10 $h(x) = \int_0^{x^2} \sqrt{1+r^3}\, dr$ [illegible]

Let $u = x^2$ $\quad \int_0^{u} \sqrt{1+r^3}\, dr$

$$\frac{d}{dx} \int_0^{x^2} \sqrt{1+r^3}\, dr = \frac{d}{dx} \int_0^{u} \sqrt{1+r^3}\, dr$$

By chain rule

$$\frac{d}{dx} \int_0^{x^2} \sqrt{1+r^3} = \frac{d}{dx}\left[\int_0^{u} \sqrt{1+r^3}\, dr\right] \frac{du}{dx}$$

$$= \sqrt{1+u^3}\,\frac{du}{dx} = \boxed{\sqrt{1+x^6} \cdot 2x}$$

12-9-09

2. $\int_0^{1/2} \sin^{-1}(x)\, dx$

$\int u\,dv = u \cdot v - \int v\,du$

$= x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$
$du = -2x$

$\int_{x=0}^{x=1/2} u\,dv = x\sin^{-1}(x) + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$

$= x\sin^{-1}(x) + \frac{1}{2}\int u^{-1/2}\,du$

$= x\sin^{-1}(x) + \frac{1}{2}\left(2u^{1/2}\right)\Big|_{x=0}^{x=1/2}$

$= x\sin^{-1}(x) + \sqrt{u}\,\Big|_{x=0}^{x=1/2}$

$= x\sin^{-1}x + \sqrt{1-x^2}\,\Big|_0^{1/2}$

$= \left[\frac{1}{2}\left(\frac{\pi}{6}\right) + \sqrt{\frac{3}{4}}\right] - [0+1]$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

$= \frac{\pi}{12} + \frac{6\sqrt{3}}{12} - \frac{12}{12}$

$= \frac{1}{12}\left(\pi + 6\sqrt{3} - 12\right)$

$u = \sin^{-1}(x)$
$v = x$
$du = \frac{1}{\sqrt{1-x^2}}\,dx$
$dv = dx$

$y = \sin^{-1}(x)$
$\sin y = x$
$\cos y \cdot y' = 1$
$y' = \frac{1}{\cos y}$
$y' = \frac{1}{\sqrt{1-\sin^2 y}}$ $\quad \sin y = x$
$y' = \frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$
$\cos^2 y = 1 - \sin^2 y$
$\cos y = \sqrt{1-\sin^2 y}$

Griffin Williams

$\boxed{7}$ $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$ $|R_n| = |S - S_n|$

$S = -\frac{2}{3} + \frac{5}{5} - \frac{8}{7} + \frac{11}{9} - \frac{14}{11} + \frac{17}{13} - \frac{20}{15} + \frac{23}{17} \ldots$

$S = .467$

$S_8 = \frac{23}{17}$ or 1.353

$|R_n| = |S - S_n|$

$|R_n| = |.467 - 1.353|$

$R_n = .886$

Eli Scandalis

.4
3

1) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

root test

8.4
#33

$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{n^2+1}{2n^2+1}\right)^n\right|}$

$\lim_{n\to\infty} \left|\frac{n^2+1}{2n^2+1}\right|$

$= \frac{1}{2} < 1$ Absolutely Converges by ratio test

Marya Blinzyuk

Solution- Problem 1

$F=kx$

$x=10$ cm$=0.1$ m

$\Rightarrow 25=k(.1) \Rightarrow k=250$ N/m $\Rightarrow F(x)=250x$

$$W=\int_{L_i}^{L_f} F(x)\,dx$$

Natural Length=20 cm, so:
L_f= 25cm-20cm= 5cm= 0.05 m
L_i=20cm-20cm=0m

$$\Rightarrow W=\int_0^{0.05} 250x\,dx=\frac{250}{2}x^2\Big]_0^{0.05}=125(0.05^2)=\boxed{0.3125\text{ J}}$$

Beau Osland Answer Sheet 8.2 (Geom.)

1. $1 + 0.4 + 0.16 + 0.064 + \ldots$

$$= \sum_{n=0}^{\infty} (0.4)^{n}$$

converges by geometric series test with $|r| = |0.4| < 1$

Converges to the following:

$$\frac{\text{first term}}{(1-r)} = \frac{1}{(1-0.4)} = \frac{1}{0.6} = \boxed{\frac{5}{3}}$$

Nick Delgado

Section 8.2

15. Determine whether the series is convergent or divergent. If it is convergent, find the sum.

$\sum_{n=1}^{\infty} \sqrt[n]{2}$ $\lim_{n\to\infty} \sqrt[n]{2} = 1 \neq 0$ nth term test

Diverges

Find the values of x for which the series converges.

$\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \left(\frac{x}{3}\right)^n$ $\frac{|x|}{3} < 1$

$|x| < 3$

Values: $-3 < x < 3$

Sum = $\dfrac{\frac{x}{3}}{1 - \frac{x}{3}} = \dfrac{x}{(3-x)}$

Alison Mackey

1. $\int x e^{x^2}\,dx$

$u = x^2$

$du = 2x$ $\qquad \frac{1}{2}\int e^u\,du$

$\frac{1}{2}\,du = x$ $\qquad = \frac{1}{2}e^u$

$\qquad = \frac{1}{2}e^{x^2}$

David Yandell

6.4 (Tables of Integrals)
Solution

1. $\int e^{2x}\sin(3x)\,dx$

Using the formula (Table #98)

$$\int e^{au}\sin(bu)\,du = \frac{e^{au}(a\sin(bu) - b\cos(bu))}{a^2+b^2} + C$$

$a=2 \qquad b=3 \qquad u=x$

therefore,

$$\int e^{2x}\sin(3x)\,dx = \frac{e^{2x}(2\sin(3x) - 3\cos(3x))}{2^2+3^2} + C$$

$$= \frac{e^{2x}(2\sin(3x) - 3\cos(3x))}{13} + C$$

Alex [illegible]

Answers

1) $$\sum_{n=0}^{\infty} \frac{n^2}{5n^2+4}$$

$$\lim_{n\to\infty} \frac{1}{5+\frac{4}{n^2}}$$

$$= \frac{1}{5} \neq 0$$

Converges by n^{th} term test

2) $$\sum_{n=0}^{\infty} \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{1}{n^2}$$

$= 0$ ← means nothing, n^{th} term test is a test for divergence [illegible] have to [illegible]

$$\sum_{n=0}^{\infty} \frac{1}{n^2}$$

Converges by p-series test $p = 2 > 1$

Kevin Forey

9) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

Use a certain test to determine if the series is convergent or divergent.

Solution:

$\dfrac{1}{n^2+n+1} < \dfrac{1}{n^2}$ so $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ converges by comparison. w/ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ which converges by p-series, $p=2>1$.

Jacob R.

8.7

$$e^{i\theta} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!}$$

$$= 1 + ix - x^2/2! - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!}$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right)$$

$$= \cos x + i \sin x$$

Alvin Javier
Math 151 02

solutions

1. $$\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$$

use ratio test

$$\lim_{n \to \infty} \left| \frac{(-3)^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-3)^n} \right|$$

$$= \lim_{n \to \infty} \left| \frac{-3n^3}{(n+1)^3} \right|$$

$$= 3 \lim_{n \to \infty} \left| \frac{n^3}{(n+1)^3} \right|$$

$$= 3 \cdot 1$$

$$= 3$$

since the limit is greater than 1, the series $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$ is divergent

