Jeffrey Ru

Solutions

Section 5.1 - #9

Lower estimate = S_R

$= 2(7.6 + 6.8 + 6.2 + 5.7 + 5.3) = 63.2$ L

Upper estimate = S_L

$= 2(8.7 + 7.6 + 6.8 + 6.2 + 5.7) = 70$ L

Section 5.2 - # 17

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

hence, our limit is simply:

$$\int_1^8 \sqrt{2x+x^2}\,dx$$

Section 5.3 - # 30

tan x is not continuous from 0 to π. The evaluation theorem cannot be applied!

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$$\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}}\,dx$$

$$= \int 2\cos x\,dx$$

$$= 2\sin x + C$$

Chapter 5.4-5.5 Answers

T.J. Reed

5.4

15. Find the average value of the function on the given interval

$f(x) = x^2, \quad [-1, 1]$

$y_{average} = \frac{1}{b-a} \int_a^b f(x)\,dx$

$y_{average} = \frac{1}{1+1} \int_{-1}^{1} x^2\,dx$

$y_{average} = \frac{1}{2}\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{2}\left[\frac{1}{3} + \frac{1}{3}\right]$

$\boxed{y_{average} = \frac{1}{3}}$

31. Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x} \qquad \text{for all } x > 0.$$

$\int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x} - 6$

$g'(x) = \int_a^x \frac{f(t)}{t^2}\,dt$

$g'(x) = 2\sqrt{x} - 6$

$g''(x) = x^{-1/2}$

$g''(x) = \frac{f(x)}{x^2}$

$\frac{f(x)}{x^2} = x^{-1/2}$

$f(x) = x^{-1/2} \cdot x^2$

$\boxed{f(x) = x^{3/2}}$

$\int_a^x \frac{t^{3/2}}{t^2}\,dt = 2\sqrt{x} - 6$

$\left[2t^{1/2}\right]_a^x = 2\sqrt{x} - 6$

$2\sqrt{x} - 2\sqrt{a} = 2\sqrt{x} - 6$

$2\sqrt{a} = 6$

$\sqrt{a} = 3$

$\boxed{a = 9}$

Chapter 5.4-5.5 Answers

T.J. Reed

Ch. 5.5

29. Evaluate the indefinite integral

$$\int \frac{e^x+1}{e^x}\,dx$$

$$= \int \left(1 + \frac{1}{e^x}\right) dx$$

$$= \int 1\,dx + \int e^{-x}\,dx$$

$$= x + (e^{-x} \cdot -1) + C$$

$$\boxed{x - e^{-x} + C}$$

41. Evaluate the definite integral

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$= \int_1^4 \left(e^{x^{1/2}} \cdot x^{-1/2}\right) dx$$

$$u = x^{1/2}, \quad du = \tfrac{1}{2}x^{-1/2}$$

$$= \int_{x=1}^{x=4} \left(e^u \cdot 2du\right)$$

$$= 2\int_{x=1}^{x=4} e^u\,du = 2\left[e^u\right]_{x=1}^{x=4}$$

$$= 2\left[e^{x^{1/2}}\right]_1^4$$

$$= 2\left[e^{\sqrt{4}} - e^{\sqrt{1}}\right]$$

$$\boxed{2(e^2 - e)}$$

$\boxed{6.1}$ solutions

Kearns Alex
Math 151

3. $\int x \cos 5x \, dx$

$u = x \qquad dv = \cos 5x \, dx$

$du = dx \qquad v = \frac{1}{5}\sin 5x$

$uv - \int v \, du$

$= \frac{x \sin 5x}{5} - \frac{1}{5}\int \sin 5x \, dx$

$= \frac{x \sin 5x}{5} + \frac{1}{5}\left(\frac{1}{5}\right)\cos 5x + c$

$= \boxed{\frac{x \sin 5x}{5} + \frac{\cos 5x}{25} + c}$

9. $\int \ln(2x+1) \, dx$

$u = \ln(2x+1) \qquad dv = dx$

$du = \frac{2}{2x+1} dx \qquad v = x$

$= x\ln(2x+1) - \int \frac{2x}{2x+1} dx$

$= x\ln(2x+1) - \int \frac{(2x+1)-1}{2x+1} dx$

$= x\ln(2x+1) - \int 1 dx - \left(-\int \frac{1}{2x+1} dx\right)$

$= x\ln(2x+1) - x + \frac{1}{2}\ln|2x+1| + c$

$= \boxed{\frac{1}{2}(2x+1)\ln(2x+1) - x + c}$

$\boxed{6.1}$ solutions

Kearns, Alex

13. $\int e^{2\theta} \sin 3\theta \, d\theta$

$u = e^{2\theta}$ $\quad dv = \sin 3\theta \, d\theta$

$du = 2e^{2\theta} d\theta$ $\quad v = -\frac{1}{3}\cos 3\theta$

$$\frac{-e^{2\theta}\cos 3\theta}{3} + \int (-\tfrac{1}{3}\cos 3\theta)(2e^{2\theta})\, d\theta$$

$$= \frac{-e^{2\theta}\cos 3\theta}{3} + \frac{2}{3}\int \cos 3\theta \, e^{2\theta} \, d\theta$$

$u = \cos 3\theta$ $\quad dv = e^{2\theta} d\theta$

$du = \frac{-\sin 3\theta}{3} d\theta$ $\quad v = \frac{1}{2} e^{2\theta}$

$$= \frac{-e^{2\theta}\cos 3\theta}{3} + \frac{2}{3}\left[\frac{\cos 3\theta \, e^{2\theta}}{2} + \frac{1}{6}\int e^{2\theta} \sin 3\theta \right]$$

$$\int e^{2\theta} \sin 3\theta \, d\theta = \frac{-e^{2\theta}\cos 3\theta}{3} + \frac{\cos 3\theta e^{2\theta}}{3} + \frac{1}{9}\int e^{2\theta} \sin 3\theta$$

?

