Problems from October 26, 2009

1. Show that $y = x - x^{-1}$ is a solution of the differential equation $xy' + y = 2x$.

2. Verify that $y = \sin(x)\cos(x) - \cos(x)$ is a solution of the initial value problem $y' + (\tan x)y = \cos^2 x$ and $y(0) = 1$ on the interval $-\pi/2 < x < \pi/2$.

3. Remember in class we showed that the spring mass system was governed by the differential equation $mx'' = -kx$.

   (a) For what values of k does the function $x = \cos(kt)$ satisfy the differential equation $4x'' = -25x$?

   (b) For those values of k, verify that every member of the family of functions $x = A\sin(kt) + B\cos(kt)$ is also a solution.

4. Which of the following functions are solutions of the differential equation $y'' + y = \sin(x)$?

   (a) $y = \sin x$

   (b) $y = \cos x$

   (c) $y = \dfrac{1}{2}x\sin x$

   (d) $y = -\dfrac{1}{2}x\cos x$