Katie Beaudoin

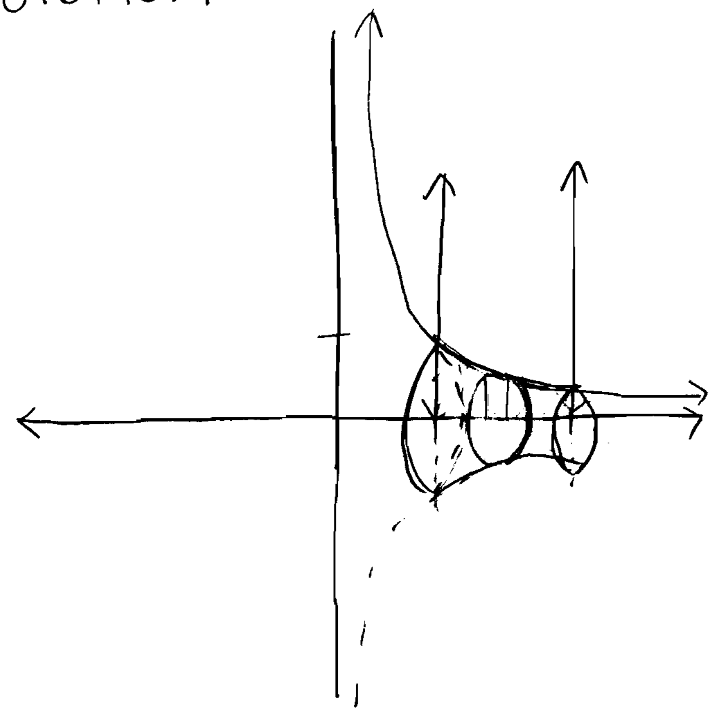
Solution

1. $y = \frac{1}{x}$

$x = 1$

$x = 2$

$y = 0$



$$V = \int_1^2 \pi \left(\frac{1}{x}\right)^2 dx$$

$$= \int_1^2 \pi \cdot \frac{1}{x^2}\, dx$$

$$= \pi \int_1^2 x^{-2}\, dx$$

$$= \pi \left[-\frac{1}{x}\right]_1^2$$

$$= \pi \left[-\frac{1}{2} - \left(-\frac{1}{1}\right)\right]$$

$$= \pi \left[-\frac{1}{2} + 1\right]$$

$$= \pi \left[\frac{1}{2}\right]$$

$$\boxed{= \frac{\pi}{2}}$$

6.2

Will Muldowne

2. $\int \sin^6 x \cos^3 x\,dx$

$= \int \sin^6 x \cos^2 x \cos x\,dx \qquad u = \sin x \quad du = \cos x\,dx$

$= \int \sin^6 x (1-\sin^2 x)\cos x\,dx$

$= \int u^6(1-u^2)\,du$

$= \int (u^6 - u^8)\,du$

$= \frac{u^7}{7} - \frac{u^9}{9} + C = \boxed{\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C}$

8.7 (e, cos, sin) Maddie Gerling

1) $\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

find $\int_0^1 x\tan^{-1}(x)\,dx$

$$x\tan^{-1}(x) = x\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)!}$$

$$\int_0^1 x\tan^{-1}(x)\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)!\,(2n+3)} = \sum_{n=0}^{\infty} \frac{(-1)^n (1)^{2n+3}}{(2n+1)!(2n+3)}$$

$$\cancel{\frac{1}{3} - \frac{1^{2+3}}{(2)(5)} + \frac{1}{(4\cdot3\cdot2)(7)} - \frac{1}{(7)!(9)} + \frac{1}{9!}}$$

$$\frac{1}{3} - \frac{1}{3!(5)} + \frac{1}{5!(7)} - \frac{1}{7!(9)} + \frac{1}{9!(11)} \cdots$$

$$\frac{1}{3} - \frac{1}{30} + \frac{1}{840} - \frac{1}{45360} + \cdots = .3012$$

Ian Mahaney

Solution to Problem (B):

$$\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx = \int \frac{(x+1)(5x-2)}{(x^2)(x+2)}\,dx = \int \frac{Ax+B}{x^2} + \frac{C}{x+2}\,dx$$

$$(Ax+B)(x+2) + Cx^2 = (x+1)(5x-2)$$

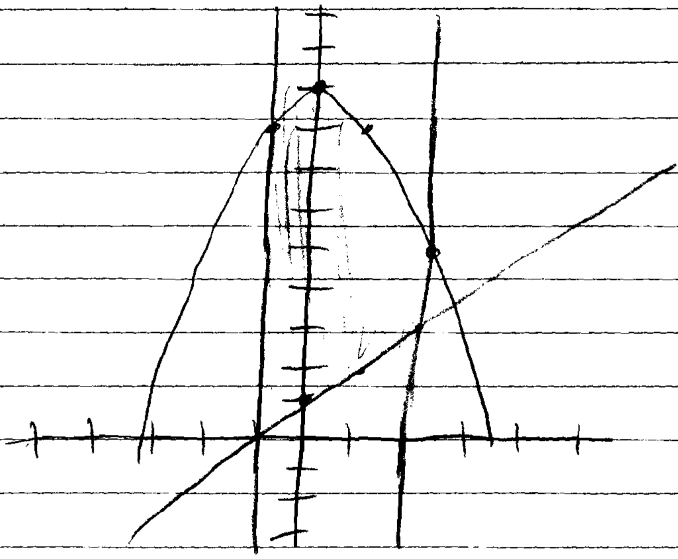
$X=0$	$x=-2$	$x=1 \quad C=3$
$2B=-2$	$4C=12$	$(A-1)3+3=6$
$B=-1$	$C=3$	$3A \cancel{-3+3} = 6$
		$A=2$

$$= \int \frac{2x-1}{x^2} + \frac{3}{x+2}\,dx = \int \frac{2}{x} - \int \frac{1}{x^2} + \int \frac{3}{x+2}$$

$$= 2\ln|x| + x^{-1} + 3\ln|x+2| + C$$

Aaron Bales

1) $y = x+1$, $y = 9-x^2$, $x = -1$, $x = 2$



$= \int_{-1}^{2} [(9-x^2) - (x+1)]\,dx$

$= \int_{-1}^{2} (-x^2 - x + 8)\,dx$

$= -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 8x \Big|_{-1}^{2}$

$= 19.5$

Lily ~~MA~~ Marucci

Solution:

13) $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du =$

$\int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du =$

$-\int_{0}^{2x} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du$

$g'(x) = -\frac{(2x)^2-1}{(2x)^2+1} \cdot \frac{d}{dx}(2x) + \frac{(3x)^2-1}{(3x)^2+1} \cdot \frac{d}{dx}(3x) =$

$$\boxed{-2\left(\frac{4x^2-1}{4x^2+1}\right) + 3\left(\frac{9x^2-1}{9x^2+1}\right)}$$

Joseph Wallace

Section 8.3 (P-Series) - Review Answers

26) $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$

<u>limit comparison test</u>

$$a_n = \frac{n+5}{\sqrt[3]{n^7+n^2}} \qquad b_n = \frac{n}{\sqrt[3]{n^7}} = \frac{1}{n^{(7/3)-1}} = \frac{1}{n^{4/3}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}} \cdot n^{4/3}$$

$$= \lim_{n\to\infty} \frac{n^{7/3} + 5n^{4/3}}{(n^7+n^2)^{1/3}}$$

$$= \lim_{n\to\infty} \frac{1 + 5n^{(4/3)-(7/3)}}{\left(1 + \frac{1}{n^5}\right)^{1/3}}$$

$$= \lim_{n\to\infty} \frac{1 + \frac{5}{n}}{\left(1 + \frac{1}{n^5}\right)^{1/3}} = 1 > 0$$

Since the limit exists, and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges since it is a p-series with $p = 4/3 > 1$.

Then $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$ also <u>converges</u> by the limit comparison test.

Kristin Crawford
12/9/09

7. $\sum_{n=1}^{\infty} \frac{1}{n^4}$ positive and decreasing on $[1, \infty)$

$f(x) = \frac{1}{x^4}$

$\int_1^\infty \frac{1}{x^4}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^4}\,dx = \lim_{t\to\infty} \frac{x^{-3}}{-3}\Big]_1^t$

$\lim_{t\to\infty} \left(\frac{t^{-3}}{3} + \frac{1}{3} \right) = \frac{1}{3}$

p-series with $p=4$, $4>1$, so the series converges

Nelson Siu

$$\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\right\}$$

$$\boxed{a_n = \left(-\frac{2}{3}\right)^{n-1}}$$

Solution

Annie Heaton

8.5

17 $\sum_{n=1}^{\infty} n!\,(2x-1)^n$

ratio test:

$$\lim_{n\to\infty}\left|\frac{(n+1)n!\,(2x-1)^{n+1}}{n!\,(2x-1)^n}\right| = \lim_{n\to\infty}\left|(n+1)\,(2x-1)\right|$$

diverges mostly
but converges if $(2x-1)=0$

$(2x-1)$ must be 0 to converge

$\boxed{R=0}$

so $2x-1=0$

$2x=1$

$x=\frac{1}{2}$ $\boxed{IOC=\left\{\frac{1}{2}\right\}}$

Scott Gump

Answer

2. $\int_0^{0.3} \frac{x^2}{1+x^4}\,dx$

- find power series representation

$x^2 \cdot \frac{1}{1-(-x^4)}$

$= x^2 \sum_{n=0}^{\infty} (-x^4)^n$

$= \sum_{n=0}^{\infty} (-1)^n x^{4n+2}$

- Integrate

$\int_0^{0.3} \sum_{n=0}^{\infty} (-1)^n x^{4n+2}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{4n+3} \Big|_0^{0.3}$

$\sum_{n=0}^{\infty} (-1)^n \frac{.3^{4n+3}}{4n+3} - 0$

n=2 for sixth decimal place.

