Bill Thompson

Sections 5.1–5.2 page 2

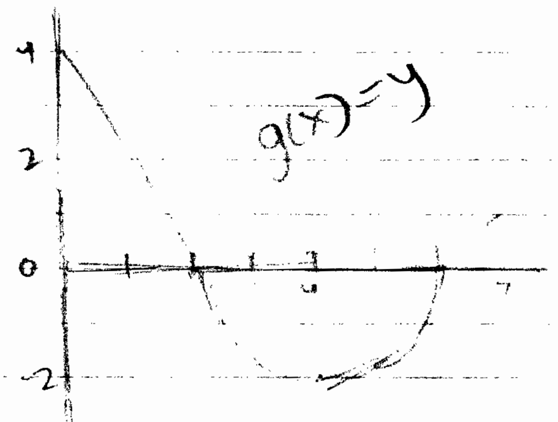
5.1 13) Use definition 2 to find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$f(x) = \sqrt[4]{x}, \ 1 \le x \le 16$

$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \dots f(x_n)\Delta x]$

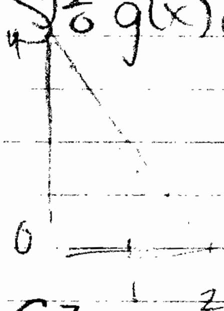
$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt[4]{1 + \frac{15i}{n}}\right)\left(\frac{15}{n}\right)$

5.2 30)



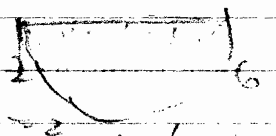
The graph of g consists of two straight lines and a semicircle. Use it to evaluate each integral.

A) $\int_0^2 g(x)\,dx$



$\frac{1}{2}(4 \cdot 2) = \boxed{4}$

B) $\int_2^6 g(x)\,dx$



$\frac{1}{2}(2^2\pi) = \boxed{2\pi}$

C) $\int_0^7 g(x)\,dx$

$\int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$

$4 \quad - 2\pi \quad + \frac{1}{2}$

$\boxed{4\tfrac{1}{2} - 2\pi}$

$1(\frac{1}{2}) = \frac{1}{2}$

Justin Levin

5.5 Answers

1) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ $\quad u=\sqrt{x}=x^{1/2}$
$du=\frac{1}{2}x^{-1/2}dx=\frac{1}{2\sqrt{x}}dx$

$\Rightarrow 2\int \sin u\,du$

$\Rightarrow -2\cos u + C \Rightarrow -2\cos\sqrt{x}+C$

2) $\int \frac{e^x}{1+e^x}dx$ $\quad u=e^x$
$du=e^x dx$

$\Rightarrow \int \frac{du}{1+u}$

$\Rightarrow \ln|1+u|+C \Rightarrow \ln|e^x+1|+C$

Tyler Stoll

Section 6.1

13) $\int e^{2\theta}\sin 3\theta\, d\theta$

Let $I = \int e^{2\theta}\sin 3\theta\, d\theta$

Let $u = e^{2\theta}$ $\quad dv = \sin 3\theta\, d\theta$

$du = 2e^{2\theta}d\theta$ $\quad v = -\frac{\cos 3\theta}{3}$

$$I = e^{2\theta}\frac{(-\cos 3\theta)}{3} - \int \frac{(-\cos 3\theta)}{3} 2e^{2\theta}d\theta$$

$$I = \frac{-e^{2\theta}\cos 3\theta}{3} + 2/9\, e^{2\theta}\sin 3\theta - 4/9 \int e^{2\theta}\sin 3\theta\, d\theta$$

$$I = \frac{-e^{2\theta}\cos 3\theta}{3} + 2/9\, e^{2\theta}\sin 3\theta - 4/9\, I$$

$$I + 4/9\, I = \frac{-e^{2\theta}(\cos 3\theta)}{3} + 2/9\, e^{2\theta}\sin 3\theta + C$$

$$13/9\, I = \frac{-e^{2\theta}(\cos 3\theta)}{3} + \frac{2}{9}e^{2\theta}\sin 3\theta + C$$

$$I = -3/13\, e^{2\theta}\cos 3\theta + \frac{2}{13}e^{2\theta}\sin 3\theta + C$$

