

Calculus
Answers

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12/7/10

$$1. \frac{[4 + 3(3+h) - (3+h)^2] - [4 + 9 - 9]}{h}$$

$$\frac{[4 + 9 + 3h - (9 + 6h + h^2)] - 4}{h}$$

$$\frac{4 + 9 + 3h - 9 - 6h - h^2 - 4}{h} = \frac{-3h - h^2}{h}$$

$$\frac{h(-3-h)}{h} = \boxed{-3-h}$$

$$2. \begin{aligned} g(x) &= x^2 + 1 \\ f(x) &= x^{10} \end{aligned}$$

$$\begin{aligned} f(g(x)) &= f(x^2 + 1) \\ &= (x^2 + 1)^{10} \checkmark \end{aligned}$$

1.4-1.5 Answers

$$1. \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x \rightarrow 2} x+3 = 2+3 = \boxed{5}$$

$$2. \lim_{x \rightarrow 0} \frac{\sin 3x}{x} = 3 \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 3 \cdot 1 = \boxed{3}$$

ANSWERS

1) Find the limit

$$\lim_{x \rightarrow \infty} \left(\frac{x^3 - 2x + 3}{5 - 2x^2} \right)$$

First divide the numerator and denominator by x^2 (the highest power of x that occurs in the denominator)

$$\lim_{x \rightarrow \infty} \left(\frac{x^3 - 2x + 3}{5 - 2x^2} \right) = \frac{x - 2/x + 3/x^2}{5/x^2 - 2}$$

$$\lim_{x \rightarrow \infty} x - 2/x + 3/x^2 = \infty \quad \text{and} \quad \lim_{x \rightarrow \infty} 5/x^2 - 2 = -2$$

$$\text{so } \lim_{x \rightarrow \infty} \left(\frac{x^3 - 2x + 3}{5 - 2x^2} \right) = \boxed{-\infty}$$

$$2) g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \quad g(x) = 1 - x^3$$

$$g'(x) = \lim_{h \rightarrow 0} \frac{1 - (x+h)^3 - (1 - x^3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1 - (x^3 + 3hx^2 + 3h^2x + h^3) - 1 + x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-3hx^2 - 3h^2x - h^3}{h}$$

$$= \lim_{h \rightarrow 0} -3x^2 - 3hx - h^2 = -3x^2 \quad g'(0) = 0$$

$$y - g(a) = g'(a)(x - a)$$

$$y - 1 = 0(x - 0)$$

$$y - 1 = 0$$

$$\boxed{y = 1}$$

$$\text{point } (0, 1)$$

$$g(a) = g(0) = 1$$

$$g'(a) = g'(0) = 0$$

Section 2.2 Answers

#2

A) $f'(0) = -2$

B) $f'(1) = 0$

C) $f'(2) = 2$

D) $f'(3) = 2$

E) $f'(4) = -\frac{3}{2}$

F) $f'(5) = -1$

#28

It is not differentiable at $f(x) = 0$
because there is an asymptote.

It is not differentiable at $f(x) = 3$
because it's a sharp point so there
is no tangent line

Section 2.3 Answers

$$\#16 \quad f'(t) = 2t^5 - 12t^3 + 1$$

#27

$$y' = 1 + \frac{1}{2\sqrt{x}}$$

$$y'(1) = 1 + \frac{1}{2\sqrt{1}} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$$

$$\frac{3}{2} = \text{slope} \quad \text{put into } y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{2}(x - 1)$$

Calc Final Review

2.4 Solutions

$$\begin{aligned}
 \#1. \quad y &= \frac{x^2}{3x^2 - 2x - 1} = \frac{(3x^2 - 2x - 1)Dx(x^2) - (x^2)Dx(3x^2 - 2x - 1)}{(3x^2 - 2x - 1)^2} \\
 &= \frac{(3x^2 - 2x - 1)(2x) - (x^2)(6x - 2)}{(3x^2 - 2x - 1)^2} \\
 &= \frac{2x(1-x)}{(3x^2 - 2x - 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 \#2. \quad y &= t^3 \cos t \\
 &= t^3 Dx \cos t + \cos t Dx t^3 \\
 &= t^3 (-\sin t) + \cos t (3t^2) \\
 &= -t^3 \sin t + 3t^2 \cos t
 \end{aligned}$$