$-\frac{.3^7}{7} + \frac{.3^{11}}{11}$

$\boxed{= -3.11 \times 10^{-5}}$

John
Nute

7.3 "Shells"

Q #1 $y = x^2,\ y = 0,\ x = 1$

$V = \int_a^b 2\pi x f(x)\,dx$

so, $V = \int_0^1 2\pi x (x^2)\,dx$

$= 2\pi \int_0^1 x^3\,dx$

$= 2\pi \left[\frac{1}{4}x^4\right]_0^1$

$= 2\pi \left[\frac{1}{4}(1)^4 - \frac{1}{4}(0)^4\right]$

$= 2\pi \left(\frac{1}{4}\right)$

$= \frac{1}{2}\pi$

Anna Yamamoto

1. (Solution)

$y = \ln(\cos x) \quad 0 \le x \le \frac{\pi}{3}$

$y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

$L = \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx$

$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$

$= \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx$

$= \int_0^{\pi/3} \sec x\, dx$

Use table #14 from the textbook!

$= \ln|\sec x + \tan x| \Big|_0^{\pi/3} \ln|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}| - \ln|\sec 0 - \tan 0|$

$= \ln(2+\sqrt{3}) - \ln(1) = \boxed{\ln(2+\sqrt{3}).}$

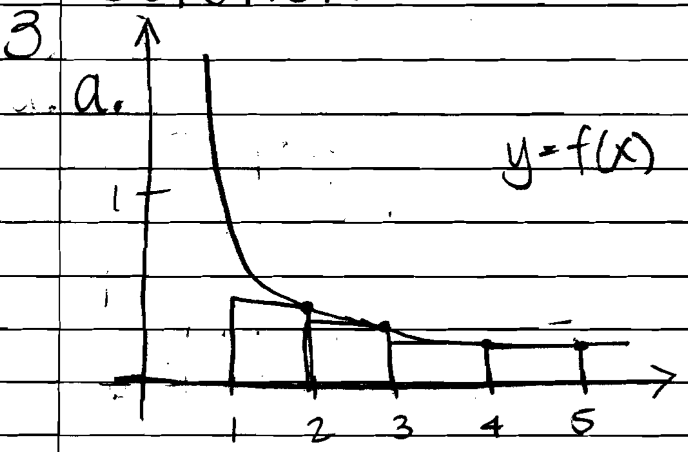
Erin Hong

5.1: Areas & Distances

Solution

3.

a.



$f(x) = \frac{1}{x}$ from $x = 1$ to $x = 5$

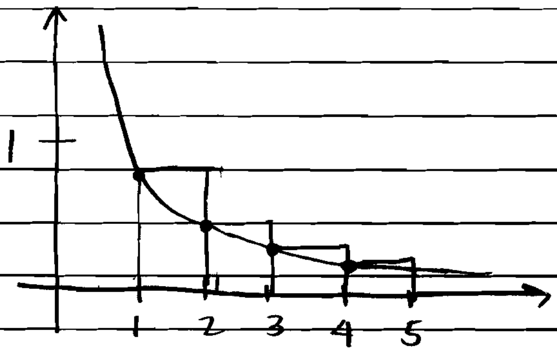
$\Delta x = 1$

$R_4 = 1\left(f(5) + f(4) + f(3) + f(2)\right)$

$R_4 = \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2}\right) = \boxed{\frac{77}{60}}$

underestimate

b.



$L_4 = 1\left(f(1) + f(2) + f(3) + f(4)\right)$

$L_4 = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \boxed{\frac{25}{12}}$

overestimate

Daniel Hagen

Solution #1

$$\sum_{n=1}^{\infty}\left(\frac{(n^2+1)}{(2n^2+1)}\right)^n$$

Root test

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n^2+1}{2n^2+1}\right)^n} = \lim_{n\to\infty}\frac{n^2+1}{2n^2+1} = 1/2$$

$1/2 < 1$ therefore the series

$\boxed{\text{converges absolutely}}$

Daniel Hagell

Solution #2

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{-1^{n+1}(n+1)}{(n+1)^2+1} \cdot \frac{n^2+1}{(-1)^n n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)(n^3+n^2+n+1)}{(n^2+2n+2)n} \right|$$

$$= \lim_{n\to\infty} \frac{n^3+n^2+n+1}{n^3+2n^2+2n} = 1 \quad \leftarrow \text{inconclusive}$$

Direct Comparison

$$\frac{n+1}{(n+1)^2+1} = \frac{n+1}{n^2+2n+2} \qquad 1) \ \frac{n+1}{n^2+2n+2} < \frac{n}{n^2+1}$$

and $2) \ \lim_{n\to\infty} \frac{n}{n^2+1} = 0$ therefore <u>converges</u>

Abs. convergence?

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} = \frac{1}{2} + \frac{2}{5} + \frac{3}{7} + \frac{4}{17} + \dots \quad \leftarrow \text{corresponds to } \sum_{n=0}^{\infty} \frac{1}{n}$$

which is divergent therefore series is

conditionally convergent

Pat Kennu

Approximate the function $f(x) = \sqrt[3]{x}$ by a Taylor polynomial of degree 2 at $a=8$.

$f(x) = \sqrt[3]{x} = x^{1/3}$ $\qquad f(8) = 2$

$f'(x) = \frac{1}{3} x^{-2/3}$ $\qquad f'(8) = \frac{1}{12}$

$f''(x) = -\frac{2}{9} x^{-5/3}$ $\qquad f''(8) = -\frac{1}{144}$

$f'''(x) = \frac{10}{27} x^{-8/3}$

$$T_2(x) = f(8) + \frac{f'(8)}{1!}(x-8) + \frac{f''(8)}{2!}(x-8)^2$$

so

$$\sqrt[3]{x} \approx T_2(x) = 2 + \frac{1}{12}(x-8) + \frac{1}{288}(x-8)^2$$

a) Approximate the function $f(x) = e^{x^2}$, $a=0$, $n=3$, $0 \le x \le 0.1$

b) Use Taylor's Formula to estimate the accuracy of the approximation $f(x) \approx T_N(x)$

Hillary
Hawkins

6.6 solution sheet 1

25. $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} = \int_{-2}^{14} (x+2)^{-1/4} \cdot dx$

$= C + \frac{(x+2)^{-1/4+1}}{(-1/4+1)}\Big|_{-2}^{14} = \frac{(x+2)^{3/4}}{3/4}\Big|_{-2}^{14} = \frac{4(x+2)^{3/4}}{3}\Big|_{-2}^{14}$

$\frac{4(14+2)^{3/4}}{3} - \frac{4(-2+2)^{3/4}}{3} = \frac{4(16^{3/4})}{3} - \frac{4(0)^{3/4}}{3}$

$= \frac{4}{3}(8) - 0 = \boxed{\frac{32}{3} = 10.6667}$ $\boxed{\text{convergent}}$

1. a) $\int_1^\infty x^4 e^{-x^4} dx$

$= \lim_{t\to\infty} \int_1^t x^4 e^{-x^4} dx = \lim_{t\to\infty} -e^{-x^4}\Big|_1^t = \lim_{t\to\infty} -e^{-t^4} - (e^{(1)^4})$

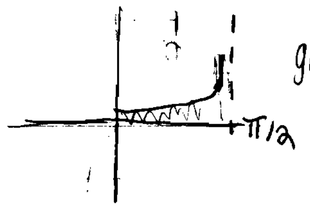
$= \lim_{t\to\infty} -e^{-t^4} + e^1 = \lim_{t\to\infty} -e^{-t^4} = \lim_{t\to\infty} -e/(t^4)$

infinite interval.

b) $\int_0^{\pi/2} \sec x \, dx = \lim_{t\to\pi/2} \int_0^t \sec x$

$= \lim_{t\to\pi/2} \ln|\sec x + \tan x|\Big|_0^t = \lim_{t\to\pi/2} \ln|\sec t + \tan t| - \ln|\sec 0 + \tan 0|$

$\sin \pi/2 = 1$
$\cos \pi/2 = 0$



graph shows undefined @ $\pi/2$

$= \lim_{t\to\pi/2} \ln|\sec t + \tan t| - \ln|1+0|$

$= \lim_{t\to\pi/2} \ln|\sec t + \tan t| - 0$

$\boxed{\text{Infinite discontinuity}}$ by def of an improper integral of type 2.

c) $\int_0^2 \frac{x \, dx}{x^2 + -5x + 6} = \int_0^2 \frac{x}{(x-3)(x-2)} dx$ $\boxed{\text{infinite discontinuity}}$ @ 2 b/c when $x=2$, $x-2=0$, meaning $x/0$, cannot divide by $\emptyset$ so discontinuity.