$\boxed{6.1}$ solutions

Kearns, Alex

28. $\int_1^4 e^{\sqrt{x}}\,dx$

$\sqrt{x} = y \quad x = y^2$

$dx = 2y\,dy$

$\int_1^4 e^y\, 2y\,dy$

$u = 2y \qquad dv = e^y\,dy$

$du = 2dy \qquad v = e^y$

$= 2ye^y - \int 2e^y\,dy$

$= 2ye^y - 2e^y$

$= 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}\Big]_1^4$

$= (2\sqrt{4}e^{\sqrt{4}} - 2e^{\sqrt{4}}) - (2\sqrt{1}e^{\sqrt{1}} - 2e^{\sqrt{1}})$

$= (4e^2 - 2e^2) - (\cancel{2e^1 - 2e^1})$

$= 4e^2 - 2e^2$

$= \boxed{2e^2}$

Amber Moser

Section 6.2 Solutions

1. $\int \cos^3 x \, dx$

$\int \cos x \cos^2 x \, dx$

$\int \cos x(1-\sin^2 x)dx$

$u = \sin x$

$du = \cos x$

$\int 1-u^2 du$

$u - \frac{u^3}{3} + C$

$\boxed{\sin x - \frac{\sin^3 x}{3} + C}$

2. $\int \frac{\sqrt{1-x^2}}{x^4} dx$

$x = \sin\theta \quad dx = \cos\theta d\theta \quad x^2 = \sin^2\theta$

$\int \frac{\sqrt{1-\sin^2\theta}}{(\sin^2\theta)^2} \cos\theta d\theta$

$\int \frac{\sqrt{\cos^2\theta} \cos\theta d\theta}{(\sin^2\theta)^2}$

$\int \frac{\cos^2\theta \, d\theta}{(\sin^2\theta)^2}$

$\int \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{d\theta}{\sin^2\theta}$

$\int \frac{\cot^2\theta \, d\theta}{\sin^2\theta}$

(next page)

Amber Mosier

Section 6.2 cont (#2)

2. (cont.)

$$\int \frac{\cot^2\theta}{\sin^2\theta}\, d\theta$$

$u = \cot\theta$

$du = -\csc^2\theta\, d\theta$

$= -\frac{1}{\sin^2\theta}$

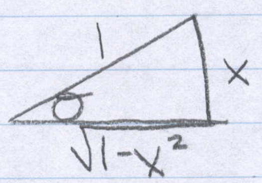
$$-\int u^2\, du$$

$= \frac{u^3}{3} + c \quad (u = \cot\theta)$

$\frac{\cot^3\theta}{3} + c$

*remember to change back x

$x = \sin\theta$



$$= \frac{\left(\frac{\sqrt{1-x^2}}{x}\right)^3}{3}$$

$$= \frac{\sqrt[3]{1-x^2}}{x^3} \cdot \frac{1}{3}$$

$$\boxed{= \frac{\sqrt[3]{1-x^2}}{3x^3}}$$

3. $\int_1^2 \frac{1}{x\sqrt{4x^2+9}}$

$2x = 3\tan\theta$

$x = \frac{3}{2}\tan\theta$

$dx = \frac{3}{2}\sec^2\theta\, d\theta$

(next page)

Amber Mosier

Section 6.2 cont. (#3)

3 $\displaystyle\int \frac{\cancel{\frac{3}{2}}\sec^2\theta\, d\theta}{\cancel{\frac{3}{2}}\tan\theta\sqrt{4\left(\frac{9}{4}\tan^2\theta\right)+9}}$

$$\int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{9\tan^2\theta+9}}$$

$$\int \frac{\sec^2\theta\, d\theta}{\tan\theta\cdot 3\sqrt{\tan^2\theta+1}}$$ pull out $\sqrt{9}$

$$\frac{1}{3}\int \frac{\sec^2\theta\, d\theta}{\tan\theta\sqrt{\sec^2\theta}}$$

$$\frac{1}{3}\int \frac{\sec^2\theta\, d\theta}{\tan\theta\sec\theta}$$

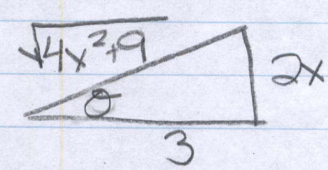
$$\frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$$\frac{1}{3}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\,d\theta = \frac{1}{3}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}\,d\theta$$

$$\frac{1}{3}\int \frac{1}{\sin\theta}\,d\theta$$

$$\frac{1}{3}\int \csc\theta\, d\theta$$

$-\dfrac{|\csc\theta+\cot\theta|}{3}$ remember $x=\frac{3}{2}\tan\theta$ $\tan\theta=\frac{2}{3}x$