$$I = \frac{1}{13}e^{2\theta}(2\sin 3\theta - 3\cos 3\theta + C)$$

Jessica Flores-Vazquez
12/7/10

Answers: Section 6.2

(#2) $\int \sin^6 x \cos^3 x \, dx$

$= \int \sin^6 x \cdot \cos \cdot \cos^2 x \, dx$

$= \int \sin^6 x \cdot (1-\sin^2 x) \cdot \cos x \, dx$

$u = \sin x$
$du = \cos x \, dx$

$= \int u^6 (1-u^2) \, du$

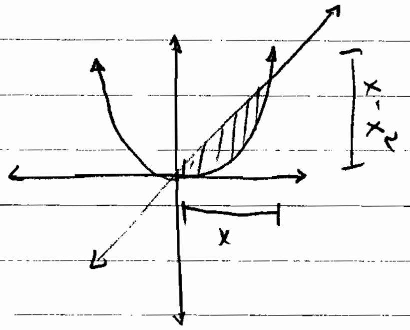
$= \int u^6 - u^8 \, du = \frac{u^7}{7} - \frac{u^9}{9} + C = \boxed{\frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C}$

(#19) $\int \tan^2 x \, dx$

$= \int 1 - \sec^2 x \, dx = \boxed{x - \tan x + C}$

TODD COPE

(1)



radius x

$C = 2\pi x$

height $x - x^2$

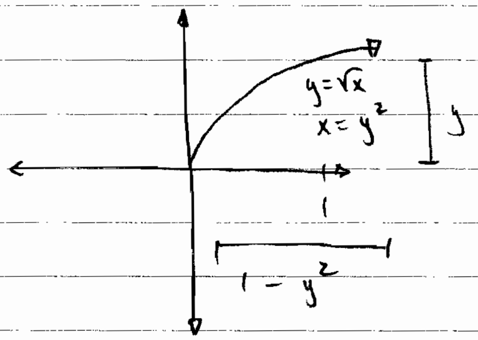
$$V = \int_0^1 (2\pi x)(x - x^2)\,dx$$

$$V = 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{\pi}{6}$$

(2)



radius y

$C = 2\pi y$

height $1 - y^2$

$$V = \int_0^1 (2\pi y)(1 - y^2)\,dy$$

$$V = 2\pi \int_0^1 (y - y^3)\,dy$$

$$= 2\pi \left[\frac{y^2}{2} - \frac{y^4}{4} \right]_0^1$$

$$= \frac{\pi}{2}$$

9. $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$

$= \displaystyle\int \left(\frac{A}{(x+5)} + \frac{B}{(x-2)}\right) dx$

$= \displaystyle\int \left[\left(\frac{A}{(x+5)} + \frac{B}{(x-2)}\right)(x+5)(x-2)\right] dx$

$= \displaystyle\int A(x-2) + B(x+5)\ dx$

$x - 9 = A(x-2) + B(x+5)$

At $x = 2$

$2 - 9 = A(2-2) + B(2+5)$

$-7 = A(0) + 7B$

$-1 = B$

At $x = -5$

$-5-9 = A(-5-2) + B(-5+5)$

$-14 = -7A + B(0)$

$2 = A$

$= \displaystyle\int \frac{2}{(x+5)} - \frac{1}{(x-2)}$

$\boxed{= 2\ln|x+5| - \ln|x-2|}$

11. $\displaystyle\int_2^3 \frac{1}{x^2-1}\,dx$

$= \displaystyle\int_2^3 \frac{1}{(x+1)(x-1)}\,dx$

$= \displaystyle\int_2^3 \left(\frac{A}{(x+1)} + \frac{B}{(x-1)}\right) dx$

$= \displaystyle\int_2^3 \left[\left(\frac{A}{(x+1)} + \frac{B}{(x-1)}\right)(x+1)(x-1)\right] dx$

$= \displaystyle\int_2^3 A(x-1) + B(x+1)\ dx$

$1 = A(x-1) + B(x+1)$

At $x = 1$

$1 = A(1-1) + B(1+1)$

$1 = A(0) + 2B$

$1/2 = A$

At $x = -1$

$1 = A(-1-1) + B(-1+1)$

$1 = -2A + B(0)$

$-1/2 = A$

$= \displaystyle\int_2^3 -\frac{1}{2}\frac{1}{(x-1)} + \frac{1}{2}\frac{1}{(x+1)}\,dx$