$$\#3. \quad f(x) = \frac{x^3}{1+x}$$

$$\begin{aligned}
 f'(x) &= \frac{(1+x)(2x) - x^2(1)}{(1+x)^2} \\
 &= \frac{2x + 2x^2 - x^2}{(1+x)^2} \\
 &= \frac{2x + x^2}{(1+x)^2}
 \end{aligned}$$

$$f''(x) = \frac{(1+x)^2(2+2x) - (2x+x^2)2(1+x)}{(1+x)^4}$$

$$f''(1) = \frac{4}{16} = \frac{1}{4}$$

$$\begin{aligned} \textcircled{1} \quad y &= (2x+1)^5 (x^3-x+1)^4 \\ y' &= (2x+1)^5 \frac{d}{dx} (x^3-x+1)^4 + (x^3-x+1)^4 \frac{d}{dx} (2x+1)^5 \\ y' &= (2x+1)^5 \cdot 4(x^3-x+1)^3 \frac{d}{dx} (x^3-x+1) + (x^3-x+1)^4 \cdot 5(2x+1)^4 \frac{d}{dx} (2) \\ y' &= 4(2x+1)^5 (x^3-x+1)^3 (3x^2-1) + 5(x^3-x+1)^4 (2x+1)^4 \cdot 2 \\ y' &= [2(2x+1)^4 (x^3-x+1)^3] (17x^3 + 6x^2 - 9x + 3) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad xy + 2x + 3x^2 &= 4 \\ \frac{d}{dx} (xy + 2x + 3x^2) &= \frac{d}{dx} (4) \\ \frac{d}{dx} (xy) + \frac{d}{dx} (2x) + \frac{d}{dx} (3x^2) &= \frac{d}{dx} (4) \\ (1 \cdot y + x \cdot \frac{dy}{dx}) + (2) + (6x) &= 0 \end{aligned}$$

$$y + x \frac{dy}{dx} + 2 + 6x = 0$$

$$\begin{array}{r} -y \\ \hline x \frac{dy}{dx} + 2 + 6x = -y \\ \quad -2 \qquad \qquad -2 \end{array}$$

$$\begin{array}{r} x \frac{dy}{dx} + 6x = -y - 2 \\ \quad -6x \qquad -6x \end{array}$$

$$\frac{x \frac{dy}{dx}}{x} = \frac{-y - 2 - 6x}{x}$$

$$\frac{dy}{dx} = \frac{-y - 2 - 6x}{x} \quad \text{or} \quad \frac{-(y + 2 + 6x)}{x}$$

2.7

Answers

5. $y = x^3 + 2x$

given $\frac{dx}{dt} = 5$

when $\dot{x} = 2$

$$\frac{dy}{dt} = \frac{d}{dt}(x^3 + 2x)$$

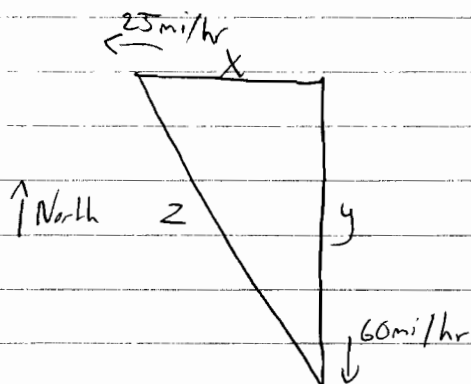
$$\frac{dy}{dt} = (3x^2 + 2) \frac{dx}{dt}$$

$$\frac{dy}{dt} = (3(2)^2 + 2)(5)$$

$$\frac{dy}{dt} = (14)(5)$$

$$\frac{dy}{dt} = 70$$

13.

