\infty} \frac{\arctan x}{e^x+2}\,dx$

Name : Varada Rohokale

Section 5.10 - Comparison Test

Q1) $\displaystyle\int_1^\infty \frac{1 + \sin^2 x}{\sqrt{x}}\, dx$

Answer 1) f and g are continuous functions such that $f(x) \geq g(x) \geq 0$ for $x \geq a$

$\therefore \quad 1 + \sin^2 x \geq 1$

$$\frac{1 + \sin^2 x}{\sqrt{x}} \geq \frac{1}{\sqrt{x}}$$

$g(x) = \dfrac{1}{\sqrt{x}}$ diverges by p-series test as $p = \frac{1}{2} < 1$

$\therefore$ by comparison test, $g(x)$ diverges so $f(x)$ must diverge too.

$f(x) = \displaystyle\int_1^\infty \frac{1 + \sin^2 x}{\sqrt{x}}\, dx$ diverges.

Name : Varada Rohokale
Section 5.10 - Comparison test

Q2) $\int_0^{\infty} \frac{\arctan x}{e^x+2} \, dx$

$= \int_0^1 \frac{\arctan x}{e^x+2} \, dx + \int_1^{\infty} \frac{\arctan x}{e^x+2}$

We know $\tan^{-1}(x) \leq \frac{\pi}{2}$

$\therefore \quad \arctan x \leq \frac{\pi}{2}$

$\frac{\arctan x}{e^x+2} \leq \frac{\frac{\pi}{2}}{e^x+2}$

$\frac{\arctan x}{e^x+2} \leq \frac{\frac{\pi}{2}}{e^x}$

$g(x) = \int_1^{\infty} \frac{\pi}{2e^x}$

$= \frac{\pi}{2} \int_1^{\infty} \frac{1}{e^x}$

$\downarrow$
convergent

$\therefore$

g(x) is convergent so f(x) must also be convergent.

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Dr. Parker
MATH-151

5.7 (Partial Fractions)

Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

23a (Exercises). $\frac{2x}{(x+3)(3x+1)}$

23b (Exercises). $\frac{1}{x3+2x^2+x}$

Example 4 (Reading). Find $\int \frac{5x-4}{2x^2+x-1}\,dx$

Derie Woo
Dr. Parker
MATH -151

5.7 (Partial Fractions) *Solutions*

23a. $\frac{2x}{(x+3)(3x+1)} = \boxed{\frac{A}{x+3} + \frac{B}{3x+1}}$

23b. $\frac{1}{x^3+2x^2+x} = \frac{1}{x(x^2+2x+1)} = \frac{1}{x(x+1)^2}$

$= \boxed{\frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}}$

Example 4.

$\int \frac{5x-4}{2x^2+x-1} \rightarrow$ We know that $2x^2+x-1 = (x+1)(2x-1) \rightarrow$ So $\frac{5x-4}{2x^2+x-1} = \frac{A}{(x+1)} + \frac{B}{(2x-1)}$

$= \frac{5x-4}{\cancel{2x^2+x-1}} = \frac{A(2x-1)+B(x+1)}{\cancel{2x^2+x-1}}$

$= 5x-4 = A(2x-1)+B(x+1) = 2Ax - A + Bx + B = (2A+B)x + (-A+B)$

This means

$5x-4 = (2A+B)x + (-A+B)$, so $\begin{cases} 2A+B=5 \\ -A+B=-4 \end{cases} \rightarrow \begin{matrix} A=3 \\ B=-1 \end{matrix}$

Lastly,

$\int \frac{5x-4}{2x^2+x-1}dx = \int \left(\frac{3}{x+1} - \frac{1}{2x-1}\right)dx = \boxed{3\ln|x+1| - \frac{1}{2}\ln|2x-1| + C}$

Patrick Matthews 5.10 (Type 1)

Determine whether the integral is Convergent or Divergent. Evaluate those that are convergent.