$= \left[-\frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1|\right]_2^3$

$= \left[\left(-\frac{1}{2}\ln|2| + \frac{1}{2}\ln|4|\right) - \left(-\frac{1}{2}\ln|1| + \frac{1}{2}\ln|3|\right)\right]$

$= \frac{1}{2}\ln|3/2|$

Section 6.4 Review

Tori Mauser-Jeppesen

9. Evaluate $\int \frac{\tan^3(1/z)\,dz}{z^2}$

Let $\frac{1}{z} = t$ then $-\frac{1}{z^2}dz = dt$

$\hookrightarrow \frac{1}{z^2}dz = -dt$

So, $\int \frac{\tan^3(1/z)\,dz}{z^2} = -\int \tan^3 t\,dt$

use table: $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

$\hookrightarrow -\int \tan^3 t\,dt = -\left(\frac{1}{2}\tan^2(1/z) + \ln|\cos 1/z|\right) + C$

$= -\frac{1}{2}\tan^2(1/z) - \ln|\cos(1/z)| + C$

17. Evaluate $\int \frac{x^4\,dx}{\sqrt{x^{10}-2}}$

Let $x^5 = t$

$\hookrightarrow 5x^4\,dx = dt$

$\hookrightarrow x^4\,dx = \frac{dt}{5}$

So, $\int \frac{x^4\,dx}{\sqrt{x^{10}-2}} = \frac{1}{5}\int \frac{dt}{\sqrt{t^2-2}}$

use table: $\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u + \sqrt{u^2+a^2}\right| + C$

$\hookrightarrow \frac{1}{5}\int \frac{dt}{\sqrt{t^2-2}} = \frac{1}{5}\ln\left|t + \sqrt{t^2-2}\right| + C$

$= \frac{1}{5}\ln\left|x^5 + \sqrt{x^{10}-2}\right| + C$

Section 6.6: Improper Integrals

Christopher

(18) $\displaystyle\int_{-\infty}^{6} re^{r/3}\,dr = \lim_{b\to-\infty}\int_b^6 re^{r/3}\,dr$

Integration by parts: Let $u = r$, $dv = e^{r/3}\,dr$, $du = dr$, $v = 3e^{r/3}$

$$= \lim_{b\to-\infty}\left[3re^{r/3}\Big|_b^6 - 3\int_b^6 e^{r/3}\,dr\right]$$

$$= \lim_{b\to-\infty}\left[\left(18e^2 - 3be^{b/3}\right) - 9\left(e^2 - e^{b/3}\right)\right]$$

$$\lim_{b\to-\infty}\frac{3b}{e^{-b/3}}$$

$\downarrow$ L'Hopital

$$\lim_{b\to-\infty}\frac{3}{-\frac{1}{3}e^{-b/3}}$$

0

$$= (18e^2 - 0) - 9(e^2 - 0)$$

$$= 9e^2$$

(37) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{b\to1^-}\int_0^b \frac{dx}{\sqrt{1-x^2}}$

$$= \lim_{b\to1^-}\left[\sin^{-1}(x)\right]\Big|_0^b$$

$$= \lim_{b\to1^-}\sin^{-1}(b) - \sin^{-1}(0)$$

$$= \frac{\pi}{2} - 0$$

$$= \frac{\pi}{2}$$

(47) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx \longrightarrow \int_0^\infty \frac{2}{1+u^2}\,du = \int_0^1 \frac{2}{1+u^2}\,du + \int_1^\infty \frac{2}{1+u^2}\,du$

Substitution: Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$$= \lim_{a\to0^+}\left[\int_a^1 \frac{2}{1+u^2}\,du\right] + \lim_{b\to\infty}\left[\int_1^b \frac{2}{1+u^2}\,du\right]$$

$$= \lim_{a\to0^+}\left[2\tan^{-1}(u)\right]\Big|_a^1 + \lim_{b\to\infty}\left[2\tan^{-1}(u)\right]\Big|_1^b$$

$$= \lim_{a\to0^+}\left[2\tan^{-1}(1) - 2\tan^{-1}(a)\right] +$$

$$\lim_{b\to\infty}\left[2\tan^{-1}(b) - 2\tan^{-1}(1)\right]$$

$$= \left[2\left(\tfrac{\pi}{4}\right) - 2(0)\right] + \left[2\left(\tfrac{\pi}{2}\right) - 2\left(\tfrac{\pi}{4}\right)\right]$$