we want to calculate $\frac{dz}{dt}$

$$z^2 = x^2 + y^2$$

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \quad \leftarrow \text{divide this side by } 2z$$

$$\frac{dz}{dt} = \frac{1}{z} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right)$$

$$\frac{dz}{dt} = \frac{1}{130} (50)(25) + 120(60)$$

$$\frac{dz}{dt} = 65 \text{ mi/hr}$$

$$z = \sqrt{x^2 + y^2}$$

$$z = \sqrt{50^2 + 120^2}$$

$$z = 130$$

2.8

SOLUTIONS

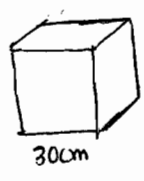
12) $\sqrt{99.8}$

$y = \sqrt{x}$ $y' = \frac{1}{2\sqrt{x}}$
 $y(100) = \sqrt{100}$ $y'(100) = \frac{1}{2\sqrt{100}}$
 $y(100) = 10$ $y'(100) = \frac{1}{20} = m$

* NOW PLUG INTO EQUATION

$y - y_1 = m(x - x_1)$
 or
 $y = m(x - x_1) + y_1$
 $y = \frac{1}{20}(x - 100) + 10$
 $= \frac{1}{20}(99.8 - 100) + 10$
 $= 9.99$

21)



30cm edge with possible error of 0.1cm

a) Volume = s^3
 $\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$
 $\frac{dV}{dt} = 3(30)^2 (\pm 0.1)$
 $\frac{dV}{dt} = 270 \text{ cm}^3$ MAX ERROR

relative error = $\frac{dV}{V}$
 $\frac{3s^2}{s^3} \frac{ds}{dt} = \frac{3 ds/dt}{s}$
 $= \frac{(3)(0.1)}{30} = 0.1$
 % error = relative error $\times 100$
 $= (0.01) \times 100$
 $= 1\%$

b) SA = $6s^2$
 $\frac{dSA}{dt} = 12s \frac{ds}{dt}$
 $= 12(30)(\pm 0.1)$
 $= 36 \text{ cm}^2$

~~0.006~~
 0.006×100
 $= 0.67\%$

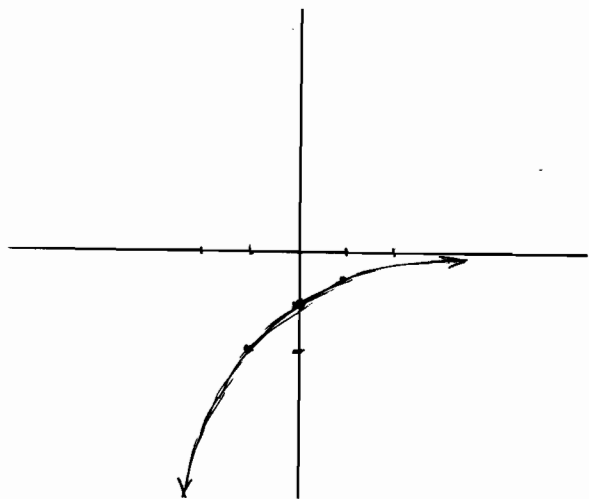
~~0.006~~
 $\frac{12s}{6s^2} \frac{ds}{dt} = \frac{2 ds}{s dt}$
 $= \frac{2(0.1)}{30}$
 $= 0.006$

3.1 Solutions

9

$$y = -2^{-x}$$

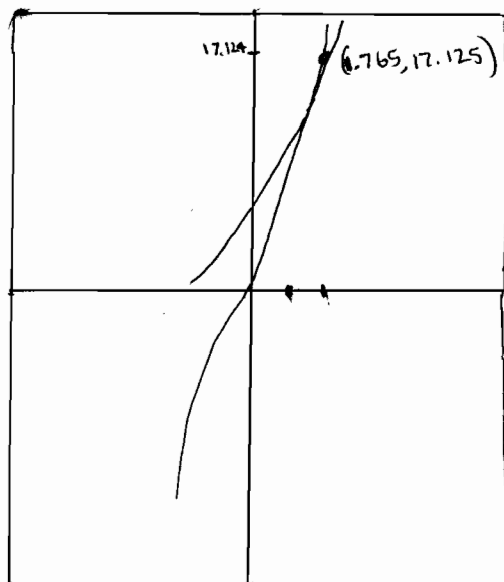
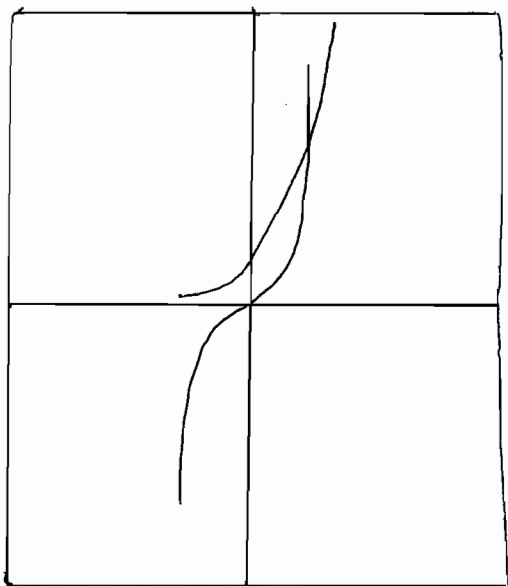
x	y
0	-1
1	-0.5
-1	-2