5.) $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$

17.) $\int_1^{\infty} \frac{e^{-1/x}}{x^2}\,dx$

5.) $\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$ Patrick Matthews 5.10 Type I

$\int_3^\infty (x-2)^{-3/2}\,dx$

$\lim_{B\to\infty} \int_3^B (x-2)^{-3/2}\,dx$

$\lim_{B\to\infty} \left[-2(x-2)^{-\frac{1}{2}}\right]_3^B$

$\lim_{B\to\infty} \left[-2\frac{1}{\sqrt{B-2}} - \left(-2\frac{1}{\sqrt{3-2}}\right)\right]$

$\lim_{B\to\infty} [0 - -2] = 2$

Convergent, 2

17.) $\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$

$\lim_{B\to\infty} \int_1^B e^{-x^{-1}} \cdot x^{-2}\,dx$

$\lim_{B\to\infty} \int_1^B e^u\,du$ $u = -x^{-1}$ $du = x^{-2}dx$

$\lim_{B\to\infty} [e^u]_1^B$

$\lim_{B\to\infty} [e^{-x^{-1}}]_1^B$

$\lim_{B\to\infty} [e^{-\frac{1}{B}} - e^{-1^{-1}}] = 1 - \frac{1}{e}$

Convergent, $1 - \frac{1}{e}$

Section 5.3 - Evaluating Definite Integrals

Caley Tamondong

1) $\int_0^1 (x^{10} + 10^x)\,dx$

2) $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

Solutions - 5.3

Caley Tamondong

1) $\int_0^1 (x^{10} + 10^x)\,dx$

$= \left[\frac{x^{11}}{11} + \frac{10^x}{\ln(10)}\right]_0^1$

$= \left(\frac{1}{11} + \frac{10}{\ln(10)}\right) - \left(0 + \frac{1}{\ln(10)}\right)$

$= \frac{1}{11} + \frac{9}{\ln(10)}$

2) $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

$= \int_0^{\pi/4} \left(\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\right) d\theta$

$= \int_0^{\pi/4} (\sec^2\theta + 1)\,d\theta$

$= \left[\tan(x) + x\right]_0^{\pi/4}$

$= \left(\tan\pi/4 + \pi/4\right) - \left(\tan(0) + 0\right)$

$= 1 + \pi/4$

Julia Marie Greely

5.2

1) If $f(x) = x^2 - 2x$, $0 \leq x \leq 3$, evaluate the Riemann sum with $n = 6$ subintervals, taking the sample points to be right endpoints. What does the Riemann sum represent? Illustrate with a diagram.

2) Use the Midpoint Rule with given value of n to approximate the integral. Round the answer to the four decimal places.

$$\int_1^5 4x^5 e^{-x}\,dx, \quad n = 4$$

Julia Marie Greely

1) If $f(x) = x^2 - 2x$, $0 \leq x \leq 3$, evaluate the Riemann sum with $n=6$ subintervals, taking the sample points to be right endpoints. What does the Riemann sum represent? Illustrate with a diagram.

$f(x) = x^2 - 2x$

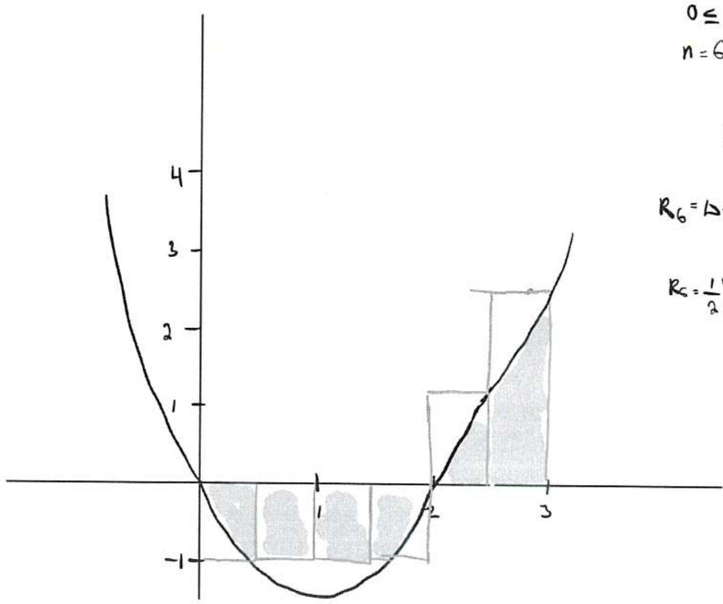
$0 \leq x \leq 3$

$n = 6 \rightarrow \Delta x = \frac{b-a}{n}$

$\Delta x = \frac{3-0}{6} = \frac{1}{2} \rightarrow \Delta x = \frac{1}{2}$

$R_6 = \Delta x\left(f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2}) + f(3)\right)$