$$= \pi$$

6.5 Solutions

(3) a) $n=4$, $a=0$, $b=1$ $\Big\rangle$ $\Delta x = \frac{1-0}{4} = .25$

so by Trapezoidal Rule,

$$\int_0^1 \cos(x^2)\,dx = \frac{.25}{2}\left[f(0) + 2f(.25) + 2f(.5) + 2f(.75) + f(1)\right]$$

$$= \frac{.25}{2}\left[\cos(0) + 2\cos(.25^2) + 2\cos(.5^2) + 2\cos(.75^2) + \cos(1^2)\right]$$

$$= \frac{.25}{2}\left[1 + 2\cos(.0625) + 2\cos(.25) + 2\cos(.5625) + \cos 1\right]$$

plug into calculator

$$\approx .895759$$

- underestimate

b) The midpoints of the four subintervals are $.125, .375, .625, .875$

so by the Midpoint Rule,

$$\int_0^1 \cos(x^2)\,dx = .25\left[f(.125) + f(.375) + f(.625) + f(.875)\right]$$

$$= .25\left[\cos(.125^2) + \cos(.375^2) + \cos(.625^2) + \cos(.875^2)\right]$$

plug into calculator

$$\approx .908906$$

- overestimate

$T_4 < I < M_4$

(15) a) $n=6$ $a=0$ $b=3$ $\Delta x = \frac{1}{2}$

$$\int_0^3 \frac{1}{1+y^5}\,dy = \frac{1}{4}\left[\frac{1}{1+0} + 2\cdot\frac{1}{1+.5^5} + 2\cdot\frac{1}{1+1^5} + 2\cdot\frac{1}{1+1.5^5} + 2\cdot\frac{1}{1+2^5} + 2\cdot\frac{1}{1+2.5^5} + \frac{1}{1+3^5}\right]$$

≈ 1.064275

b) 6 midpoints: $.25, .75, 1.25, 1.75, 2.25, 2.75$

$$\int_0^3 \frac{1}{1+y^5}\,dy = \frac{1}{2}\left(\frac{1}{1+.25^5} + \frac{1}{1+.75^5} + \frac{1}{1+1.25^5} + \frac{1}{1+1.75^2} + \frac{1}{1+2.25^5} + \frac{1}{1+2.75^3}\right)$$

≈ 1.067416

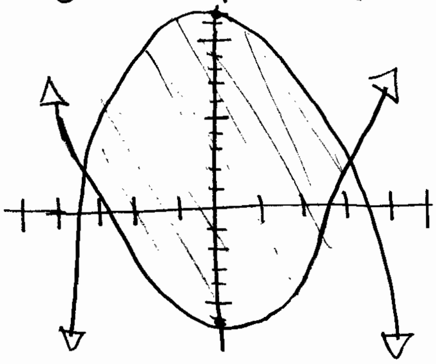
c) $$\int_0^3 \frac{1}{1+y^5}\,dy = \frac{1}{6}\left[\frac{1}{1+0^5} + \frac{4}{1+.5^5} + \frac{2}{1+1^5} + \frac{4}{1+1.5^5} + \frac{2}{1+2^5} + \frac{4}{1+2.5^5} + \frac{1}{1+3^5}\right]$$

≈ 1.074915

REVIEW PROBLEMS SOLUTIONS

Diane Lawrence

1.



$$A = \int_{-3}^{3} (12 - x^2) - (x^2 - 6)\,dx$$

$$A = \int_{-3}^{3} 18 - 2x^2\,dx$$

$$A = \left[18x - \frac{2}{3}x^3\right]_{-3}^{3}$$

$$A = \left(18(3) - \frac{2}{3}(3)^3\right) - \left(18(-3) - \frac{2}{3}(-3)^3\right)$$

$$A = (54 - 18) - (-54 + 18)$$

$$A = 72$$

3. $y = 1 + 6x^{3/2}$

$$y' = 9x^{1/2}$$

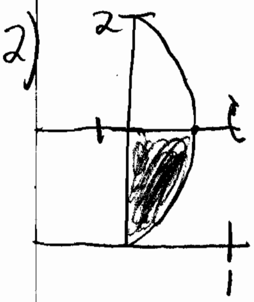
$$L = \int_{0}^{1} \sqrt{(9x^{1/2})^2 + 1}\,dx$$

$$L = \int_{0}^{1} \sqrt{81x^{1/4} + 1}\,dx$$

Sean Urban

Review Answers 7.2

2)