20

$$f(x) = x^5 \quad g(x) = 5^x$$

Point of Intersection
(1.765, 17.125)



Solutions: 3.2-3.3

MJ Marconi

21. $f(x) = \sqrt{10-3x}$
 $y = \sqrt{10-3x}$
 $x^2 = 10-3y$
 $x^2 = 10-3y$
 $x-10 = 3y$
 $1 = \frac{x^2+10}{3}$
 $f'(x) = \frac{x^2+10}{3}$

43. a) $\frac{\log_2 6^4}{\log_2 2^6} = \boxed{6}$

b) $\frac{\log_6 \frac{1}{36}}{\log_6 6^{-2}} = \boxed{-2}$

Answers

$$1.) \lim_{t \rightarrow 0} \frac{e^t - 1}{t^3} = \frac{e^0 - 1}{0^3} = \frac{1 - 1}{0} = \frac{0}{0}$$

Use l'Hospital's Rule

$$\frac{d_x (e^t - 1)}{d_x t^3} = \frac{e^t}{3t^2} = \frac{e^0}{3(0)^2} = \frac{1}{0} \leftarrow \text{getting closer to } 0,$$

so $\boxed{\infty}$

$$2.) \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sin(x)} = \frac{(2)^2 - 4}{\sin(2)} = \frac{4 - 4}{\sin(2)} = \frac{0}{\sin(2)} = \boxed{0}$$

cannot use l'Hospital's rule because not $\frac{0}{0}$!

Joe · Answer Sheet 1

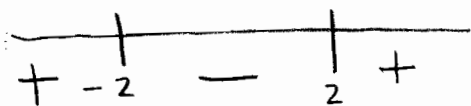
Section 4.3-4.4

4.3

11 $f(x) = x^3 - 12x + 1$

(a) Intervals increasing/decreasing

$$\begin{aligned} f'(x) &= 3x^2 - 12 \\ 0 &= 3(x^2 - 4) \\ &= 3(x+2)(x-2) \\ x &= -2, 2 \end{aligned}$$



Increasing on $(-\infty, -2] \cup [2, \infty)$
decreasing on $(-2, 2)$

(b) $f(-2) = (-2)^3 - 12(-2) + 1$
 $-8 + 24 + 1 = -7 + 24 = 17 \rightarrow (-2, 17)$
 $f(2) = (2)^3 - 12(2) + 1$
 $8 - 24 + 1 = 9 - 24 = -15 \rightarrow (2, -15)$