Hillary Hawkins

Soln 6.6 sheet2

1) d) $\int_{-\infty}^{0} \frac{1}{x^2+5}\,dx = \int_{-\infty}^{0} (x^2+5)^{-1}\,dx = \lim_{b\to-\infty} \int_{b}^{0} (x^2+5)^{-1}$

$\boxed{\text{infinite interval}}$, makes it an improper integral

$= \lim_{b\to-\infty} \frac{\ln|x^2+5|}{2x}\Big|_b^0 = \lim_{b\to-\infty} \frac{\ln|0+5|}{2(0)} - \frac{\ln|b^2+5|}{2b} = \lim_{b\to-\infty} \frac{\ln 5}{0} - \frac{\ln|b^2+5|}{2b}$

can't divide by Ø

$= \lim_{b\to-\infty} (\ln 5)/0 - \lim_{b\to-\infty} \frac{\ln|b^2+5|}{2b}$

$=$

Ben Brantley

1) $y = 12 - x^2$ $\quad y = x^2 - 6$

$12 - x^2 = x^2 - 6$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

$\int_{-3}^{3} (12 - x^2) - (x^2 - 6)\, dx$

$\int_{-3}^{3} 12 - x^2 - x^2 + 6\, dx$

$\int_{-3}^{3} 18 - 2x^2\, dx$

$18x - \frac{2}{3}x^3 \Big|_{-3}^{3}$

$18(3) - \frac{2}{3}(27) - \left(18(-3) - \frac{2}{3}(-27)\right)$

$54 - 18 - (-54 + 18)$

$36 - (-36)$

$\boxed{72}$

Ellen Stidham

$$\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\, d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\cos^2\theta}\right) d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta + 1)\, d\theta$$

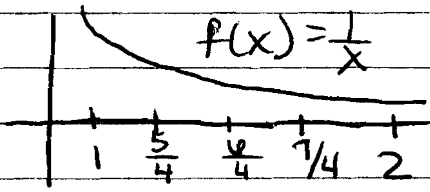
$$= \left[\tan\theta + \theta\right]_0^{\pi/4}$$

$$= (\tan \pi/4 + \pi/4) - (0+0)$$

$$\boxed{= 1 + \pi/4}$$

Kate Partynski

1) $\int_1^2 \frac{1}{x}\,dx \qquad n=4$



a) Midpoint Rule

$M_4 = \frac{1}{4}\left(\frac{8}{9} + \frac{8}{11} + \frac{8}{13} + \frac{8}{15}\right) \approx 0.6912$

b) Trapezoidal Rule

$T_4 = \frac{1}{8}\left(1 + 2\left(\frac{4}{5}\right) + 2\left(\frac{4}{6}\right) + 2\left(\frac{4}{7}\right) + \frac{1}{2}\right) \approx 0.6970$

c) $S_4 = \frac{1}{12}\left(1 + 4\left(\frac{4}{5}\right) + 2\left(\frac{4}{6}\right) + 4\left(\frac{4}{7}\right) + \frac{1}{2}\right) \approx 0.6933$

answer

Jamie Shea

1 a) $P_d(x) = y = -0.1x^2 - 10x + 200$

$R(x) = x \cdot P_d(x)$

$= -0.1x^3 - 10x^2 + 200x$

$R'(x) = -0.3x^2 - 20x + 200$

$-0.3x^2 - 20x + 200 = 0$

$x = 8.83$

$P(8.83) = -0.1(8.83)^2 - 10(8.83) + 200$

$= \$110.92$

b) consumer surplus $= \int_0^x [p_d(x) - p_0]\, dx$

$\int_0^{8.83} (-0.1)x^2 - 10x + 200 - 110.92 \;\; dx$

$= \left. \frac{-1}{30}x^3 - 5x^2 + 89.08x \right]_0^{8.83}$

$= \$373.18$

44 answer

Kelly Fromm
8.7

$$\int \frac{\sin x}{x} dx$$

$$\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!\, x}$$

$R=\infty$

$$\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}}$$

$\boxed{R=\infty}$

Samantha Baumstark

$$\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$$

let $a = 4$

$x = a\sin\theta$

$dx = a\cos\theta\,d\theta$

$a\cos\theta = \sqrt{a^2 - a^2\sin^2\theta}$

$$\int_0^{2\sqrt{3}} \frac{(4\sin\theta)^3}{\sqrt{16-(4\sin\theta)^2}} \cdot 4\cos\theta\,d\theta$$

$$\int_0^{2\sqrt{3}} \frac{(4\sin\theta)^3\, \cancel{4\cos\theta}}{\cancel{4\cos\theta}}\,d\theta$$

$$\int_0^{2\sqrt{3}} (4\sin\theta)^3\,d\theta$$

$$64\int_0^{2\sqrt{3}} (1-\cos^2\theta)\sin\theta\,d\theta$$

$u = \cos\theta$

$-du = \sin\theta\,d\theta$

$$-64\int_0^{2\sqrt{3}} 1-u^2\,du$$

$$-64\left[u - \frac{u^3}{3}\right]_0^{2\sqrt{3}}$$

$$-64\left[\cos(2\sqrt{3}) - \frac{(\cos(2\sqrt{3}))^3}{3} - \cos(0) + \frac{(\cos(0))^3}{3}\right]$$

$$= \boxed{-1.33}$$

Andrew McCraven
Solutions

8.3 Direct Comparison Test

9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

$1 \le 1$

$n^2+n+1 \ge n^2$

$\dfrac{1}{n^2+n+1} \le \dfrac{1}{n^2}$

compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2 \leftarrow p}}$

converges by the p-series test because $p > 1$

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ converges by direct comparison test since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} \ge \sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

Duggwn Clem

16) EXpress the limit as a definite integral

$$\lim_{n \to \infty} \sum_{i=1}^{\infty} \frac{e^{x_i}}{1+x_i} \Delta x \quad [1,5]$$

Well thrm 4 states that:

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{\infty} f(x_i)\,dx$$

where $\Delta x = \frac{b-a}{n}$ $\quad x_i = a + i\Delta x$

So If we use this formula and replace $\frac{e^{x_i}}{1+x_i}$ with $\frac{e^x}{1+x}$

We get $\int_1^5 \frac{e^x}{1+x}\,dx$

Note: we replaced Δx with dx

so $\lim_{n \to \infty} \sum_{i=1}^{\infty} \frac{e^{x_i}}{1+x_i} \Delta x \; [1,5] = \int_1^5 \frac{e^x}{1+x}\,dx$

Heidi Hirsh (solution)

51. $\lim_{x \to 0} \dfrac{x - \tan^{-1} x}{x^3}$

known:

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3} = \frac{\cancel{x} - \left(\cancel{x} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{x^{3-3}}{3} - \frac{x^{5-3}}{5} + \frac{x^{7-3}}{7} - \cdots}{x^3}$$

$$= \lim_{x \to 0} \frac{1}{3} - \frac{x^2}{5} + \frac{x^4}{7} = 1/3$$

$\emptyset$ $\emptyset$

10 $h(x) = \int_0^{x^2} \sqrt{1+r^3}\, dr$ [illegible]

Let $u = x^2$ $\quad \int_0^{u} \sqrt{1+r^3}\, dr$

$$\frac{d}{dx} \int_0^{x^2} \sqrt{1+r^3}\, dr = \frac{d}{dx} \int_0^{u} \sqrt{1+r^3}\, dr$$