$R_6 = \frac{1}{2}\left[-\frac{3}{4} - 1 - \frac{3}{4} + 0 + \frac{5}{4} + 3\right] = \boxed{\frac{7}{8}}$



2) Use the Midpoint Rule with given value of n to approximate the integral. Round the answer to the four decimal places.

$\int_1^5 4x^5 e^{-x}\,dx$, $n=4$

Midpoint rule:

$\int_a^b f(x)dx \approx h\left[f(x_0^*) + f(x_1^*) + \ldots\right]$

where $h = \frac{b-a}{n}$ and x^*_0 = mid point of subinterval

$h = \frac{5-1}{4} = 1$

Subintervals: $(1,2), (2,3), (3,4), (4,5)$

$M_4 = \int_1^5 4x^5 e^{-x}dx \approx h\left[4(1.5)^5 e^{-1.5} + 4(2.5)^5 e^{-2.5} + 4(3.5)^5 e^{-3.5} + 4(4.5)^5 e^{-4.5}\right]$

$= 1\left[6.7775 + 32.064 + 63.44 + 81.996\right]$

$= \boxed{184.2775}$

Anthony Bonilla Sect. 5.1

1. Let $f(x) = \frac{1}{x}$. Estimate the area of the region bounded by the graph of f, the x-axis, and the lines $x=1$ & $x=2$ using four rectangles & right and left endpoints.

2. The hyperloop, designed by Elon Musk, is a type of bullet train w/ pods carrying passengers through underground tubes. Recently, a team of students designed a pod that reached a top speed of 463 km/h in the annual Space X Hyperloop pod competition. Suppose the table below gives the velocity of the pod at various times. Use the data to estimate the distance the pod traveled in the first 30 seconds (3 midpoints).

Time (s)	Velocity (ft/s)
0	0
5	179
10	239
15	277
20	304
25	326
30	343

Anthony Bonilla Sect. 5.1

1. $f(x) = 1/x$ $\quad \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$

$x = 1$

$x = 2$ $\quad x_0 = 1 \quad x_2 = 1.5 \quad x_4 = 2$

four rectagles $\quad x_1 = 1.25 \quad x_3 = 1.75$

a) Right Endpoints

$$\frac{1}{4}\left[f(1.25) + f(1.5) + f(1.75) + f(2)\right]$$

$$= \frac{1}{4}\left[4/5 + \frac{2}{3} + 4/7 + \frac{1}{2}\right] =$$

$$= \frac{1}{4}\left[\frac{533}{210}\right] = \frac{533}{840}$$

$$\boxed{A_R \approx \frac{533}{840} \text{ or } 0.635}$$

b) Left Endpoints

$$\frac{1}{4}\left[f(1) + f(1.25) + f(1.5) + f(1.75)\right]$$

$$= \frac{1}{4}\left[1 + 4/5 + 2/3 + 4/7\right]$$

$$= \frac{1}{4}\left[\frac{319}{105}\right] = \frac{319}{420}$$

$$\boxed{A_L \approx \frac{319}{420} \text{ or } 0.76}$$

Anthony Bonilla Sect. 5.1

2. $x = 0$ $\frac{30-0}{3} = 10$

$x = 30$

3 midpoint rectangles

$x = 0 \rightarrow x = 10$

$x = 5$

$x = 10 \rightarrow x = 20$

$x = 15$

$x = 20 \rightarrow x = 30$

$x = 25$

$x_0 = 5 \quad x_1 = 15 \quad x_2 = 25$

$$10[f(5) + f(15) + f(25)]$$

$$= 10[179 + 277 + 326]$$

$$= 10[782] = 7820$$

$A_m \approx 7820$

Using midpoint rectangles, the estimated distance the pod traveled in the first 30 seconds is 7820 ft.