$$-\frac{1}{3}\left|\frac{\sqrt{4x^2+9}}{2x}+\frac{3}{2x}\right|\Bigg|_1^2$$

$$-\frac{1}{3}\left|\frac{\sqrt{4(4)+9}}{4}+\frac{3}{4}\right|-\left(-\frac{1}{3}\left|\frac{\sqrt{4+9}}{2}+\frac{3}{2}\right|\right)$$

$$=-\frac{1}{3}\left|\frac{5}{4}+\frac{3}{4}\right|+\frac{1}{3}\left(\frac{\sqrt{13}}{2}+\frac{3}{2}\right)=\frac{\sqrt{13}+3}{6}-\frac{8}{12}=\boxed{\frac{\sqrt{13}+3}{6}-\frac{2}{3}}$$

Grace Daly

34. $\int \frac{x^4+1}{x(x^2+1)^2}\, dx$

$$\frac{x^4+1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$x^4+1 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$$
$$= Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Cx^3 + Cx + Dx^2 + Ex$$
$$= (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A$$

so: $A+B=1$ $\quad C=0 \quad 2A+B+D=0 \quad C+E=0 \quad A=1$

$1+B=1 \qquad 2(1)+0+D=0 \qquad 0+E=0$

$B=0 \qquad 2+D=0 \qquad E=0$

$D=-2$

$$\int \frac{x^4+1}{x(x^2+1)^2}\, dx = \int \frac{1}{x} + \frac{0}{x^2+1} + \frac{-2x}{(x^2+1)^2}\, dx = \int \frac{1}{x} - \frac{2x}{(x^2+1)^2}\, dx$$

$= \ln x - \int \frac{2x}{(x^2+1)^2}\, dx$ let $u = x^2+1$

$du = 2x\,dx$

so $\int \frac{2x}{(x^2+1)^2}\, dx = \int \frac{du}{u^2}$

$= \int u^{-2}\, du$

$= -u^{-1}$

$= -\frac{1}{u}$

$= -\frac{1}{x^2+1}$

so $\ln x - \left(-\frac{1}{x^2+1}\right)$

$$\boxed{= \ln x + \frac{1}{x^2+1} + C}$$

Grace Daly

6.3 & 6.4 Problems

Solve the integrals using partial fractions.

16. $\int \frac{x^3 - 4x - 10}{x^2 - x - 6} \, dx$

34. $\int \frac{x^4 + 1}{x(x^2+1)^2} \, dx$

Use the table of integrals to evaluate the integral.

16. $\int_0^2 x^3 \sqrt{4x^2 - x^4} \, dx$

19. $\int \frac{\sqrt{4 + (\ln x)^2}}{x} \, dx$

Heleen Bennett
12-6-11

3. $\int_0^1 \cos(x^2)\,dx \quad n=4$

a. Trapezoidal Rule $= T_n = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+\ldots+2f(x_{n-1})+f(x_n)]$

$\Delta x = \frac{(b-a)}{n}$ so $\Delta x = \frac{1-0}{4} = \frac{1}{4}$

$T_4 = \frac{.25}{2}[f(0)+2f(.25)+2f(.5)+2f(.75)+f(1)]$

$= .125[\cos(0)+2\cos(.0625)+2\cos(.25)+2\cos(.5625)$
$+ \cos(1)]$

$= .125[1+2\cos(.0625)+2\cos(.25)+2\cos(.5625)+\cos(1)]$

$= .125[7.1660711169] \approx .8957588961 \approx .895759$

underestimate

b. Midpoint Rule $= M_n = \Delta x[f(\bar{x}_1)+f(\bar{x}_2)+\ldots+f(\bar{x}_n)]$

$\Delta x = \frac{(b-a)}{n}$ so $\Delta x = \frac{1-0}{4} = \frac{1}{4}$

$M_4 = .25[f(.125)+f(.375)+f(.625)+f(.875)]$

$= .25[\cos(.015625)+\cos(.140625)+\cos(.390625)+$
$\cos(.765625)]$

$= .25[3.6356271163] = .9089067907 \approx .908907$

overestimate

$T_4 <$ true value $< M_4$

5. a. $M_n = \frac{\pi}{8}[f(\frac{\pi}{16})+f(\frac{3\pi}{16})+f(\frac{5\pi}{16})+f(\frac{7\pi}{16})+f(\frac{9\pi}{16})+f(\frac{11\pi}{16})+$
$\hookrightarrow f(\frac{13\pi}{16})+f(\frac{15\pi}{16})]$

$\Delta x = \frac{b-a}{n} = \frac{\pi-0}{8} = \frac{\pi}{8} \qquad \int_0^\pi x^2 \sin x\,dx$

$M_n = \frac{\pi}{8}[(\frac{\pi^2}{256}\sin\frac{\pi}{16})+(\frac{9\pi^2}{256}\sin\frac{3\pi}{16})+(\frac{25\pi^2}{256}\sin\frac{5\pi}{16})+(\frac{49\pi^2}{256}\sin\frac{7\pi}{16})$
$+(\frac{81\pi^2}{256}\sin\frac{9\pi}{16})+(\frac{121\pi^2}{256}\sin\frac{11\pi}{16})+(\frac{169\pi^2}{256}\sin\frac{13\pi}{16})+$
$(\frac{225\pi^2}{256}\sin\frac{15\pi}{16})]$