max = $(-2, 17)$

min = $(2, -15)$

(c) $f'(x) = 3x^2 - 12$
 $f''(x) = 6x$
 $x = 0$

$(-\infty, 0] =$ concave down

$[0, \infty) =$ concave up

Inflection point @ zero, one
 $(0, 1)$



Joe = Answer Sheet 2

Section 4.4

1) $y = x^3 + x$

A = Domain: $\{x \mid \mathbb{R}\}$

B = Intercept: zero

C = Symmetry = about the x axis

D = Asymptotes = none

E = Intervals of Decrease or Increase =

$$y = x^3 + x$$

$$y' = 3x^2 + 1$$

$$\begin{array}{c} \hline + \quad | \quad + \\ \hline \end{array}$$

F max & min: none

G Concavity

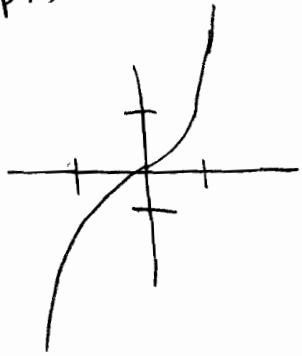
$$y'' = 6x$$

$$\begin{array}{c} \hline - \quad | \quad + \\ \hline \end{array}$$

Joe · Answer Sheet 3

H Inflection Point: $(0,0)$

I Graph



Problem # 57 \rightarrow 4.4

$$f(x) = 1 + \frac{1}{x} + \frac{8}{x^2} + \frac{1}{x^3}$$

Maggie Hartz

December 5, 2010

Final Review (4.1 - 4.2)

4.1

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = x^3 - 6x^2 + 9x + 2, \quad [-1, 4]$$

$$f'(x) = 3x^2 - 12x + 9$$

$$0 = \frac{3x^2 - 12x + 9}{3}$$

$$0 = x^2 - 4x + 3$$

$$(x-1)(x-3) = 0$$

$$x-1=0 \quad x-3=0$$

$$x=1, 3$$

$$f(-1) = -14$$

$$f(1) = 6$$

$$f(3) = 2$$

$$f(4) = 6$$

$$\text{abs max: } f(4) = 6 \text{ ; } f(1) = 6$$

$$\text{abs min: } f(-1) = -14$$

4.2

Verify that the function satisfies the hypotheses of the mean value theorem on the given interval. Then find all numbers c that satisfy the conclusion of the mean value theorem.

$$f(x) = e^{-2x}, \quad [0, 3]$$

$$f(0) = e^{-2(0)} = e^0 = 1$$

$$f(3) = e^{-2(3)} = e^{-6}$$

$$f'(x) = -2e^{2x}$$

$$(e^{-6} - 1) = -2e^{2x}(3)$$

$$\frac{(e^{-6} - 1)}{-6} = \frac{-6e^{2x}}{-6}$$

$$\frac{1}{6}(1 - e^{-6}) = e^{2x}$$

$$\ln\left(\frac{1}{6}(1 - e^{-6})\right) = \ln(e^{2x})$$

$$\ln\left(\frac{1}{6}(1 - e^{-6})\right) = 2x$$

2

$$\frac{1}{2} \ln\left(\frac{1}{6}(1 - e^{-6})\right) = x$$

$$\textcircled{1} f''(x) = 24x^2 + 2x + 10$$

$$f'(x) = (24)\left(\frac{1}{3}\right)x^3 + (2)\left(\frac{1}{2}\right)x^2 + 10x$$

$$= 8x^3 + x^2 + 10x + C$$

$$-3 = 8(1)^3 + (1)^2 + 10(1) + C$$

$$-3 = 8 + 1 + 10 + C$$

$$-3 = 19 + C$$

$$-22 = C$$

$$f'(x) = 8x^3 + x^2 + 10x - 22$$

$$f(x) = (8)\left(\frac{1}{4}\right)x^4 + \frac{1}{3}x^3 + (10)\left(\frac{1}{2}\right)x^2 - 22x$$

$$= 2x^4 + \frac{1}{3}x^3 + \frac{10}{2}x^2 - 22x + C$$

$$5 = 2(1)^4 + \frac{1}{3}(1)^3 + 5(1)^2 - 22(1) + C$$

$$5 = 2 + \frac{1}{3} + 5 - 22 + C$$

$$\frac{59}{3} = C$$

$$f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + \frac{59}{3}$$

$$\textcircled{2} g(\theta) = \cos\theta - 5\sin\theta$$

$$\sin\theta + 5\cos\theta + C$$

$$\textcircled{3} \text{ Lower estimate} = 34.7 \text{ ft}$$

$$\text{upper estimate} = 44.8 \text{ ft}$$

FINAL REVIEW

Section 5.2 Answers

Chris Melvin

$$19) \int_{-1}^5 (1+3x) dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5-(-1)}{n} = \frac{6}{n}$$

$$x_i = a + i\Delta x$$

$$x_i = -1 + \frac{6i}{n}$$

$$\int_{-1}^5 (1+3x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[1 + 3\left(-1 + \frac{6i}{n}\right) \right] \cdot \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[-2 + \frac{18i}{n} \right] \cdot \frac{6}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{12}{n} \sum_{i=1}^n \left[\frac{9i}{n} - 1 \right]$$