By chain rule

$$\frac{d}{dx} \int_0^{x^2} \sqrt{1+r^3} = \frac{d}{dx}\left[\int_0^{u} \sqrt{1+r^3}\, dr\right] \frac{du}{dx}$$

$$= \sqrt{1+u^3}\, \frac{du}{dx} = \boxed{\sqrt{1+x^6} \cdot 2x}$$

12-9-09

2. $\int_0^{1/2} \sin^{-1}(x)\,dx$

$u = \sin^{-1}(x)$

$v = x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$dv = dx$

$\int u\,dv = u \cdot v - \int v\,du$

$= x\sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = 1 - x^2$

$du = -2x$

$\int_{x=0}^{x=1/2} u\,dv = x\sin^{-1}(x) + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$

$= x\sin^{-1}(x) + \frac{1}{2}\int u^{-1/2}\,du$

$= x\sin^{-1}(x) + \frac{1}{2}(2u^{1/2}) \Big|_{x=0}^{x=1/2}$

$= x\sin^{-1}(x) + \sqrt{u} \Big|_{x=0}^{x=1/2}$

$= x\sin^{-1}x + \sqrt{1-x^2} \Big|_0^{1/2}$

$= \left[\frac{1}{2}\left(\frac{\pi}{6}\right) + \sqrt{\frac{3}{4}}\right] - [0+1]$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

$= \frac{\pi}{12} + \frac{6\sqrt{3}}{12} - \frac{12}{12}$

$= \frac{1}{12}(\pi + 6\sqrt{3} - 12)$

$y = \sin^{-1}(x)$

$\sin y = x$

$\cos y \cdot y' = 1$

$y' = \frac{1}{\cos y}$

$y' = \frac{1}{\sqrt{1-\sin^2 y}}$ $\quad \sin y = x$

$y' = \frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1-\sin^2 y}$

Griffin Williams

$\boxed{7}$ $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$ $\quad |R_n| = |S - S_n|$

$S = -\frac{2}{3} + \frac{5}{5} - \frac{8}{7} + \frac{11}{9} - \frac{14}{11} + \frac{17}{13} - \frac{20}{15} + \frac{23}{17} \dots$

$S = .467$

$S_8 = \frac{23}{17}$ or 1.353

$|R_n| = |S - S_n|$

$|R_n| = |.467 - 1.353|$

$R_n = .886$

Eli Scandalis

.4
33

8.4
#33

1) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

root test

$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{n^2+1}{2n^2+1}\right)^n\right|}$

$\lim_{n\to\infty} \left|\frac{n^2+1}{2n^2+1}\right|$

$= \frac{1}{2} < 1$ Absolutely Converges by ratio test

Marya Blinzyuk

Solution- Problem 1

F=kx

$\Rightarrow$ 25= k(.1) $\Rightarrow$ k=250 N/m $\Rightarrow$ F(x)=250x

x=10 cm= 0.1 m

$$W=\int_{L_i}^{L_f} F(x)dx$$

$$\Rightarrow W=\int_0^{0.05} 250x\,dx=\frac{250}{2}x^2\Big]_0^{0.05}=125(0.05^2)=\boxed{0.3125\text{ J}}$$

Natural Length=20 cm, so:
L_f= 25cm-20cm= 5cm= 0.05 m
L_i=20cm-20cm=0m

Beau Osland Answer Sheet 8.2 (Geom.)

1. $1 + 0.4 + 0.16 + 0.064 + \ldots$

$$= \sum_{n=0}^{\infty} (0.4)^{n}$$

converges by geometric series test with $|r| = |0.4| < 1$

Converges to the following:

$$\frac{\text{first term}}{(1-r)} = \frac{1}{(1-0.4)} = \frac{1}{0.6} = \boxed{\frac{5}{3}}$$

Nick Delgado

Section 8.2

15. Determine whether the series is convergent or divergent. If it is convergent, find the sum.

$\sum_{n=1}^{\infty} \sqrt[n]{2}$ $\quad \lim_{n\to\infty} \sqrt[n]{2} = 1 \neq 0$ nth term test

Diverges

[illegible] Find the values of x for which the series converges.

$\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \left(\frac{x}{3}\right)^n \quad \left|\frac{x}{3}\right| < 1$

$|x| < 3$

Values: $-3 < x < 3$

Sum $= \dfrac{\frac{x}{3}}{1 - \frac{x}{3}} = \dfrac{x}{(3-x)}$

Alison Mackey

1. $\int xe^{x^2}\ dx$

$u = x^2$

$du = 2x$ $\qquad \frac{1}{2}\int e^u\ du$

$\frac{1}{2}\ du = x$ $\qquad = \frac{1}{2}e^u$

$\qquad = \frac{1}{2}e^{x^2}$

David Yandell

6.4 (Tables of Integrals) Solution

1. $\int e^{2x} \sin(3x)\, dx$

Using the formula (Table # 98)

$$\int e^{au} \sin(bu)\, du = \frac{e^{au}(a\sin(bu) - b\cos(bu))}{a^2 + b^2} + C$$

$a = 2 \qquad b = 3 \qquad u = x$

therefore,

$$\int e^{2x} \sin(3x)\, dx = \frac{e^{2x}(2\sin(3x) - 3\cos(3x))}{2^2 + 3^2} + C$$

$$= \frac{e^{2x}(2\sin(3x) - 3\cos(3x))}{13} + C$$

Ale... S...

Answers

1) $\sum_{n=0}^{\infty} \frac{n^2}{5n^2+4}$

$\lim_{n\to\infty} \frac{1}{5n^2+4}$

$= \frac{1}{5} \neq 0$

Diverges by n^{th} term test

2) $\sum_{n=0}^{\infty} \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{1}{n^2}$

$= 0$ ← means nothing, n^{th} term test is a test for divergence only, you have to [illegible]

$\sum_{n=0}^{\infty} \frac{1}{n^2}$

Converges by p-series test $p = 2 > 1$

Kevin Forey

9) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

Use a certain test to determine if the series is convergent or divergent.

Solution:

$$\frac{1}{n^2+n+1} < \frac{1}{n^2}$$

so $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ converges by comparison. w/ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ which converges by p-series, $p=2>1$.

Jacob R.

8.7

$$e^{i\theta} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!}$$

$$= 1 + ix - x^2/2! - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!}$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right)$$

$$= \cos x + i\sin x$$

Alvin Javier
Math 151 02

Solutions

1. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

use ratio test

$\lim_{n\to\infty} \left| \frac{(-3)^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-3)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{-3n^3}{(n+1)^3} \right|$

$= 3 \lim_{n\to\infty} \left| \frac{n^3}{(n+1)^3} \right|$

$= 3 \cdot 1$

$= 3$

since the limit is greater than 1, the series

$\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$ is divergent

Katie Beaudoin

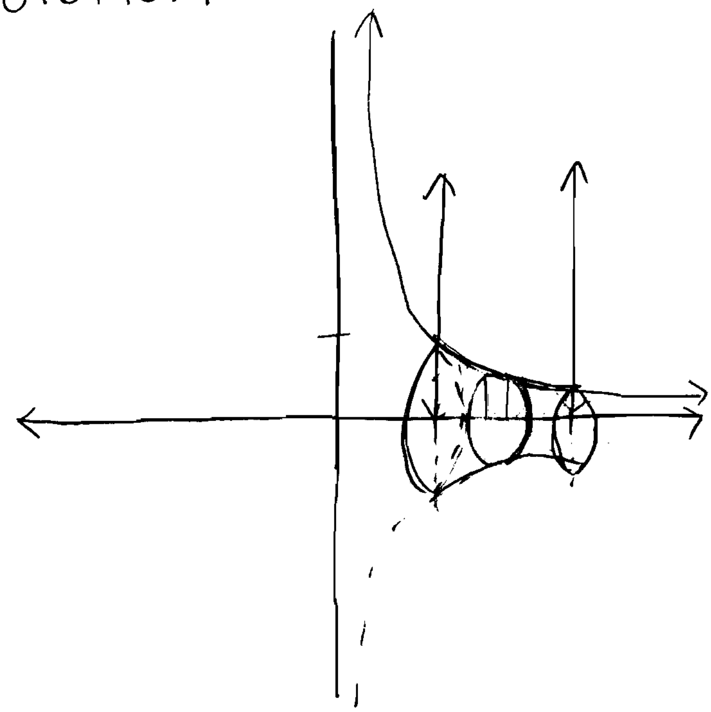
Solution

1. $y = \frac{1}{x}$

$x = 1$

$x = 2$

$y = 0$



$$V = \int_1^2 \pi \left(\frac{1}{x}\right)^2 dx$$

$$= \int_1^2 \pi \cdot \frac{1}{x^2}\, dx$$

$$= \pi \int_1^2 x^{-2}\, dx$$

$$= \pi \left[-\frac{1}{x}\right]_1^2$$

$$= \pi \left[-\frac{1}{2} - \left(-\frac{1}{1}\right)\right]$$

$$= \pi \left[-\frac{1}{2} + 1\right]$$

$$= \pi \left[\frac{1}{2}\right]$$

$$\boxed{= \frac{\pi}{2}}$$

Will Muldowne

6.2

2. $\int \sin^6 x \cos^3 x \, dx$

$= \int \sin^6 x \cos^2 x \cos x \, dx \qquad u = \sin x \quad du = \cos x \, dx$