$M_n = \frac{\pi}{8}[15.10815094] \approx 5.932957$

$\hookrightarrow$ Error $= -.063353$

Integrate by parts to get actual value $\qquad$ 5.869604401 ↑

$\int_0^\pi x^2 \sin x\,dx \qquad u=x^2 \quad du=2x\,dx \qquad dv=\sin x \quad v=\cos x$

$-x^2\cos x + 2\int \cos x \cdot x\,dx \rightarrow (-\pi^2\cos\pi x + 0\cos 0)+2[\pi\sin\pi + 0\sin 0 + \cos\pi - \cos 0) = \pi^2 - 4$

Heleen Bennett
12-6-11

5. b. Simpson's Rule $= S_n = \frac{\Delta x}{3}[f(x_0)+4(fx_1)+2f(x_2)+\ldots 2f(x_{n-2})+ 4f(x_{n-1})+f(x_n)]$

(cont'd.)

$S_n = \frac{\pi}{24}[0+4(\frac{\pi^2}{256}\sin\frac{\pi}{16})+2(\frac{9\pi^2}{256}\sin\frac{3\pi}{16})+4(\frac{25\pi^2}{256}\sin\frac{5\pi}{16}) + 2(\frac{49\pi^2}{256}\sin\frac{7\pi}{16}+4(\frac{81\pi^2}{256}\sin\frac{9\pi}{16})+2(\frac{121\pi^2}{256}\sin\frac{11\pi}{16})+ 4(\frac{169\pi}{256}\sin\frac{13\pi}{16})+(\frac{225\pi^2}{256}\sin\frac{15\pi}{16})]$

$\approx 5.869247 \quad E = .000357$

13. $\int_1^5 \frac{\cos x}{x}\,dx \quad n=8 \quad \Delta x = \frac{b-a}{n} = \frac{5-1}{8} = \frac{4}{8} = \frac{1}{2}$

a. $T_8 = \frac{.5}{2}[f(1)+2f(1.5)+2f(2)+2f(2.5)+2f(3)+2f(3.5)+ 2f(4)+2f(4.5)+2f(5)]$

$= .25[\cos(1)+2(\frac{\cos 1.5}{1.5})+2(\frac{\cos 2}{2})+2(\frac{\cos 2.5}{2.5})+2(\frac{\cos 3}{3})+ 2(\frac{\cos 3.5}{3.5})+2(\frac{\cos 4}{4})+2(\frac{\cos 4.5}{4.5})+\cos(5)]$

$\approx -.495333$

b. $M_8 = .5[f(1.25)+f(1.75)+f(2.25)+f(2.75)+f(3.25)+ f(3.75)+f(4.25)+f(4.75)]$

$= .5[\frac{\cos 1.25}{1.25}+\frac{\cos 1.75}{1.75}+\frac{\cos 2.25}{2.25}+\frac{\cos 2.75}{2.75}+ \frac{\cos 3.25}{3.25}+\frac{\cos 3.75}{3.75}+\frac{\cos 4.25}{4.25}+\frac{\cos 4.75}{4.75}]$

$= -.543321$

c. $S_8 = \frac{.5}{3}[f(1)+4f(1.5)+2f(2)+4f(2.5)+2f(3)+4f(3.5)+ 2f(4)+4f(4.5)+f(5)]$

$S_8 = \frac{.5}{3}[\cos(1)+4(\frac{\cos 1.5}{1.5})+2(\frac{\cos 2}{2})+4(\frac{\cos 2.5}{2.5})+2(\frac{\cos 3}{3})+ 4(\frac{\cos(3.5)}{3.5})+2(\frac{\cos 4}{4})+4(\frac{\cos 4.5}{45})+\cos(5)]$

$\approx -.526123$

Eric Fallon

6.6 Improper Integrals: Answers

1. $\int_1^\infty \frac{1}{(3x+1)^2}\,dx = \lim_{B\to\infty}\int_1^B \frac{1}{(3x+1)^2}\,dx$

$= \lim_{B\to\infty}\int_1^B (3x+1)^{-2}\,dx$

$= \lim_{B\to\infty} -(3x+1)^{-1}\left(\frac{1}{3}\right)\Big]_1^B$

$= \lim_{B\to\infty}\left(-\frac{1}{3(3x+1)}\right)\Big|_1^B$

$= \lim_{B\to\infty}\left[-\frac{1}{3}\left(\frac{1}{3B+1}-\frac{1}{4}\right)\right]$

$= -\frac{1}{3}\left(\frac{1}{\infty}-\frac{1}{4}\right) = -\frac{1}{3}\left(0-\frac{1}{4}\right)$

$= \frac{1}{12}$

Thus the given integral is convergent because its limit exists

2. $\int_1^\infty \frac{\ln(x)}{x}\,dx = \lim_{B\to\infty}\int_1^B \ln(x)\left(\frac{1}{x}\right)dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$

$= \lim_{B\to\infty}\int_1^B u\,du$

$= \lim_{B\to\infty}\frac{1}{2}u^2\Big|_1^B$

$= \lim_{B\to\infty}\frac{1}{2}(\ln x)^2\Big|_1^B$

$= \frac{1}{2}\lim_{B\to\infty}\left((\ln B)^2-(\ln 1)^2\right)$

$= \frac{1}{2}\lim_{B\to\infty}(\ln B)^2 = \infty$

Thus the given integral diverges because the limit does not exist.