Use the midpoint rule, Simpson's rule to approximate the given integral with the specified value of n. Round to 6 decimal places. Compare your results to the actual value to determine the error in each approximation.

1. $\int_0^2 \frac{x}{1+x^2}\,dx \quad n=10$

Use the Trapezoidal Rule, the Midpoint Rule and Simpson's Rule to approximate the given integral with the specified value of n. Round your answer to six decimal places.

2. $\int_0^2 \frac{\ln x}{1+x}\,dx \quad n=10$

Rebecca Leiva Section 5.9

1. $M_{10} \approx .806598$

$S_{10} \approx .804779$

$\int_0^2 \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln 5 \approx .804719$

$E_M \approx -.001879$ $E_S \approx -0.000060$

2. $T_{10} \approx .146879$

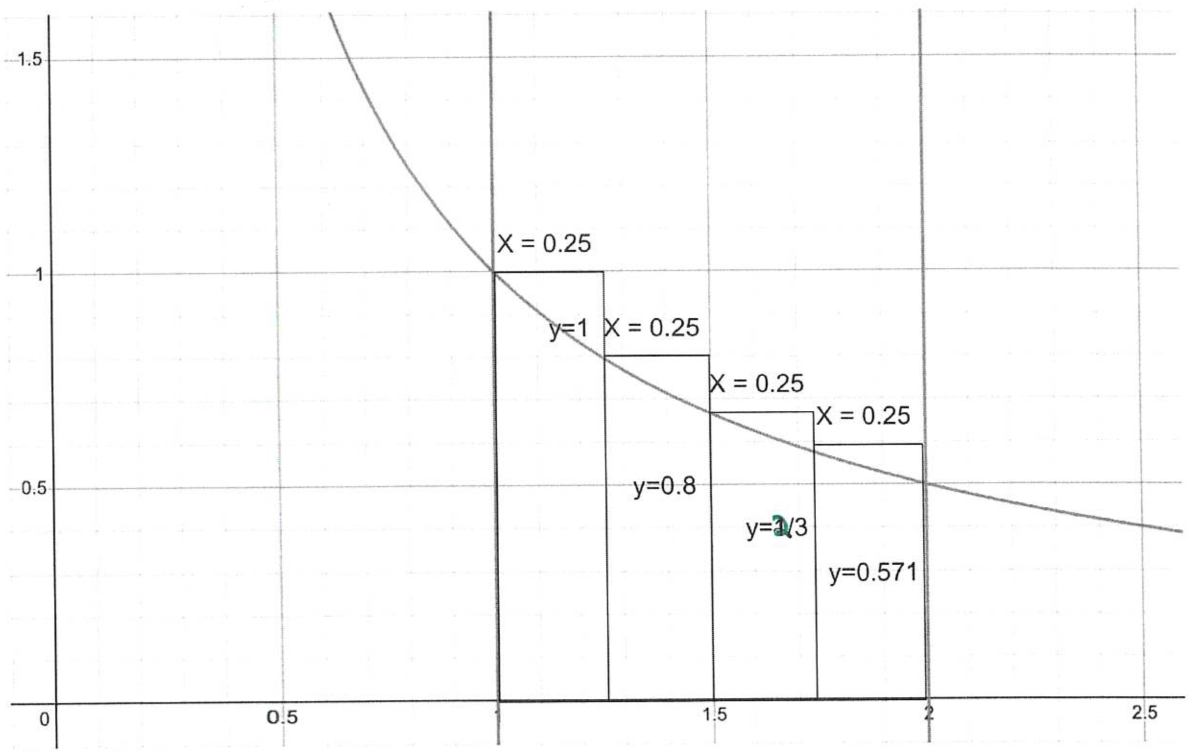
$M_{10} \approx .147391$

$S_{10} \approx .147219$

Christopher Lee
Chapter 5.1
Section 1
3. Left $f(x)=\frac{1}{x}$

Finding the Area:
Work Right Endpoint:

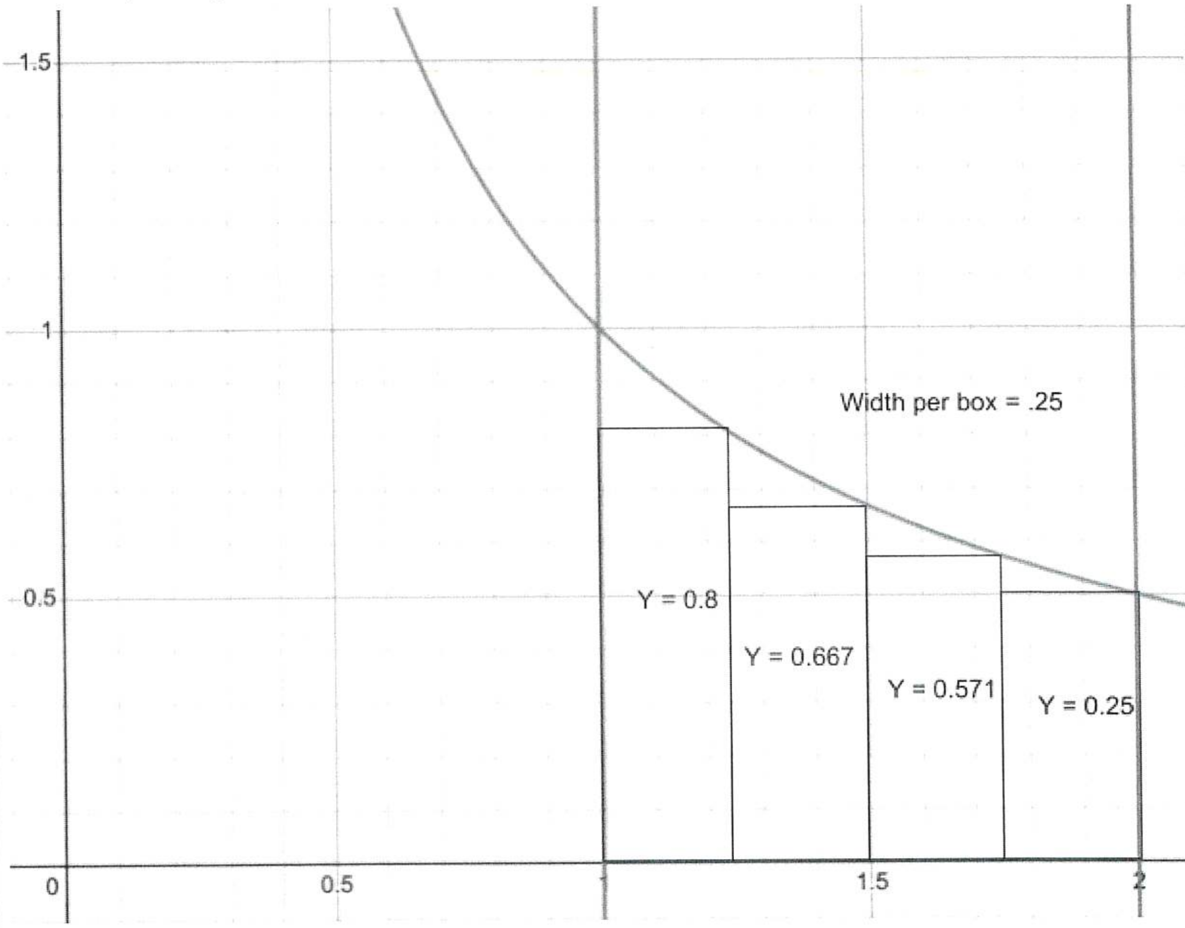
a. Estimate the area of the region bounded by the graph of f, the x-axis, and the lines x = 1 and x = 2 using four rectangles and right endpoints. Sketch the graph of f and the rectangles. Is your estimate an underestimate or an overestimate of the true area? Explain your reasoning.



The answer Above would be an overestimation as it's messing the graph from above the line. Thus you would get an overestimation of the graph.

Christopher Lee
Chapter 5.1
Section 1

Work Left Endpoint:
b. Repeat part(a) using left endpoints.



The left endpoints method is an underestimation of the graph as it measures below the line on the graph.