$= \int \sin^6 x (1 - \sin^2 x) \cos x \, dx$

$= \int u^6 (1 - u^2) \, du$

$= \int (u^6 - u^8) \, du$

$= \frac{u^7}{7} - \frac{u^9}{9} + C = \boxed{\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C}$

8.7 (e, cos, sin) Maddie Gerling

1) $\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

find $\int_0^1 x\tan^{-1}(x)\,dx$

$$x\tan^{-1}(x) = x\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)!}$$

$$\int_0^1 x\tan^{-1}(x)\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)!\,(2n+3)} = \sum_{n=0}^{\infty} \frac{(-1)^n (1)^{2n+3}}{(2n+1)!(2n+3)}$$

$$\cancel{\frac{1}{3} - \frac{1^{2+3}}{(2)(5)} + \frac{1}{(4\cdot3\cdot2)(7)} - \frac{1}{(7)!(9)} + \frac{1}{9!}}$$

$$\frac{1}{3} - \frac{1}{3!(5)} + \frac{1}{5!(7)} - \frac{1}{7!(9)} + \frac{1}{9!(11)} \cdots$$

$$\frac{1}{3} - \frac{1}{30} + \frac{1}{840} - \frac{1}{45360} + \cdots = .3012$$

Ian Mahaney

Solution to Problem (B):

$$\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx = \int \frac{(x+1)(5x-2)}{(x^2)(x+2)}\,dx = \int \frac{Ax+B}{x^2} + \frac{C}{x+2}\,dx$$

$$(Ax+B)(x+2) + Cx^2 = (x+1)(5x-2)$$

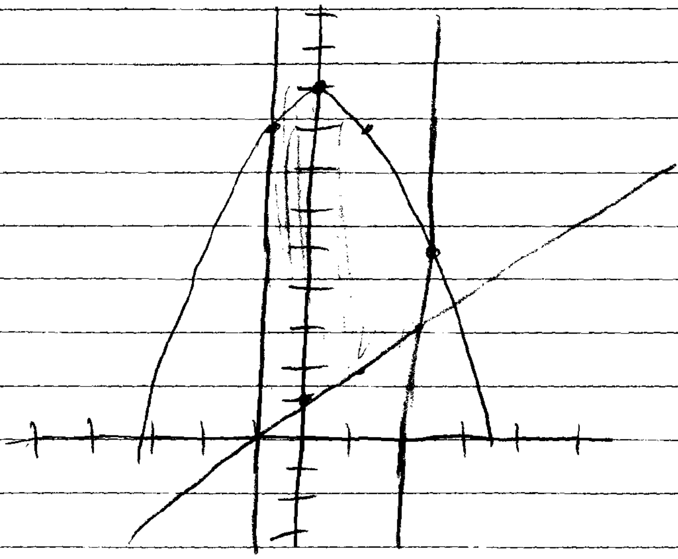
$X = 0$ | $x = -2$ | $x = 1 \quad C = 3$
$2B = -2$ | $4C = 12$ | $(A-1)3 + 3 = 6$
$B = -1$ | $C = 3$ | $3A \cancel{-3+3} = 6$
 | | $A = 2$

$$= \int \frac{2x-1}{x^2} + \frac{3}{x+2}\,dx = \int \frac{2}{x} - \int \frac{1}{x^2} + \int \frac{3}{x+2}$$

$$= 2\ln|x| + x^{-1} + 3\ln|x+2| + C$$

Aaron Bales

1) $y = x+1$, $y = 9 - x^2$, $x = -1$, $x = 2$



$= \int_{-1}^{2} [(9-x^2) - (x+1)]\,dx$

$= \int_{-1}^{2} (-x^2 - x + 8)\,dx$

$= -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 8x \Big|_{-1}^{2}$

$= 19.5$

Lily Marucci

Solution:

13) $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du =$

$\int_{2x}^{0} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du =$

$-\int_{0}^{2x} \frac{u^2-1}{u^2+1}\,du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\,du$

$g'(x) = -\frac{(2x)^2-1}{(2x)^2+1} \cdot \frac{d}{dx}(2x) + \frac{(3x)^2-1}{(3x)^2+1} \cdot \frac{d}{dx}(3x) =$

$\boxed{-2\left(\frac{4x^2-1}{4x^2+1}\right) + 3\left(\frac{9x^2-1}{9x^2+1}\right)}$

Joseph Wallace

Section 8.3 (P-Series) - Review Answers

26) $$\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$$

<u>limit comparison test</u>

$$a_n = \frac{n+5}{\sqrt[3]{n^7+n^2}} \qquad b_n = \frac{n}{\sqrt[3]{n^7}} = \frac{1}{n^{(7/3)-1}} = \frac{1}{n^{4/3}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}} \cdot n^{4/3}$$

$$= \lim_{n\to\infty} \frac{n^{7/3}+5n^{4/3}}{(n^7+n^2)^{1/3}}$$

$$= \lim_{n\to\infty} \frac{1+5n^{(4/3)-(7/3)}}{\left(1+\frac{1}{n^5}\right)^{1/3}}$$

$$= \lim_{n\to\infty} \frac{1+\frac{5}{n}}{\left(1+\frac{1}{n^5}\right)^{1/3}} = 1 > 0$$

Since the limit exists, and $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges since it is a p-series with $p = 4/3 > 1$.

Then $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$ also <u>converges</u> by the limit comparison test.

Kristin Crawford
12/9/09

7. $\sum_{n=1}^{\infty} \frac{1}{n^4}$ positive and decreasing on $[1, \infty)$

$f(x) = \frac{1}{x^4}$

$$\int_1^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \frac{x^{-3}}{-3}\Big]_1^{\infty}$$

$$\lim_{t\to\infty} \left(\frac{t^{-3}}{3} + \frac{1}{3} \right) = \frac{1}{3}$$

P-series with $P=4$, $4>1$, so the series converges

Nelson Siu

$$\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\right\}$$

$$\boxed{a_n = \left(-\frac{2}{3}\right)^{n-1}}$$

Solution

Annie Heaton
8.5

17 $\sum_{n=1}^{\infty} n!\,(2x-1)^n$

ratio test:

$$\lim_{n\to\infty}\left|\frac{(n+1)n!\,(2x-1)^{n+1}}{n!\,(2x-1)^n}\right| = \lim_{n\to\infty}\left|(n+1)\,(2x-1)\right|$$

diverges mostly
but converges if $(2x-1)=0$

$(2x-1)$ must be 0 to converge

so $2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$\boxed{R=0}$

$\boxed{IOC=\left\{\frac{1}{2}\right\}}$

Scott Gump

Answer

2. $\int_0^{0.3} \frac{x^2}{1+x^4}\,dx$

- find power series representation

$x^2 \cdot \frac{1}{1-(-x^4)}$

$= x^2 \sum_{n=0}^{\infty} (-x^4)^n$

$= \sum_{n=0}^{\infty} (-1)^n x^{4n+2}$

- Integrate

$\int_0^{0.3} \sum_{n=0}^{\infty} (-1)^n x^{4n+2}\,dx$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{4n+3} \Big|_0^{0.3}$

$\sum_{n=0}^{\infty} (-1)^n \frac{.3^{4n+3}}{4n+3} - 0$

n=2 for sixth decimal place.

