If you deposit \$100 at the end of every month (1)
into an account that pays 3% Interest compounded monthly, the amount of total interest accumulated after n months is given by the Sequence

$$I_n = 100\left(\frac{1.0025^n - 1}{0.0025} - n\right)$$

a. Find the first six terms of the Sequence.

b. How much interest will you have earned after 2 years?

8.1 Josiah Lee

Solution

a)

$$I_1 = 100\left(\frac{1.0025^1 - 1}{0.0025} - 1\right) = 0$$

$$I_2 = 100\left(\frac{1.0025^2 - 1}{0.0025} - 2\right) = 0.25$$

$$I_3 = 100\left(\frac{1.0025^3 - 1}{0.0025} - 3\right) = 0.751$$

$$I_4 = 1.5025$$

$$I_5 = 2.5063$$

$$I_6 = 3.7625$$

B)

$$I_{24} = 100\left(\frac{1.0025^{24} - 1}{0.0025} - 24\right)$$

$$= 70.282$$

8.1 Josiah Lee

Determine whether the given sequence converges or diverges. If it converges, find the limit. ②

$$a_n = \left\{ \frac{(2n-1)!}{(2n+1)!} \right\}$$

8.1 Josiah Lee

Solution

②

$$a_n = \frac{(2n-1)!}{(2n+1)!}$$

$$\frac{(2n-1)!}{(2n+1)!} = \frac{\cancel{(2n-1)!}}{(2n+1)(2n)\cancel{(2n-1)!}} = \frac{1}{(2n+1)(2n)} \rightarrow \frac{1}{2x(2x+1)}$$

$$\lim_{x \to 0} \frac{1}{2x(2x+1)} = 0$$

The Solution <u>Converges to 0</u>

8.1 Josiah

Use the direct comparison test to determine convergence

1) $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

2) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n+1}$

Emily Citin section 2
8.3

1) $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

$$\frac{2n^3+1}{n} > \frac{n^3}{n}$$

$$\frac{n}{2n^3+1} < \frac{n}{n^3} = \frac{1}{n^2}$$

this series converges

2) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n+1}$

$$\frac{0 \leq \cos^2 n \leq 1}{n+1}$$

$$\frac{1}{n^2+1} < \frac{1}{n^2}$$

convergent by p-series test

Final Review Section 8.4 Absolute/Conditional Convergence

(Hunter Lysholm)

1) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

#27

2) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n}\right)^{n}$

Final Review Section 8.4 Absolute/Conditional Convergence

(Hunter Lysholm)

1) #27 $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} = \sum \left| \frac{(-1)^{n-1}}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$; Divergent due to the p-series test $(P \leq 1)$

- $\lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0$ ✓

- Decreasing; $b_n = \frac{1}{n^{1/2}}$ $\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \ldots$ ✓

Conditionally Convergent.

2) $\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n} \right)^n = \lim_{n \to \infty} \left[\left(\frac{1-n}{2+3n} \right)^n \right]^{1/n}$ Use Root test

$\frac{1}{n} = 0$

$= \lim_{n \to \infty} \left| \left(\frac{n(\frac{1}{n}-1)}{n(\frac{2}{n}+3)} \right) \right|$

$= \lim_{n \to \infty} \left| -\frac{1}{3} \right|$

$L < 1$ = Converges
$L > 1$ = Diverges
$L = 1$ = Fails

$= \lim_{n \to \infty} a_n = \frac{1}{3} < 1$

Converges Absoluvetly by the Root test

Noemi
Hernandez

8.2:

28) Determine if the series is convergent or divergent. If it is convergent, find its sum.