Eric fallon

6.6 Improper Integrals: Answers cont.

3. $\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx$ for $x \geq 1$

$$\frac{\cos^2 x}{1+x^2} < \frac{1}{1+x^2} < \frac{1}{x^2}$$

So $\int_1^\infty \frac{1}{x^2}\,dx$ is convergent with $p=2$

So $\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx$ converges by comparison theorem.

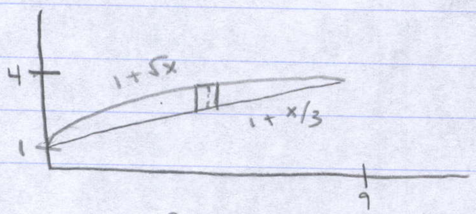
Sarah Baker

7.1 & 7.2
Solutions to Sample Problems

1. $A \approx \frac{2}{3}[0+4(6.2)+2(7.2)+4(6.8)+2(5.6)+4(5.0)+2(4.8)+4(4.8)+0]$

$A \approx \boxed{84.267m}$

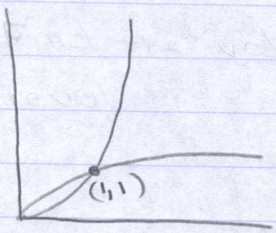
2.



width = Δx
height = $1+\sqrt{x}-(1+x/3) = \sqrt{x} - x/3$

$$A = \int_0^9 \sqrt{x} - x/3 \; dx$$
$$= \frac{2}{3}x^{3/2} - \frac{x^2}{6} \Big]_0^9$$
$$= 18 - 13.5$$
$$= \boxed{4.5}$$

3.



area of cross section $= \pi\sqrt{x}^2 - \pi(x^3)^2$
volume of cross section $= \pi x - \pi x^6 \, \Delta x$
volume of region: $\int_0^1 \pi x - \pi x^6 \, dx$
$$= \frac{\pi x^2}{2} - \frac{\pi x^7}{7}\Big]_0^1$$
$$= \pi/2 - \pi/7$$
$$= \boxed{5\pi/14}$$

4. $x^2 = r^2 - y^2$
$$V = \pi \int_h^{r-h} r^2 - y^2 \, dy$$
$$= \pi(r^2y - y^3/3)\Big]_{r-h}^{r}$$
$$= \pi r^3 - \pi r^3/3 - (\pi r^2(r-h) - \pi/3(r-h)^3)$$
$$= 2\pi r^3/3 - \pi r^3 + \pi r^2 h + \pi/3(r^3 - 3r^2h + 3rh^2 - h^3)$$
$$= -\pi r^3/3 + \pi r^2 h + \pi r^3/3 - \pi r^2 h + \pi r h^2 - \pi h^3/3$$
$$= (\pi h^2/3)(3r-h)$$
$$= \boxed{(\pi h^2)(r - h/3)}$$

Sam Honch

7.4 - Arc Length & 7.5 - Applications to Physics and Engineering (work)

1) $y = 1 + 6x^{3/2}, \quad 0 \le x \le 1$

$\frac{dy}{dx} = \frac{3}{2} \cdot 6x^{\frac{1}{2}} = 9x^{\frac{1}{2}} = 9\sqrt{x}$

arc length $= \int_0^1 \sqrt{1 + (9\sqrt{x})^2}\, dx$

$= \int_0^1 \sqrt{1 + 81x}\, dx$

$= \left[\frac{(1+81x)^{3/2}}{\frac{3}{2} \cdot 81} \right]_0^1$

$= \frac{82^{3/2}}{\frac{3}{2} \cdot 81} - \frac{1}{\frac{3}{2} \cdot 81} = \boxed{\frac{2(82\sqrt{82} - 1)}{243}}$

2) $x = \frac{1}{3}\sqrt{y}\,(y-3), \quad 1 \le y \le 9$

$\frac{dx}{dy} = \frac{1}{6} y^{-\frac{1}{2}}(y-3) + \frac{1}{3} y^{\frac{1}{2}} \cdot 1$

$= \frac{1}{3}\left(\frac{y-3}{2\sqrt{y}} + \sqrt{y} \right)$

$= \frac{1}{3}\left(\frac{y-3+2y}{2\sqrt{y}} \right)$

$= \frac{y-1}{2\sqrt{y}}$

arc Length $= \int_1^9 \sqrt{1 + \left(\frac{y-1}{2\sqrt{y}}\right)^2}$

$= \int_1^9 \sqrt{1 + \frac{y^2 - 2y + 1}{4y}}$

$= \int_1^9 \sqrt{\frac{y^2 + 2y + 1}{4y}}$

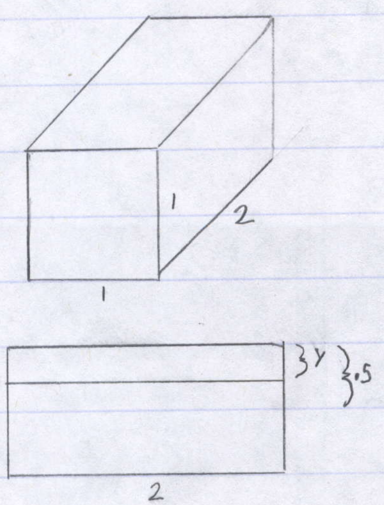
$= \int_1^9 \frac{y+1}{2\sqrt{y}} = \int_1^9 \frac{y}{2\sqrt{y}} + \int_1^9 \frac{1}{2\sqrt{y}}$