$- \frac{.3^7}{7} + \frac{.3^{11}}{11}$

$\boxed{= -3.11 \times 10^{-5}}$

John
Nute

7.3 "Shells"

Q#1 $y = x^2, \quad y = 0, \quad x = 1$

$V = \int_a^b 2\pi x f(x)\,dx$

so, $V = \int_0^1 2\pi x (x^2)\,dx$

$= 2\pi \int_0^1 x^3\,dx$

$= 2\pi \left[\frac{1}{4}x^4\right]_0^1$

$= 2\pi \left[\frac{1}{4}(1)^4 - \frac{1}{4}(0)^4\right]$

$= 2\pi \left(\frac{1}{4}\right)$

$= \frac{1}{2}\pi$

Anna Yamamoto

1. (Solution)

$y = \ln(\cos x) \quad 0 \le x \le \frac{\pi}{3}$

$y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

$L = \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\ dx$

$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\ dx$

$= \int_0^{\pi/3} \sqrt{\sec^2 x}\ dx$

$= \int_0^{\pi/3} \sec x\ dx$ Use table #14 from the textbook!

$= \ln|\sec x + \tan x| \Big|_0^{\pi/3} \ln|\sec \frac{\pi}{3} + \tan \frac{\pi}{3}| - \ln|\sec 0 - \tan 0|$

$= \ln(2 + \sqrt{3}) - \ln(1) = \boxed{\ln(2 + \sqrt{3}).}$

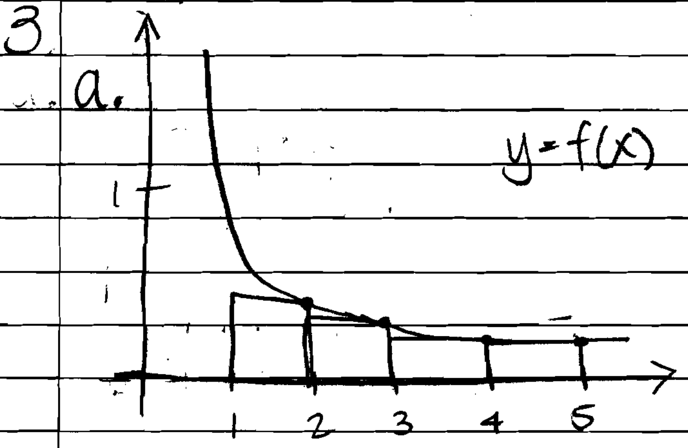
Erin Hong

5.1: Areas & Distances

Solution

3.

a.



$f(x) = \frac{1}{x}$ from $x=1$ to $x=5$

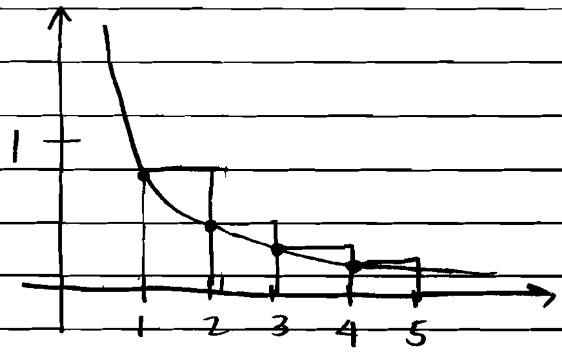
$\Delta x = 1$

$R_4 = 1\left(f(5) + f(4) + f(3) + f(2)\right)$

$R_4 = \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2}\right) = \boxed{\frac{77}{60}}$

underestimate

b.



$L_4 = 1\left(f(1) + f(2) + f(3) + f(4)\right)$

$L_4 = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \boxed{\frac{25}{12}}$

overestimate

Daniel Hagen

Solution #1

$$\sum_{n=1}^{\infty}\left(\frac{(n^2+1)}{(2n^2+1)}\right)^n$$

Root test

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n^2+1}{2n^2+1}\right)^n} = \lim_{n\to\infty}\frac{n^2+1}{2n^2+1} = 1/2$$

$1/2 < 1$ therefore the series

$\boxed{\text{converges absolutely}}$

Daniel Hagen

Solution #2

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{-1^{n+1}(n+1)}{(n+1)^2+1} \cdot \frac{n^2+1}{(-1)^n n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)(n^3+n^2+n+1)}{(n^2+2n+2)n} \right|$$

$$= \lim_{n\to\infty} \frac{n^3+n^2+n+1}{n^3+2n^2+2n} = 1 \quad \leftarrow \text{inconclusive}$$

Direct Comparison

$$\frac{n+1}{(n+1)^2+1} = \frac{n+1}{n^2+2n+2} \qquad 1)\ \frac{n+1}{n^2+2n+2} < \frac{n}{n^2+1}$$

and $2)\ \lim_{n\to\infty} \frac{n}{n^2+1} = 0$ therefore <u>converges</u>

Abs. convergence?

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} = \frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{4}{17} + \ldots \quad \leftarrow \text{corresponds to } \sum_{n=0}^{\infty} \frac{1}{n}$$

which is divergent therefore series is

$\boxed{\text{conditionally convergent}}$

Pat Kennu

. Approximate the function $f(x) = \sqrt[3]{x}$ by a Taylor polynomial of degree 2 at $a = 8$.

$f(x) = \sqrt[3]{x} = x^{1/3}$ $\qquad f(8) = 2$

$f'(x) = \frac{1}{3} x^{-2/3}$ $\qquad f'(8) = \frac{1}{12}$

$f''(x) = -\frac{2}{9} x^{-5/3}$ $\qquad f''(8) = -\frac{1}{144}$

$f'''(x) = \frac{10}{27} x^{-8/3}$

$$T_2(x) = f(8) + \frac{f'(8)}{1!}(x-8) + \frac{f''(8)}{2!}(x-8)^2$$

so

$$\sqrt[3]{x} \approx T_2(x) = 2 + \frac{1}{12}(x-8) + \frac{1}{288}(x-8)^2$$

a) Approximate the function $f(x) = e^{x^2}$, $a = 0$, $n = 3$, $0 \leq x \leq 0.1$

b) Use Taylor's Formula to estimate the accuracy of the approximation $f(x) \approx T_N(x)$

Hillary
Hawkins

6.6 solution sheet 1

25. $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} = \int_{-2}^{14} (x+2)^{-1/4} \cdot dx$

$= C + \frac{(x+2)^{-1/4+1}}{(-1/4+1)} \Big|_{-2}^{14} = \frac{(x+2)^{3/4}}{3/4} \Big|_{-2}^{14} = \frac{4(x+2)^{3/4}}{3} \Big|_{-2}^{14}$

$\frac{4(14+2)^{3/4}}{3} - \frac{4(-2+2)^{3/4}}{3} = \frac{4(16^{3/4})}{3} - \frac{4(0)^{3/4}}{3}$

$= \frac{4}{3}(8) - 0 = \boxed{\frac{32}{3} = 10.6667}$ $\boxed{\text{convergent}}$

1 a) $\int_1^{\infty} x^4 e^{-x^4} dx$

$= \lim_{t\to\infty} \int_1^t x^4 e^{-x^4} dx = \lim_{t\to\infty} -e^{-x^4} \Big|_1^t = \lim_{t\to\infty} -e^{-t^4} - (e^{(1)^4})$

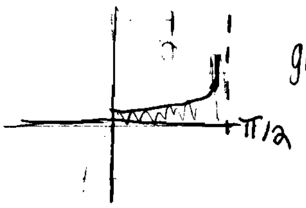
$= \lim_{t\to\infty} -e^{-t^4} + e^1 = \lim_{t\to\infty} -e^{-t^4} = \lim_{t\to\infty} -e/(t^4)$

infinite interval.

b) $\int_0^{\pi/2} \sec x \, dx = \lim_{t\to\pi/2} \int_0^t \sec x$

$= \lim_{t\to\pi/2} \ln|\sec x + \tan x| \Big|_0^t = \lim_{t\to\pi/2} \ln|\sec t + \tan t| - \ln|\sec 0 + \tan 0|$

$\sin \pi/2 = 1$
$\cos \pi/2 = 0$



graph shows undefined @ π/2

$= \lim_{t\to\pi/2} \ln|\sec t + \tan t| - \ln|1+0|$

$= \lim_{t\to\pi/2} \ln|\sec t + \tan t| - 0$

$\boxed{\text{Infinite discontinuity}}$ by def of an improper integral of type 2.

c) $\int_0^2 \frac{x \quad dx}{x^2 + -5x + 6} = \int_0^2 \frac{x}{(x-3)(x-2)} dx$ $\boxed{\text{infinite discontinuity}}$
@ 2 b/c when $x = 2$, $x - 2 = 0$, meaning $x/0$, cannot divide by $\emptyset$ so discontinuity.