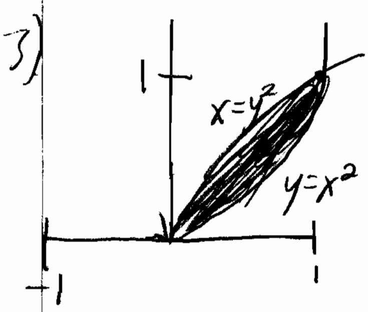
$\pi\int(\text{outer} - \text{inner})^2\,dx$

outer - $y=1$

inner - $\tan^3 x$

from 0 to $\pi/4$ $\boxed{\pi\int_0^{\pi/4}(1-\tan^3 x)^2\,dx}$

3)



$\pi\int(\text{outer})^2-(\text{inner})^2\,dy$

outer - $1+\sqrt{y}$

inner - $1+y^2$

from 0 to 1

$\pi\int_0^1 (1+\sqrt{y})^2-(1+y^2)^2\,dy$

$\pi\int_0 1+2\sqrt{y}+y-1-2y^2-y^4\,dy$

$\pi\int_0 2\sqrt{y}+y-2y^2-y^4\,dy$

$\pi\left(\frac{4}{3}y^{3/2}+\frac{1}{2}y^2-\frac{2}{3}y^3-\frac{1}{5}y^5\right)\Big|_0^1$

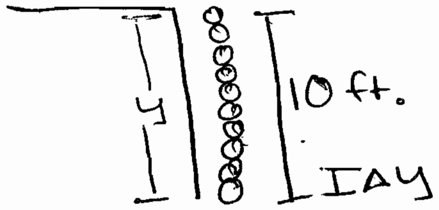
$\pi\left(\frac{4}{3}+\frac{1}{2}-\frac{2}{3}-\frac{1}{5}\right)$

$\frac{40}{30}+\frac{15}{30}-\frac{20}{30}-\frac{6}{30}=\left(\frac{29}{30}\right)\pi=\boxed{\frac{29\pi}{30}}$

Tawni Paradise

Section 7.5 Solutions

14)



$10-2y$ = distance chain travels

$$\frac{25 \text{ lbs}}{10 \text{ ft}} = 2.5 \frac{\text{lbs}}{\text{ft}} \cdot \text{DISTANCE}$$

when $y=5$ chain is at top so distance is 0
$10-2(5)=0$ ✓

you can only pull up half the rope to top $\frac{10}{2}=5$

$$\int_0^5 2.5(10-2y)\,\Delta y$$

$$= \int_0^5 25-5y\,\Delta y = 25y - \frac{5y^2}{2}\Big|_0^5$$

$$= 25(5) - \frac{5(5^2)}{2} - 0 =$$

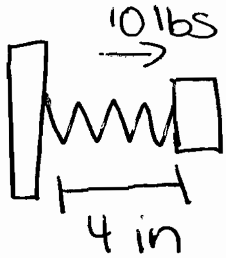
$$= 125 - \frac{125}{2} = \frac{125}{2} = 62.5 \text{ lbs}\cdot\text{ft}$$

Tawni Paradise

Section 7.5 Solution

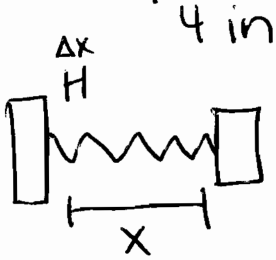
5) $\dfrac{10\text{ lb}}{\left(\frac{1}{3}\text{ ft}\right)}$ $\quad 4\text{ in} = \frac{1}{3}\text{ ft}$