$= \int_1^9 \frac{y^{1/2}}{2} + \int_1^9 \frac{1}{2} y^{-\frac{1}{2}}$

$= \left[\frac{y^{3/2}}{3} \right]_1^9 + \left[y^{1/2} \right]_1^9$

$9 - \frac{1}{3} + 3 - 1 = 10\frac{2}{3} = \boxed{\frac{32}{3}}$

3)



Side view

Volume of slice $= (2\text{m} \times 1\text{m})\,\Delta y$

mass of slice $= (1000 \frac{\text{kg}}{\text{m}^3})(2\text{m}^2)\,\Delta y$

Weight of slice $= (9.8\,\text{m/s}^2)(1000\,\text{kg/m}^3)(2\text{m}^2)\,\Delta y$

work to move slice $= (9.8\frac{\text{m}}{\text{s}^2})(1000\frac{\text{kg}}{\text{m}^3})(2\text{m}^2)(.5-y)\,\Delta y$

total work $= \int_0^{.5} (9.8\frac{\text{m}}{\text{s}^2})(2000\frac{\text{kg}}{\text{m}})(.5-y)\,\Delta y$

$= \int_0^{.5} 9800 - 19600y \;\Delta y$

$= \left[9800y - 9800y^2 \right]_0^{.5}$

$= (4900 - 2450) - 0$

$= \boxed{2450\text{ J}}$

8.1)

Dan Partynski

1) $\left\{ \frac{1}{1}, -\frac{2}{3}, \frac{4}{9} \ldots \right\}$

$$a_n = \frac{2^{n-1}}{3^{n-1}}(-1)^{n-1}$$

$$a_n = \left(-\frac{2}{3}\right)^{n-1}$$

2) $\frac{2^n}{3^{n+1}} = \frac{1}{3} \cdot \frac{2^n}{3^n} = \frac{1}{3} \cdot \left(\frac{2}{3}\right)^n$ let $f(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x$

$\lim\limits_{x \to \infty} = 0$, so it converges to 0

3) $1 \leq \cos^2 n \leq 1$

$$\frac{1}{2^n} \leq \frac{\cos^2 n}{2^n} \leq \frac{1}{2^n}$$

let $f(x) = \frac{1}{2^x}$

$\lim\limits_{x \to \infty} f(x) = 0$, so by squeeze theorem,

a_n converges to 0

8.2) Dan Partynski

1) $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1} = \frac{a_1}{1-r}$

$a_1 = 5 \quad r = \frac{2}{3}$

$\frac{5}{1-\frac{2}{3}} = 15$

2) $\sum_{n=1}^{\infty} \frac{2^n+1}{3^n} = \sum_{n=1}^{\infty} \frac{2^n}{3^n} + \frac{1}{3^n}$

$= \frac{\frac{2}{3}}{1-\frac{2}{3}} + \frac{\frac{1}{3}}{1-\frac{1}{3}} = 2 + \frac{1}{2} = \frac{5}{2}$

3) $\frac{2}{n^2-1} = \frac{2}{(n+1)(n-1)} \quad \frac{A}{n+1} + \frac{B}{n-1} = \frac{2}{n^2-1}$

$A(n-1) + B(n+1) = 2 \quad B = 1 \quad A = -1$

$\frac{2}{n^2-1} = \frac{1}{n-1} - \frac{1}{n+1}$

$\sum_{n=2}^{\infty} \frac{1}{n-1} - \frac{1}{n+1} = \left(1 - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right)$

$\left(\ldots + \cancel{\frac{1}{n+1}} - \cancel{\frac{1}{n+1}}\right)$

$= \lim_{n \to \infty} 1\frac{1}{2} = \frac{3}{2}$

Shannon Palka Section 8.3

8. $\sum_{n=1}^{\infty} \frac{1}{n^2+1} = a_n = f(n)$

$f(n)$ is continuous on $[1, \infty)$,
positive on $[1, \infty)$, &
ultimately decreasing on $[1, \infty)$.

$$\int_1^{\infty} \frac{dx}{x^2+1} = \lim_{B \to \infty} \int_1^{B} \frac{dx}{x^2+1}$$
$$= \lim_{B \to \infty} (\tan^{-1} x) \Big|_1^B$$
$$= \lim_{B \to \infty} (\tan^{-1} B) - (\tan^{-1} (1))$$
$$= \frac{\pi}{2} - \frac{\pi}{4}$$
$$= \frac{\pi}{4}$$

Since $\int_1^{\infty} f(x)\,dx$ is convergent, $\sum_{n=1}^{\infty} a_n$ is convergent.

10. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1} = a_n$, $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} = b_n$

$a_n \geq b_n$

the series b_n diverges by the p-series test.
Since $a_n \geq b_n$ & b_n diverges, a_n also diverges by the direct comparison test.

25. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}} = a_n$, $\frac{1}{n^{\frac{3}{2}}} \leq a_n \leq \frac{3}{n^{3/2}}$, $b_n = \frac{1}{n^{\frac{3}{2}}}$

$a_n \geq b_n$

b_n diverges by the p-series test.
since $a_n \geq b_n$ & b_n diverges, a_n also diverges by the direct comparison test.

Troy Zawlacki
Math 151

8.5 Power Series Solutions

Find the radius of convergence and interval for:

① $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n^3}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^n x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-1)^{n-1} x^n} \right|$$

$$= |x| \lim_{n\to\infty} \frac{n^3}{(n+1)^3}$$

$= |x|$ converges when $|x| < 1$, so $\boxed{R = 1}$ with the temporary interval $(-1, 1)$

test -1 | $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n^3} = \sum_{n=1}^{\infty} \frac{-1}{n^3}$ which converges by p-series test

test $+1$ | $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ converges by alt. series test since $\frac{1}{x^3}$ is decreasing, positive and approaches 0

Therefore, the interval of convergence is $\boxed{[-1, 1]}$

② $\sum_{n=1}^{\infty} \frac{x^n(-2)^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n x^n (2^n)}{\sqrt[4]{n}}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1} (2^{n+1})}{\sqrt[4]{n+1}} \cdot \frac{\sqrt[4]{n}}{(-1)^n x^n (2^n)} \right| = |x| \lim_{n\to\infty} \frac{2\sqrt[4]{n}}{\sqrt[4]{n+1}} = 2|x|$$

converges when $2|x| < 1$ $|x| < \frac{1}{2}$ $\boxed{R = \frac{1}{2}}$ temp interval $(-\frac{1}{2}, \frac{1}{2})$

test $-1/2$ | $\sum_{n=1}^{\infty} \frac{(-1/2)^n(-2)^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$ which diverges by p-series test

test $1/2$ | $\sum_{n=1}^{\infty} \frac{(1/2)^n(-2)^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$ converges by alt. series test b/c $1/\sqrt[4]{n}$ is positive, dec., and approaches 0

Therefore, the interval of convergence is $\boxed{(-\frac{1}{2}, \frac{1}{2}]}$

③ on next page

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Math 151

8.5 Power Series Solutions

Find the radius of convergence and interval for:

(3) $\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n \quad b>0$

$\lim_{n\to\infty} \left| \frac{n+1}{b^{\cancel{n+1}}} \cdot (x-a)^{\cancel{n+1}} \cdot \frac{\cancel{b^n}}{(n)\cancel{(x-a)^n}} \right| = |x-a| \lim_{n\to\infty} \frac{n+1}{(n)(b)}$

$\lim_{n\to\infty} = \frac{|x-a|}{b}$ converges when $\frac{|x-a|}{b} < 1 \quad |x-a| < b \quad \boxed{R=b}$

$x-a<b$ I.O.C = $(a-b, a+b)$
$x-a>-b$

test $a-b$ | $\sum_{n=1}^{\infty} \frac{n}{b^n}(a-b-a)^n = \sum_{n=1}^{\infty} (-1)^n n$ diverges since $\lim_{n\to\infty}$ equals ∞

test $a+b$ | $\sum_{n=1}^{\infty} \frac{n}{b^n}(a+b-a)^n = \sum_{n=1}^{\infty} n$ diverges

Therefore, the I.O.C. is $\boxed{(a-b, a+b)}$

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12/3/11

8.5 & 8.6 Answers

1. $f(x) = \frac{x}{4+x^2} = (x)\frac{1}{4-(-x^2)} = (x)\cdot\frac{1}{4\left(1-\left(-\frac{x^2}{4}\right)\right)} =$

$\frac{x}{4}\sum_{n=0}^{\infty}\left(\frac{-x^2}{4}\right)^n = \frac{x}{4}\sum_{n=0}^{\infty}\frac{(-1)^n(x^{2n})}{4^n} = \boxed{\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{4^{n+1}}}$

$\left|\frac{x}{4}\right|<1$ $\quad |x|<4$ converges $\boxed{[-4,4]}$ check endpoints

$x=-4$

$\sum_{n=0}^{\infty}\frac{(-1)^n(-4)^{2n+1}}{4^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n(-1)\,4^{2n+1}}{4^{n+1}} = \sum_{n=0}^{\infty}4^n$ converges by convergence test

$x=4$ $\quad \sum_{n=0}^{\infty}\frac{(-1)^n(4)^{2n+1}}{4^{n+1}} = \sum_{n=0}^{\infty}(-1)^n(4)^n$ converges by alt. series Test

2. $f(x) = \ln(4+x) \qquad f'(x) = \frac{1}{4+x}$

$\frac{1}{4+x} = \frac{1}{4(1+\frac{x}{4})} = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{x}{4}\right)^n = \boxed{\sum_{n=0}^{\infty}\frac{x^n}{4^{n+1}}}$ $\quad \left|\frac{x}{4}\right|<1$ converge

$|x|<4$ $\quad \boxed{[-4,4]}$ check endpoints

$x=4$ $\quad \sum_{n=0}^{\infty}\frac{4^n}{4^{n+1}} = \sum_{n=0}^{\infty}\frac{1}{4}$ converges by convergence Test

$x=-4$ $\quad \sum_{n=0}^{\infty}\frac{(-4)^n}{4^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{4}$ converges by Alt. series Test