Hillary Hawkins

Soln 6.6 sheet2

1) d) $\int_{-\infty}^{0} \frac{1}{x^2+5}\,dx = \int_{-\infty}^{0} (x^2+5)^{-1}\,dx = \lim_{b \to -\infty} \int_{b}^{0} (x^2+5)^{-1}$

$\boxed{\text{infinite interval}}$, makes it an improper integral

$= \lim_{b \to -\infty} \frac{\ln|x^2+5|}{2x} \Big|_{b}^{0} = \lim_{b \to -\infty} \frac{\ln|0+5|}{2(0)} - \frac{\ln|b^2+5|}{2b} = \lim_{b \to -\infty} \frac{\ln 5}{0} - \frac{\ln|b^2+5|}{2b}$

can't divide by ∅

$= \lim_{b \to -\infty} (\ln 5)/0 - \lim_{b \to -\infty} \frac{\ln|b^2+5|}{2b}$

$=$

Ben Brantley

1) $y = 12 - x^2 \qquad y = x^2 - 6$

$12 - x^2 = x^2 - 6$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

$\int_{-3}^{3} (12 - x^2) - (x^2 - 6)\, dx$

$\int_{-3}^{3} 12 - x^2 - x^2 + 6\, dx$

$\int_{-3}^{3} 18 - 2x^2\, dx$

$18x - \frac{2}{3}x^3 \Big|_{-3}^{3}$

$18(3) - \frac{2}{3}(27) - \left(18(-3) - \frac{2}{3}(-27)\right)$

$54 - 18 - (-54 + 18)$

$36 - (-36)$

$\boxed{72}$

Ellen Stidham

$$\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\, d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\cos^2\theta} \right) d\theta$$

$$= \int_0^{\pi/4} (\sec^2\theta + 1)\, d\theta$$

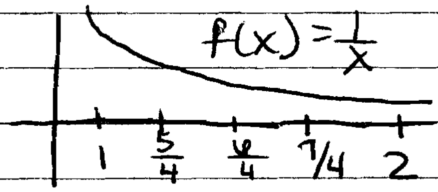
$$= \left[\tan\theta + \theta \right]_0^{\pi/4}$$

$$= (\tan \pi/4 + \pi/4) - (0+0)$$

$$\boxed{= 1 + \pi/4}$$

Kate
Partynski

1) $\int_1^2 \frac{1}{x}\,dx \qquad n=4$



a) Midpoint Rule

$M_4 = \frac{1}{4}\left(\frac{8}{9}+\frac{8}{11}+\frac{8}{13}+\frac{8}{15}\right) \approx 0.6912$

b) Trapezoidal Rule

$T_4 = \frac{1}{8}\left(1+2\left(\frac{4}{5}\right)+2\left(\frac{4}{6}\right)+2\left(\frac{4}{7}\right)+\frac{1}{2}\right) \approx 0.6970$

c) $S_4 = \frac{1}{12}\left(1+4\left(\frac{4}{5}\right)+2\left(\frac{4}{6}\right)+4\left(\frac{4}{7}\right)+\frac{1}{2}\right) \approx 0.6933$

answer

Jamie Shea

1 a) $P_d(x) = y = -0.1x^2 - 10x + 200$

$R(x) = x \cdot P_d(x)$

$= -0.1x^3 - 10x^2 + 200x$

$R'(x) = -0.3x^2 - 20x + 200$

$-0.3x^2 - 20x + 200 = 0$

$x = 8.83$

$P(8.83) = -0.1(8.83)^2 - 10(8.83) + 200$

$= \$110.92$

b) consumer surplus $= \int_0^x [pd(x) - p_0]\, dx$

$\int_0^{8.83} (-0.1)x^2 - 10x + 200 - 110.92 \quad dx$

$= \left. \frac{-1}{30}x^3 - 5x^2 + 89.08x \right]_0^{8.83}$

$= \$373.78$

44 answer

Kelly Fromm
8.7

$$\int \frac{\sin x}{x} dx$$

$$\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!\, x}$$

$R=\infty$

$$\int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}}$$

$\boxed{R=\infty}$

Samantha Baumstark

$$\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$$

let $a = 4$

$x = a\sin\theta$

$dx = a\cos\theta\,d\theta$

$a\cos\theta = \sqrt{a^2 - a^2\sin^2\theta}$

$$\int_0^{2\sqrt{3}} \frac{(4\sin\theta)^3}{\sqrt{16-(4\sin\theta)^2}} \cdot 4\cos\theta\,d\theta$$

$$\int_0^{2\sqrt{3}} \frac{(4\sin\theta)^3\, 4\cos\theta}{4\cos\theta}\,d\theta$$

$$\int_0^{2\sqrt{3}} (4\sin\theta)^3\,d\theta$$

$$64\int_0^{2\sqrt{3}} (1-\cos^2\theta)\sin\theta\,d\theta$$

$u = \cos\theta$

$-du = \sin\theta\,d\theta$

$$-64\int_0^{2\sqrt{3}} 1-u^2\,du$$

$$-64\left[u - \frac{u^3}{3}\right]_0^{2\sqrt{3}}$$

$$-64\left[\cos(2\sqrt{3}) - \frac{(\cos(2\sqrt{3}))^3}{3} - \cos(0) + \frac{(\cos(0))^3}{3}\right]$$

$$= \boxed{-1.33}$$

Andrew McCraven Solutions

8.3 Direct Comparison Test

9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

$1 \leq 1$

$n^2+n+1 \geq n^2$

$\dfrac{1}{n^2+n+1} \leq \dfrac{1}{n^2}$

compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2 \leftarrow p}}$ converges by the p-series test because $p>1$

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ converges by direct comparison test since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} \geq \sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

Duggan Clem

16) EXpress the limit as a definite integral

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{e^{x_i}}{1+x_i} \Delta x \quad [1,5]$$

Well thrm 4 states that:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i)\,dx$$

where $\Delta x = \frac{b-a}{n}$ $\quad x_i = a + i\Delta x$

So If we use this formula and replace $\frac{e^{x_i}}{1+x_i}$ with $\frac{e^x}{1+x}$

We get $\int_1^5 \frac{e^x}{1+x}\,dx$

Note: we replaced Δx with dx

So $\lim_{n\to\infty} \sum_{i=1}^{\infty} \frac{e^{x_i}}{1+x_i} \Delta x \overset{[1,5]}{=} \int_1^5 \frac{e^x}{1+x}\,dx$

Heidi Hirsh (solution)

51. $\lim_{x \to 0} \dfrac{x - \tan^{-1} x}{x^3}$

known:

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3} = \frac{\cancel{x} - \left(\cancel{x} - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{x^{3-3}}{3} - \frac{x^{5-3}}{5} + \frac{x^{7-3}}{7} - \cdots}{x^3}$$

$$= \lim_{x \to 0} \frac{1}{3} - \frac{x^2}{5} + \frac{x^4}{7} = 1/3$$

$\emptyset$ $\emptyset$

Brooke Candee

Final Review 12/09/09

Chapter 6.1 ~ Solutions

7. $\int x^2 \sin \pi x \, dx$

$u = x^2 \qquad dv = \sin(\pi x)\,dx$

$du = 2x\,dx \qquad v = -\dfrac{\cos(\pi x)}{\pi}$

$$= -x^2 \cdot \frac{\cos(\pi x)}{\pi} + \int \frac{2x\cos(\pi x)}{\pi}\,dx$$

parts again

$u = \dfrac{2x}{\pi} \qquad dv = \cos(\pi x)\,dx$

$du = \dfrac{2}{\pi}\,dx \qquad v = \dfrac{\sin(\pi x)}{\pi}$

$$= -\frac{x^2\cos(\pi x)}{\pi} + \frac{2x\sin(\pi x)}{\pi^2} - \int \frac{2\sin(\pi x)}{\pi^2}$$

$$\boxed{= -\frac{x^2\cos(\pi x)}{\pi} + \frac{2x\sin(\pi x)}{\pi^2} - \frac{2\cos(\pi x)}{\pi^3} + C}$$