$= 30\ \dfrac{\text{lb}}{\text{ft}}$



DISTANCE = X

Δx = stretching distance



$6\text{ in} = \frac{1}{2}\text{ ft}$

6 in beyond natural length

$\displaystyle\int_0^{\frac{1}{2}} 30x\,\Delta x$

$= \dfrac{30x^2}{2}\Big|_0^{1/2} = 15x^2\Big|_0^{1/2}$

$= 15\left(\frac{1}{4}\right) - 0$

$\boxed{= \frac{15}{4}\text{ lbs}\cdot\text{ft}}$

Austin Maul

$\boxed{\text{Ch 7.6 Differential Equations}}$

3 $(x^2+1)y' = xy$

$(x^2+1)\frac{dy}{dx} = xy$

$\frac{dy}{dx} = \frac{xy}{(x^2+1)}$

$\int \frac{1}{y}\, dy = \int \frac{x}{x^2+1}\, dx$

$\ln|y| = \frac{1}{2}\ln|x^2+1| + C$

$\ln|y| = \ln|\sqrt{x^2+1}| + C$

$y = \sqrt{x^2+1} + e^C$

$y = K\sqrt{x^2+1}$

5 $(1+\tan y)\,y' = x^2+1$

$(1+\tan y)\frac{dy}{dx} = x^2+1$

$\frac{dy}{dx} = \frac{x^2+1}{1+\tan y}$

$\int 1+\tan y\, dy = \int x^2+1\, dx$

$y + \ln|\sec y| = \frac{x^3}{3} + x + C$

or $y - \ln|\cos y| = \frac{x^3}{3} + x + C$

19 $y' = \frac{\sin x}{\sin y}$ for $y(0) = \pi/2$

7.6 | Solutions for Differential Equations

Johan Tillekeratne

1) $y' = y^2 \sin x$

$$\frac{dy}{dx} = \frac{\sin x}{y^{-2}}$$

$$y^{-2}\,dy = \sin x\,dx$$

$$\int y^{-2}\,dy = \int \sin x\,dx$$

$$-y^{-1} + C_1 = -\cos x + C_2$$

$$-y^{-1} = -\cos x + K$$

$$\boxed{y = \frac{-1}{(K - \cos x)}}$$

3) $\frac{dy}{dx} = \frac{y\cos x}{1+y^2}$, $y(0)=1$

$$\frac{dy}{dx} = \frac{\cos x}{y^{-1}(1+y^2)}$$

$$y^{-1}(1+y^2)\,dy = \cos x\,dx$$

$$\int (y^{-1} + y^{-1})\,dy = \int \cos x\,dx$$

$$2\ln y + C_1 = \sin x + C_2$$

$$\ln y = \frac{\sin x + K}{2}$$

$$y = e^{\frac{\sin x}{2} + K}$$

$$K = 0 \longrightarrow \boxed{y = e^{\frac{\sin x}{2}}}$$

Luke Nicol
8.2 Review
MATH 151 Fall '10

17. $\sum_{n=1}^{\infty} \arctan(n)$

$$\lim_{n\to\infty} \arctan(n) = \frac{\pi}{2}$$

Since $\lim_{n\to\infty} \arctan(n) \neq 0$ then $\sum_{n=1}^{\infty} \arctan(n)$ must diverge by the n^{th} term test.

25. $3.\overline{417} = 3.417417417\ldots$

$$= 3 + \frac{417}{10^3} + \frac{417}{10^6} + \frac{417}{10^9} + \ldots$$

$$a = \frac{417}{10^3} \qquad r = \frac{1}{10^3}$$

$$\frac{a}{1-r}$$

$$3 + \frac{\frac{417}{10^3}}{1 - \frac{1}{10^3}}$$

$$= 3 + \frac{\frac{417}{1000}}{\frac{999}{1000}}$$

$$= 3 + \frac{417}{1000} \cdot \frac{1000}{999}$$

$$= \frac{2997}{999} + \frac{417}{999}$$

$$= \frac{3414}{999}$$

$$= \frac{1138}{333}$$

Alex Demler

8.3 Answers

(9) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

$$n^2 \le n^2+n+1$$

$$\frac{1}{n^2} \ge \frac{1}{n^2+n+1}$$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series test since $p = 2 > 1$

- Since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges and $\frac{1}{n^2} \ge \frac{1}{n^2+n+1}$, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$ also converges by comparison test.

(15) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ we can use integral test since $\frac{1}{n \ln n}$ is positive and decreasing

$$\int_2^{\infty} \frac{dn}{n \ln n} \qquad u = \ln n, \quad du = \frac{dn}{n}$$

$$= \lim_{B\to\infty} \int_2^{B} \frac{du}{u}$$

$$= \lim_{B\to\infty} \Big[\ln u\Big]_2^{B}$$

$$= \lim_{B\to\infty} (\ln B - \ln 2)$$

$$= \ln \infty - \ln 2$$

$$= \infty - \ln 2$$

diverges

Since $\displaystyle\int_2^{\infty} \frac{dn}{n \ln n}$ diverges, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is also divergent by integral test.