Christopher Lee
Chapter 5.1
Section 1

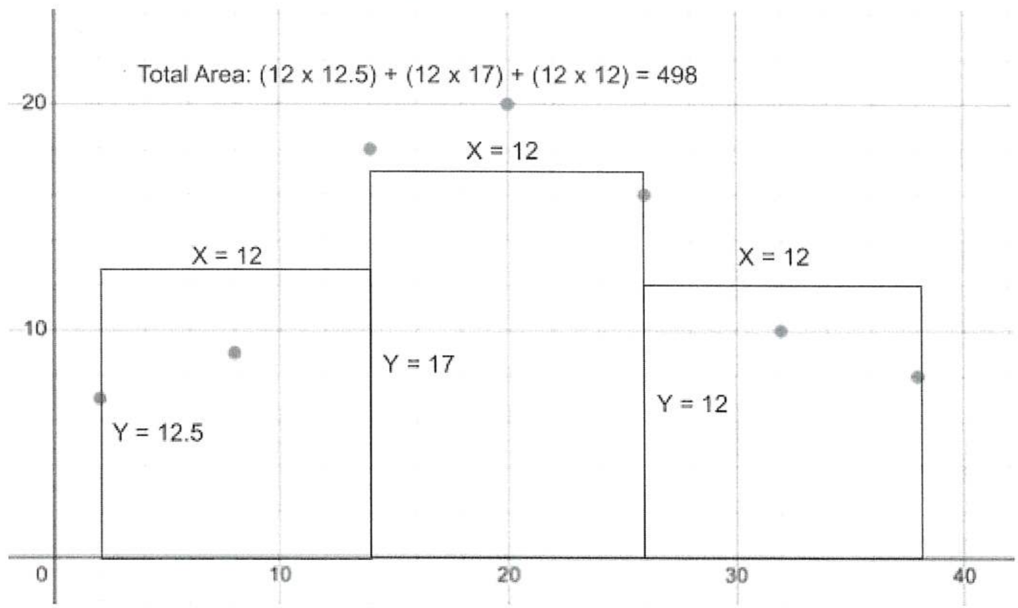
14. The function f is positive and selected values of f(x) over the interval [2,38] are given in the table:

x	2	8	14	20	26	32	38
f(x)	7	9	18	20	16	10	8

Suppose this table is used to compute an estimate of the area under the graph of f from x = 2 to x = 38 using three rectangles and midpoints. Find the value of this approximation.

<u>USING 3 RECTANGLES with Midpoint:</u>

Visual Representation/Work:



12 12 12

x	2	8	14	20	26	32	38
f(x)	7	9	18	20	16	10	8

(18+7)/2 = 12.5 (16+18)/2 = 17 (16+8)/2 = 12

(12.5)
(17)
+(12)

= 41.5
x 12

=498

Christopher Lee
Chapter 5.1
Section 1

dx

5.10 (Type 2)

$$\int_2^5 \frac{1}{\sqrt{x-2}}\, dx$$

J+

5.10 (Type 2)

$$\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$$

$$= \lim_{t \to 2^+} \int_t^5 \frac{1}{\sqrt{x-2}}\,dx$$

$$= \lim_{t \to 2^+} \left[2\sqrt{x-2} \right]_t^5$$

$$= \lim_{t \to 2^+} 2\left[\sqrt{3} - \sqrt{t-2} \right]$$

$$= 2\sqrt{3}$$

33

5.10 (Type 2)

$$\int_{-2}^{3} \frac{1}{x^4}\,dx$$

JX

5.10 (Type 2)

$$\int_{-2}^{3} \frac{1}{x^4}\,dx$$

$$= \lim_{a \to 0^-} \left[-\frac{1}{3x^3}\right]_{-2}^{a} + \lim_{b \to 0^+} \left[-\frac{1}{3x^3}\right]_{b}^{3}$$

$$= \underbrace{\lim_{a \to 0^-} -\frac{1}{3a^3}}_{\infty} + \frac{1}{3 \cdot -8}$$

$$\to -\frac{1}{3 \cdot 27} + \underbrace{\lim_{b \to 0^+} \frac{1}{3b^3}}_{\infty}$$