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \cdots$$

46) Determine whether the series is convergent or divergent by expressing the n^{th} partial sum S_n as a telescoping sum. If it's convergent find its sum

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

Noemi
Hernandez

28)

$$\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\dots$$

$$\sum_{n=1}^{\infty}\frac{1}{3n}=\frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$$

This series diverges b/c $\sum_{n=1}^{\infty}\frac{1}{n}$ is a harmonic Series

46) $\sum_{n=1}^{\infty}\frac{3}{n(n+3)}$

$$\frac{3}{n(n+3)}=\frac{A}{n}+\frac{B}{n+3} \Rightarrow 3=A(n+3)+Bn$$

$n=0$: $3=3A \Rightarrow A=1$

$n=-3$: $3=-3B \Rightarrow B=-1$

$$\sum_{n=1}^{\infty}\frac{1}{n}-\frac{1}{n+3}$$

$$S_n=\sum_{i=1}^{n}\frac{1}{i}-\frac{1}{i+3}=\left(1-\frac{1}{4}\right)+\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{3}-\frac{1}{6}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\dots$$

$$\sum_{n=1}^{\infty}\frac{3}{n(n+3)}=\lim_{n\to\infty}S_n=1+\frac{1}{2}+\frac{1}{3}=\boxed{\frac{11}{6}}$$

Jen T. Sect. 2

8.2 Geometric Series

$\boxed{1}$ $3 - \frac{3}{5} + \frac{3}{25} - \frac{3}{125} + \ldots$

Does the series converge or diverge?

$\boxed{2}$ Rewrite the number $2.3\overline{17}$ as a ratio of integers.

Jen T. sect 2
Answers 8.2

$\boxed{1}$ $3 - \frac{3}{5} + \frac{3}{25} - \frac{3}{125} + \dots$ → geometric series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3}{5^{n-1}}$

$r = \frac{1}{5}$

$|r| < 1$: converges

$|r| \geq 1$: diverges

∴ the series converges by geometric series test $r = \frac{1}{5}$.

$\boxed{2}$ $2.3\overline{17} = 2.3 + \frac{17}{10^3} + \frac{17}{10^5} + \frac{17}{10^7} + \dots$

After the first term, it is a geometric series

$a = \frac{17}{10^3}$ and $r = \frac{1}{10^2} < 1$

- geometric series converges to $\frac{a}{1-r}$

$$2.3\overline{17} = 2.3 + \frac{\frac{17}{10^3}}{1 - \frac{1}{10^2}} = 2.3 + \frac{\frac{17}{1000}}{\frac{99}{100}}$$

∴ $2.3\overline{17} = \frac{1147}{495}$

EXAMPLE #1 : USE MACLAURIN SERIES TO EVALUATE THE LIMIT

$$\lim_{x \to 0} \frac{\sin x - x - \frac{1}{6}x^3}{x^5}$$

EXAMPLE #2 : USE MACLAURIN SERIES TO EVALUATE THE LIMIT

$$\lim_{x \to 0} \frac{e^{x^2}\sin x - x}{x^3}$$

EXAMPLE #1 SOLUTION ON PAGE #2
EXAMPLE #2 SOLUTION ON PAGE #3

EXAMPLE #1: USE MACLAURIN SERIES TO EVALUATE THE LIMIT:

$$\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

1) REMEMBER:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

MACLAURIN SERIES FOR SIN x

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots$$

2). PLUG IN FIRST FEW TERMS FOR SINx IN THE LIMIT

$$\lim_{x \to 0} \left[\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \right] - x + \frac{1}{6}x^3 \right] \frac{1}{x^5}$$

3) CANCEL OUT NECESSARY TERMS

$$\lim_{x \to 0} \left[\left[\cancel{x} - \cancel{\frac{x^3}{3!}} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \right] - \cancel{x} + \cancel{\frac{1}{6}x^3} \right] \frac{1}{x^5}$$

3! EQUALS 6, SO WE CAN CANCEL OUT

$$= \lim_{x \to 0} \left[\frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots \right] \cdot \frac{1}{x^5}$$

4) DISTRIBUTE THE $\frac{1}{x^5}$ INTO THE SERIES

$$= \lim_{x \to 0} \left[\frac{1}{5!} - \frac{x^2}{7!} + \frac{x^4}{9!} - \frac{x^6}{11!} + \cdots \right]$$