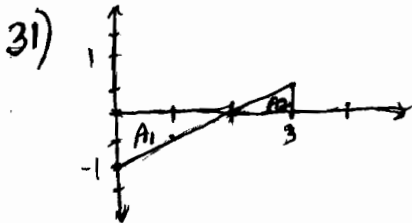
$$= \lim_{n \rightarrow \infty} \frac{12}{n} \left[\frac{9}{n} \sum_{i=1}^n i - \sum_{i=1}^n 1 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{12}{n} \left[\frac{9}{n} \cdot \frac{n(n+1)}{2} - n \right]$$

$$= \lim_{n \rightarrow \infty} \frac{12}{n} \left[\frac{9}{2}(n+1) - n \right]$$

$$= \lim_{n \rightarrow \infty} 54(n+1) - 12$$

$$= \lim_{n \rightarrow \infty} 54n + 54 - 12 = \lim_{n \rightarrow \infty} 54n + 42 = \boxed{42}$$



$$\int_0^3 \left(\frac{1}{2}x - 1\right) dx = A_1 + A_2$$

$$= \left(\frac{1}{2}b_1h_1\right) + \left(\frac{1}{2}b_2h_2\right)$$

$$= \left(\frac{1}{2}(2)(1)\right) + \left(\frac{1}{2}(1)\left(\frac{1}{2}\right)\right)$$

$$= -1 + \frac{1}{4}$$

$$= \boxed{-\frac{3}{4}}$$

Made by Hector Martinez
Solved Problems

From 5.3

$$\#14. \int_1^a \frac{3x-2}{\sqrt{x}} dx = \frac{3x}{\sqrt{x}} - \frac{2}{\sqrt{x}} = \frac{3x}{x^{\frac{1}{2}}} - \frac{2}{x^{\frac{1}{2}}}$$

$$3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = 2x^{\frac{3}{2}} - 4x^{\frac{1}{2}}$$

$$3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} = \left(2(9)^{\frac{3}{2}} - 4(9)^{\frac{1}{2}} \right) - \left(2(1)^{\frac{3}{2}} - 4(1)^{\frac{1}{2}} \right)$$

$$\left(2(27) - 4(3) \right) - \left(2 - 4 \right)$$

$$\frac{3}{1} \cdot \frac{2}{3} = 2 \quad q^3 = 729 \quad (54 - 12) - (2 - 4)$$

$$q^3 = 81$$

$$\begin{array}{r} \times q \\ 81 \\ \hline 729 \end{array}$$

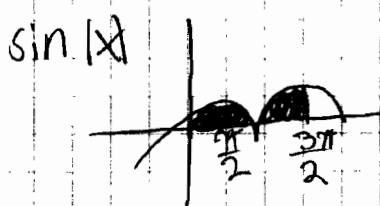
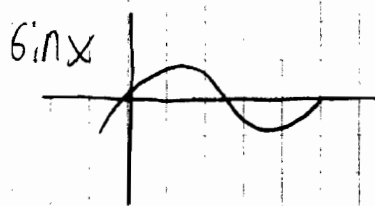
$$\begin{array}{r} \times 27 \\ 27 \\ \hline 189 \\ 540 \\ \hline 729 \end{array}$$

$$\sqrt{729} = 27$$

$$(42) - (-2)$$

$$42 + 2 = \boxed{44}$$

$$\#28. \int_0^{\frac{3\pi}{2}} |\sin(x)| dx = |\cos(x)|$$



$$\left| \sin\left(\frac{3\pi}{2}\right) \right| - \left| \sin(0) \right|$$

$$-1 - 0 = -1 + \frac{1}{2} = \boxed{\frac{3}{2}}$$