Alex Demler

8.3 Answers (Continued)

(21) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ we can use limit comparison test

$a_n = \dfrac{1}{\sqrt{n^2+1}}$

$b_n = \dfrac{1}{\sqrt{n^2}} = \dfrac{1}{n}$

b_n diverges because it's harmonic.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} \left(\frac{\sqrt{\frac{1}{n^2}}}{\sqrt{\frac{1}{n^2}}} \right)$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = 1 > 0$$

Since $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} > 0$, and b_n diverges, then $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$ must also diverge by limit comparison test.

Kayla Costello

Section 8.4

Approximate the sum of the series correct to four decimal places:

1. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n^2)}{10^n} = \underbrace{\frac{1}{10} - \frac{2^2}{10^2} + \frac{3^2}{10^3} - \frac{4^2}{10^4} + \frac{5^2}{10^5} - \frac{6^2}{10^6}}_{\text{add}} + \ldots$

$n=7 \quad \frac{7^2}{10^7} = 4.9 \times 10^{-6}$

add $= \boxed{.0676}$

Determine if the series is absolutely convergent, conditionally convergent or divergent

2. $\sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{3})}{n!} \qquad \frac{|\cos(\frac{n\pi}{3})|}{n!} \leq \frac{1}{n!}$

$\sum_{n=1}^{\infty} \frac{1}{n!}$, $b_n = \frac{1}{n!}$, $\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = \lim_{n\to\infty} \left|\frac{1}{(n+1)!} \cdot \frac{n!}{1}\right|$

$= \lim_{n\to\infty} \left|\frac{1}{n+1}\right| = 0 < 1$ $\boxed{\text{By Ratio Test } \sum_{n=1}^{\infty} \frac{1}{n!} \text{ is convergent}}$

$\boxed{\text{By comparison test, } \sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{3})}{n!} \text{ is absolutely convergent}}$

3. $\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}} \qquad \lim_{n\to\infty} \sqrt[n]{\left|\frac{n^n}{3^{1+3n}}\right|} = \lim_{n\to\infty} \left|\frac{n}{3^{\frac{1}{n}} \cdot 3^3}\right| = \left|\frac{\infty}{3^0 \cdot 27}\right|$

$\infty > 1$ $\boxed{\text{Thus, by the Root Test } \sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}} \text{ diverges}}$

Kaitlin Soper

Calculus II
Final Exam Review
Power Series

Chapter 8.5 - Solutions

① $\displaystyle\sum_{n=0}^{\infty} \frac{(x-5)^n}{2^n n}$

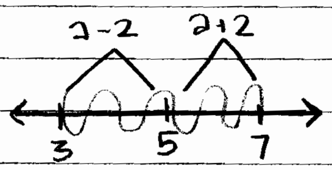
tep #1: USE RATIO TEST

$$\lim_{n\to\infty} \left| \frac{(x-5)^{n+1}}{2^{n+1}\, n+1} \cdot \frac{2^n n}{(x-5)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-5)\,n}{2(n+1)} \right| = \frac{|x-5|}{2} \lim_{n\to\infty} \left| \frac{n}{n+1} \right|$$

$$= \frac{|x-5|}{2} < 1$$

$|x-5| < 2$ so Radius of Convergence $= 2$



5 is center because
a = 5

tep #2: check endpoints: (3, 5) put into original eqn.

$\boxed{x=3}$

$$\sum_{n=0}^{\infty} \frac{-2^n}{2^n n} = \sum_{n=0}^{\infty} \frac{-1^n}{n} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{n}$$

Alt. Series Test:
$b_n = \frac{1}{n}$
• Deacreasing? YES
• $\lim_{n\to\infty} = 0$? YES

$\therefore$ CONVERGES

$\boxed{x=7}$

$$\sum_{n=0}^{\infty} \frac{2^n}{2^n n} = \sum_{n=0}^{\infty} \frac{1}{n}$$

Harmonic Series $\therefore$ DIVERGES

Therefore, the interval of convergence
$[3, 7)$

Kaitlin Soper

Chapter 8.5 - Solutions cntd.