5) EVALUATE WITH $x \to 0$

$$\lim_{x \to 0} \left[\frac{1}{5!} \underbrace{- \frac{x^2}{7!} + \frac{x^4}{9!} - \frac{x^6}{11!} + \cdots}_{} \right] = \frac{1}{5!} = \boxed{\frac{1}{120}}$$

WHEN $x \to 0$, THESE WILL ALL EQUAL ZERO, MEANING THEY DO NOT HAVE ANY VALUE TO US

EXAMPLE 2: USE MACLAURIN SERIES TO EVALUATE THE LIMIT:

$$\lim_{x \to 0} \frac{e^{x^2} \sin x - x}{x^3}$$

1) REMEMBER:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

THEREFORE:

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots$$

AND

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

2) MULTIPLY e^{x^2} AND $\sin x$

$$e^{x^2} \sin x = \left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} \dots\right)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots\right)$$

↳ DISTRIBUTE EACH TERM, TYPICAL INFINITE FOIL

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + x^3 - \frac{x^5}{3!} + \dots \frac{x^5}{2!} + \dots$$

3) INPUT INTO THE LIMIT EQUATION AND CANCEL

$$\lim_{x \to 0} \left[\left[\cancel{x} - \frac{\cancel{x^3}}{3!} + \cancel{x^3} + \frac{x^5}{5!} - \frac{x^5}{3!} + \dots\right] - \cancel{x}\right] \cdot \frac{1}{\cancel{x^3}}\right]$$

4) SOLVE

$$= \lim_{x \to 0} \left[-\frac{1}{3!} + 1 + \underbrace{\frac{x^2}{5!} - \frac{x^2}{3!} + \dots}\right]$$

$$= \frac{-1}{3!} + 1 = \boxed{\frac{5}{6}}$$

THESE WILL ALL GO TO ZERO, SO WE DON'T HAVE TO WORRY ABOUT THEM

1.) Determine whether the series is absolutely convergent.

$$\sum_{n=1}^{\infty} \frac{(-2)^n n!}{(2n)!}$$

Felipe Gil
MATH-151-02
8.4 (ratio/root test)

2.) Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$$

Felipe Gil
MATH-151-02
8.4 (ratio/root test)

1.) $\sum_{n=1}^{\infty} \frac{(-2)^n n!}{(2n)!}$

Ratio Test:

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n \to \infty} \left| \frac{(-2)^{n+1} (n+1)!}{(2(n+1))!} \cdot \frac{(2n)!}{(-2)^n n!} \right|$$

$$= \lim_{n \to \infty} \left| \frac{(-2)(n+1)(2n)!}{(2n+2)!} \right|$$

b/c $\frac{(-2)^{n+1}}{(-2)^n} = (-2)^{n+1-n} = (-2)^1$

and $\frac{(n+1)!}{n!} = \frac{(n+1)(n!)}{n!} = n+1$

$$= \lim_{n \to \infty} \frac{2(n+1)}{(2n+2)(2n+1)}$$

b/c $\frac{(2n)!}{(2n+2)!} = \frac{(2n)!}{(2n+2)(2n+1)(2n)!} = \frac{1}{(2n+2)(2n+1)}$

$$= \lim_{n \to \infty} \frac{2n+2}{(2n+2)(2n+1)}$$

$$= \lim_{n \to \infty} \frac{1}{2n+1} = 0$$

b/c denominator gets infinitely large while nomerator remains constant.

Thus, since $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 < 1$, then by the ratio test the series is absolutely convergent.