$= \infty +$ arbitrary $-$ arbitrary $+ \infty$

$= \infty$

divergent

5.3 evaluate the integral

#4 $\int_0^3 (1+6w^2-10w^4)\,dw$

Find the general indefinite integral

#55 $\int \sec t(\sec t + \tan t)\,dt$

5.3 #4 evaluate the integral

$$\int_0^3 (1+6w^2-10u^4)\,dw$$

$$\int 1+6w^2-10w^4\,dw$$

$$\int 1\,dw+\int 6w^2\,dw-\int 10w^4\,dw$$

$$w+2w^3-2w^5$$

$$(w+2w^3-2w^5)\Big|_0^3$$

$$3+2\cdot 3^3-2\cdot 3^5-(0+2\cdot 0^3-2\cdot 0^5)$$

$$=-429$$

5.3 #55 Find the general indefinit integral

$$\int \sec t(\sec t + \tan t)\,dt$$

$$\int \frac{1}{\cos(t)}\left(\frac{1}{\cos(t)} + \tan(t)\right)dt$$

$$= \int \frac{1 + \tan(t)\cos(t)}{\cos^2(t)}\,dt$$

$$= \int \frac{1}{\cos^2(t)}\,dt + \frac{\tan(t)}{\cos(t)}\,dt$$

$$= \int \frac{1}{\cos^2(t)}\,dt \quad = \tan(t)$$

$$= \int \frac{\tan(t)}{\cos(t)} \quad = \sec(t)$$

$$= \tan(t) + \sec(t) + C$$

Evaluate the indefinite integral 5.5

$$\int \cos^3\theta \sin\theta \, d\theta$$

$$\int \frac{\sec^2 x}{\tan x} \, dx$$

Solution: 5.5

$$\int \cos^3\theta \sin\theta \, d\theta$$

$U = \cos\theta d\theta$

$\frac{du}{-\sin\theta} = \frac{-\sin\theta d\theta}{-\sin\theta}$

$$\int u^3 \sin\theta \frac{du}{-\sin\theta} = \int u^3 \cancel{\sin\theta} \frac{du}{\cancel{-\sin\theta}}$$

$-1\int u^3 du \rightarrow -1\left(\frac{u^4}{4}\right)+C \rightarrow$ Plug $\cos\theta$ back in $\rightarrow$

$$\underbrace{= -1\left(\frac{\cos^4\theta}{4}\right)+C}$$

Solution: 5.5

$$\int \frac{\sec^2 x}{\tan x}\,dx \qquad \begin{aligned} u &= \tan x \\ \frac{du}{\sec^2 x} &= \frac{\sec^2 x\,dx}{\sec^2 x} \end{aligned}$$

$$\int \frac{\sec^2 x}{u} \cdot \frac{du}{\sec^2 x} = \int \frac{\cancel{\sec^2 x}}{u} \cdot \frac{du}{\cancel{\sec^2 x}} \rightarrow$$

$\int \frac{1}{u}\,du \rightarrow \ln(u)+c \rightarrow$ Subshitute tanx back in.

$$= \ln|\tan x| + c$$

3. evaluate the integral

$$\int x \cos 5x \, dx$$

41. $\int e^{\sqrt{x}} \, dx$

$$\int x \cos 5x \, dx$$

Section 5.6
Angelica Click

$u = x$

$du = 1\,dx$

$dv = \cos(5x)$

$v = (\frac{1}{5})\sin(5x)$

$uv - \int v \cdot du$

$x \cdot (\frac{1}{5})\sin(5x) - \int \frac{1}{5}\sin(5x)\, dx$

$$\boxed{\frac{1}{5} x \sin 5x + \frac{1}{25}\cos 5x + C}$$

41. $\int e^{\sqrt{x}} \; dx$

$\int e^{\sqrt{x}} dx = \int e^{t} \cdot 2t\,dt = 2\int t \cdot e^{t} dt =$

$t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\, dx = \frac{1}{2t}\, dx$

$2t\,dt = dx$

$\int u\,dv = uv - \int v\,du$

$u = t$

$du = dt$

$dv = e^{t} dt$

$v = e^{t}$

$= 2(t \cdot e^{t} - e^{t}) = 2e^{t} \cdot (t-1) = \boxed{2e^{\sqrt{x}} \cdot (\sqrt{x}-1) + C}$