(2) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

Step #1: use RATIO TEST

$$\lim_{n\to\infty}\left|\frac{x^{2(n+1)}}{(2n+2)!}\cdot\frac{(2n)!}{x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{x^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^2}{(2n+2)(2n+1)}\right|$$

$$= |x^2| \lim_{n\to\infty}\left|\frac{1}{(2n+2)(2n+1)}\right|$$

$0<1$; series will converge for ALL VALUES OF X

So, the radius of convergence is $R=\infty$, making the interval of convergence $(-\infty, \infty)$

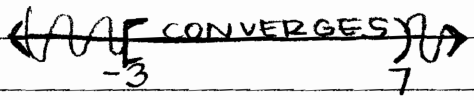
(3)

(a) CONVERGE
because $x=1$, inside convergence

(b) DIVERGE
because $x>7$

(c) DIVERGE
because $x<-3$

(d) CONVERGE
because $x=-2$, inside convergence



8.6

Jackie Watson

5.

Find a power series representation for the function and determine the radius of convergence

$f(x) = \ln(5-x)$

$f'(x) = -\frac{1}{5-x}$

since $\frac{1}{5-\cancel{x}} = \frac{1}{5(1-\frac{x}{5})} = \frac{1}{5}\sum_{n=0}^{\infty}(x/5)^n \qquad |x/5|<1$

$= \sum_{n=0}^{\infty}\frac{x^n}{5^{n+1}} \qquad |x|<5$

8.6

Jackie Watson

27.

Use a power series to approximate the definite integral to six decimal places.

$$\int_0^{.2} \frac{1}{1+x^2}\,dx$$

$$\frac{1}{1+x^5} = \frac{1}{1-(-x^5)} = \sum_{n=0}^{\infty} (-x^5)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{5n}$$

$$\left(1 + (-3.2\times10^{-4}) + (1.024\times10^{-7})\right) + (-3.2768\times10^{-11})$$

$$= .999680$$

Patrick Smith

psmith@sandiego.edu

8.7

7. $f(x)=e^{5x}$ $f(0)=1$

$f'(x)=5e^{5x}$ $f'(0)=5$

$f''(x)=25e^{5x}$ $f''(0)=25$

$f'''(x)=125e^{5x}$ $f'''(0)=125$

$f(x)=f(0)+\frac{f'(0)}{n!}x+\frac{f''(0)}{n!}x^2+\frac{f'''(0)}{n!}x^3+\cdots$

$=1+\frac{5}{n!}x+\frac{25}{n!}x^2+\frac{125}{n!}x^3+\cdots=\boxed{\sum_{n=0}^{\infty}\frac{5^n}{n!}x^n}$

$\lim_{n\to\infty}\left|\frac{5^{n+1}x^{n+1}}{(n+1)!}\cdot\frac{n!}{5^n x^n}\right|=|5x|\lim_{n\to\infty}\left|\frac{1}{(n+1)!}\right|<1$

$|5x|<\infty$ $|x|<\infty$ $\boxed{R=\infty}$

27. $f(x)=\cos(\pi x)$

We already know that $\cos(x)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

We now substitute πx in for x in the original equation:

$\cos(\pi x)=\sum_{n=0}^{\infty}(-1)^n\frac{(\pi x)^{2n}}{(2n)!}=\boxed{\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{(2n)!}x^{2n}}$

$\lim_{n\to\infty}\left|\frac{\pi^{2n+2}x^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{\pi^{2n}x^{2n}}\right|=|\pi^2x^2|\lim_{n\to\infty}\left|\frac{1}{(2n+1)(2n+2)}\right|<1$

$\pi^2x^2<\infty$ $x^2<\infty$ $x<\infty$ $\boxed{R=\infty}$

Kimberly Su
Math 151-53

Section 8.7 Solutions

(1) $f(x) = 1+x+x^2,\ a=2$

$f(x) = 1+x+x^2$ $\qquad f(2) = 1+2+4 = 7$

$f'(x) = 1+2x$ $\qquad f'(2) = 1+4 = 5$

$f''(x) = 2$ $\qquad f''(2) = 2$

$f'''(x) = 0$ $\qquad f'''(2) = 0$

$$f(x) = 1+x+x^2 = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 + 0 =$$

$$7+5(x-2) + \frac{2(x-2)^2}{2!} + 0 = \boxed{7+5(x-2)+(x-2)^2}$$

$\boxed{\text{radius of convergence: } \infty}$

(3) $$\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$$

$$e^x = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$$

$$e^{x^4} = \sum_{n=0}^{\infty} (-1)^n \frac{(x^4)^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$$

$\boxed{\text{sum of series} = e^{x^4}}$