Felipe Gil
MATH-151-02
8.4 (ratio/root test)

2.) $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$

Root Test:

$\lim_{n \to \infty} \sqrt[n]{|a_n|}$

$= \lim_{n \to \infty} \sqrt[n]{\left|\left(\frac{-2n}{n+1}\right)^{5n}\right|}$

$= \lim_{n \to \infty} \left|\left(\frac{-2n}{n+1}\right)^{5}\right|$ b/c $\sqrt[a]{x^b} = x^{\frac{b}{a}}$

$= \lim_{n \to \infty} \left(\frac{2n}{n+1}\right)^{5}$ b/c $\left|\left(\frac{-2n}{n+1}\right)^{5}\right|$ will always be positive

$= \lim_{n \to \infty} \frac{(2n)^5}{(n+1)^5}$

$= \lim_{n \to \infty} \frac{32n^5}{n^5}$ b/c since n is getting arbitrarily large, we can focus on the terms of highest degree.

$= \lim_{n \to \infty} 32$

b/c $\frac{32n^5 \cdot \frac{1}{n^5}}{n^5 \cdot \frac{1}{n^5}} = 32$

$= 32$

Thus, since $\lim_{n \to \infty} \sqrt[n]{|a_n|} = 32 > 1$, then by the root test the series diverges.

Kaitlin Divinsky, 8.3 (Integral Test)

Use the Integral Test to determine whether the series is convergent or divergent.

1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

2) $\sum_{n=1}^{\infty} \frac{1}{(3n-1)^4}$

Kaitlin Divinsky, 8.3 (Integral Test)

Use the Integral Test to determine whether the series is convergent or divergent.

1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

$$\int_1^{\infty} \frac{1}{\sqrt[3]{x}}\,dx \longrightarrow \int_1^{\infty} x^{-1/3}\,dx$$

$$\lim_{B\to\infty} \int_1^{B} x^{-1/3}\,dx$$

$$\lim_{B\to\infty} \frac{3}{2}x^{2/3}\Big|_1^B$$

$\boxed{\text{divergent}}$

2) $\sum_{n=1}^{\infty} \frac{1}{(3n-1)^4}$

$$\int_1^{\infty} \frac{1}{(3x-1)^4}\,dx \longrightarrow \int_1^{\infty} (3x-1)^{-4}\,dx$$

$$\lim_{B\to\infty} \int_1^{B} (3x-1)^{-4}\,dx$$

$$\lim_{B\to\infty} -\frac{1}{9(3x-1)^3}\Big|_1^B$$

$\boxed{\text{convergent}}$

$\int (3x-1)^{-4}\,dx$ $\quad u = 3x-1$

$= \frac{1}{3}\int u^{-4}\,dx$ $\quad du = 3\,dx$

$\quad \frac{1}{3}du = dx$

$= -\frac{1}{3}\,\frac{u^{-3}}{3}$

$= -\frac{1}{3}\,\frac{(3x-1)^{-3}}{3} = -\frac{1}{9(3x-1)^3}$

Noah Huang

Review 8.4: Alternating Series

If the series

$$\sum_{n=0}^{\infty} (-1)^n a_n = a_0 - a_1 + a_2 - a_3 + \dots \qquad a_n > 0$$

satisfies both

1.) $a_n \geq a_{n+1}$, eventually

2.) $\lim_{n \to \infty} a_n = 0$

The series converges

Ex.)

1.) $$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

2.) $$-\frac{2}{5} + \frac{4}{6} - \frac{6}{7} + \frac{8}{8} - \frac{10}{9} + \dots$$

Noah Huang

Review 8.4: Alternating Series

1.) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

Alternating Series test

Is $a_{n+1} \leq a_n$?

doesn't increase

increased by 1

$\frac{1}{n+1} \leq \frac{1}{n}$ yes ✓

Is $\lim_{n\to\infty} a_n = 0$?

$\lim_{n\to\infty} \frac{1}{n} = 0$ yes ✓

Therefore, this series converges

2.) $-\frac{2}{5} + \frac{4}{6} - \frac{6}{7} + \frac{8}{8} - \frac{10}{9} + \ldots$

First find the series $\sum_{n=0}^{\infty} a_n$

$\sum_{n=1}^{\infty} \frac{(-1)^n 2n}{n+4}$

Alternating series test

Is $a_{n+1} \leq a_n$?

$\frac{2n}{n+4} \geq \frac{2n+2}{n+5}$ no X

increased by 2

increased by 1

Is $\lim_{n\to\infty} a_n = 0$?

$\lim_{n\to\infty} \frac{2n}{n+4} \rightarrow$ L'Hopital $\lim_{n\to\infty} \frac{2}{1} = 2 \neq 0$ no X

Therefore, this series diverges

8.6 Representations of Functions as a power series

Alissa Vidal

Find a powerseries representation for the function and determine the interval of convergance.

6. $f(x) = \dfrac{2}{3-x}$

Evaluate the indefinite integral as a power series. Find the radius of convergance.

25. $\displaystyle\int \frac{x}{1+x^3}\,dx$

---

solution: 6. $f(x) = \frac{2}{3}\left(\dfrac{1}{1-\frac{x}{3}}\right) = \frac{2}{3}\displaystyle\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n$

$$= 2\sum_{n=0}^{\infty}\frac{1}{3^{n+1}}x^n$$

$\left|\frac{x}{3}\right| < 1$ $\quad |x| < 3$ $\quad R = 3$ $\quad -3 < x < 3$ $\quad I = (-3, 3)$

solution: 25. $\displaystyle\int \frac{x}{1+x^3}\,dx \rightarrow \frac{x}{1+x^3} = x \cdot \frac{1}{1-(-x^3)} = x\sum_{n=0}^{\infty}(-x^3)^n$

$|-x^3| < 1$ $\quad |x| < 1$ $\quad R = 1$

$\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{3n+1}$ $\qquad \displaystyle\int \frac{x}{1+x^3}\,dx = C + \sum_{n=0}^{\infty}(-1)^n\frac{x^{3n+2}}{3n+2}$ $\quad R = 1$

~~Find a pow~~

4. $f(x)$

using geometric series + ~~differentiation and integration~~ power series

~~8.6 Representations of Functions~~ as power series

Sydney, 8.7 Taylor and MacLaurin Series

Find the Taylor Series

$f(x) = \sin\left(\frac{\pi x}{4}\right)$

Solve the indefinite integral

$\int x \sin\left(\frac{x}{4}\right) dx$

Sydney, 8.7 Taylor and MacLaurin Series

$$f(x) = \sin\left(\frac{\pi x}{4}\right)$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin\left(\frac{\pi x}{4}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1} \pi^{2n+1}}{(2n+1)!\, 4^{2n+1}}$$

$$\int x \sin\left(\frac{x^2}{4}\right) dx$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin\left(\frac{x^2}{4}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!\, 4^{2n+1}}$$

$$x\sin\left(\frac{x^2}{4}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)!\, 4^{2n+1}}$$

$$\int x\sin\left(\frac{x^2}{4}\right) = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\, 4^{2n+1}} \cdot \frac{x^{4n+3}}{(4n+3)}$$

8.7 Euler's Theorem

Isabella Pardo

question 1: calculate the following

$$e^{\pi i/6}$$

question 2: show double angle formulas using Euler's theorem

$$e^{i(2x)}$$

8.7 Euler's Theorem

(solutions) Isabella Pardo

$e^{\pi i/6}$

$e^{ix} = \cos x + i \sin x$

$e^{i\pi/6} = \cos(\pi/6) + i \sin(\pi/6)$

$= \frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2} + \frac{i}{2} = \boxed{\frac{\sqrt{3}+i}{2}}$

$e^{i(2x)}$

$e^{ix} = \cos x + i \sin x$

$e^{i(2x)} = \cos(2x)\, i \sin(2x)$

$e^{i(2x)} = e^{(ix)^2}.$

$=(\cos(x) + i\sin(x))^2$

$= \cos^2(x) - \sin^2(x) + 2\cos(x)\, i \sin(x)$

$= (\cos^2(x) - \sin(x)) + i(2\cos(x)\sin(x))$

so, $\cos(2x) = \cos^2(x) - \sin^2(x)$ & $\sin(2x) = 2\sin(x)\cos(x)$