3. $\int\frac{x^2}{1-x^7}\,dx = x^2\int\sum_{n=0}^{\infty}x^{7n} = \int\sum_{n=0}^{\infty}x^{7n+2} = \sum_{n=0}^{\infty}\frac{x^{7n+3}}{7n+3}$

Solutions

Section 8.8

22) Use the Alternating Series Estimation Theorem or Taylor's Formula to estimate the range of values of x for which the given approximation is accurate to within the stated error.

$$\cos x \approx 1 - x^2/2 + x^4/24 \quad (|\text{error}| < 0.005)$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad R = \infty$$

$$\text{let } b_n = \frac{x^{2n}}{(2n)!}$$

$$b_{n+1} \leq b_n : x^2 \leq (2n+2)(2n+1)$$

$$x^{2n+2} \leq x^{2n}(2n+2)(2n+1)$$

$$x^{2n+2}(2n)! \leq x^{2n}(2n+2)!$$

$$\frac{x^{2n+2}}{(2n+2)!} \leq \frac{x^{2n}}{(2n)!}$$

for any x and for all n

$$\lim_{n\to\infty} b_n = 0 : \lim_{n\to\infty} \frac{x^{2n}}{(2n)!} \Rightarrow \frac{0}{\infty} = 0$$

$\therefore$ converges by alternating series estimation theorem

given $\cos x \approx S_2 = 1 - x^2/2 + x^4/24$

$$|S - S_2| \leq b_{n+1}$$

$$b_{n+1} = b_3 \leq .005$$

$$x^6/6! \leq 1/200$$

$$x^6 \leq 6!/200$$

$$(-1.238,\ 1.238)$$

Danielle Riethmiller

Solutions

Section 8.7

13) Find the Taylor Series for f(x) centered at the given value of a. [Assume that f has a power series expansion. Do not show $R_n(x) \to 0$.]

$f(x) = e^x$, $a = 3$ $\qquad$ $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$f(x) = e^x$ $\qquad$ $f(a) = e^3$

$f'(x) = e^x$ $\qquad$ $f'(a) = e^3$

$f^n(x) = e^x$ $\qquad$ $f^n(a) = e^3$

$$\sum_{n=0}^{\infty} e^3/n!\,(x-3)^n$$

43) Evaluate the indefinite integral as an infinite series.

$\int x\cos(x^3)\,dx$

$$\cos x \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(x^3) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

$$x\cos(x^3) \Rightarrow x\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{(2n)!}$$

$$\int x\cos(x^3)\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{(2n)!}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+2}}{(6n+2)(2n)!} + C$$

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Fall 2011

Assignment 44: 8.8 #3, 4, 6

3. $f(x) = \sin x, \quad a = \frac{\pi}{6}, \quad n = 3$

$$f(\tfrac{\pi}{6}) = \sin(\tfrac{\pi}{6}) = \frac{1}{2}$$
$$f'(\tfrac{\pi}{6}) = \cos(\tfrac{\pi}{6}) = \frac{\sqrt{3}}{2}$$
$$f''(\tfrac{\pi}{6}) = -\sin(\tfrac{\pi}{6}) = -\frac{1}{2}$$
$$f'''(\tfrac{\pi}{6}) = -\cos(\tfrac{\pi}{6}) = -\frac{\sqrt{3}}{2}$$

$$T_3(\tfrac{\pi}{6}) = \frac{1}{2} + \frac{\frac{\sqrt{3}}{2}}{1}(x-\tfrac{\pi}{6}) - \frac{\frac{1}{2}}{2}(x-\tfrac{\pi}{6})^2 - \frac{\frac{\sqrt{3}}{2}}{6}$$

$$= \frac{1}{2} + \frac{\sqrt{3}(x-\frac{\pi}{6})}{2} - \frac{(x-\frac{\pi}{6})^2}{4} - \frac{\sqrt{3}(x-\frac{\pi}{6})^3}{12}$$

4. $f(x) = e^x, \quad a = 2, \quad n = 3$

$$f(x) = e^x = e^2$$
$$f'(x) = e^x = e^2$$
$$f''(x) = e^x = e^2$$
$$f'''(x) = e^2$$

$$T_3(2) = e^2 + e^2(x-2) + \frac{e^2}{2}(x-2)^2 + \frac{e^2}{6}(x-2)^3$$

6. $f(x) = \frac{\ln x}{x}, \quad a = 1, \quad n = 3$

$$f(x) = \frac{\ln x}{x} = 0$$

$$f'(x) = \frac{\frac{x}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2} = 1$$

$$f''(x) = \frac{(0 - \frac{1}{x})(x^2) - (1 - \ln x)(2x)}{x^4}$$

$$= \frac{(0-1)(1) - (1-0)(2)}{1} = -3$$

$$f'\left[\frac{(-x) - (2x - 2x\ln x)}{x^4}\right]$$

$$f'\left[\frac{-3x + 2x\ln x}{x^4}\right]$$

$$f'''(x) = \frac{(-3 + (2\ln x + 2))x^4 - 4x^3(-3x + 2\ln x)}{x^4}$$

$$= \frac{(2\ln x - 1)x^4 + 12x^4(-3x\,2\ln x)}{x^4} = 11$$

$$T_3(1) = (x-1) - \frac{3(x-1)^2}{2} + \frac{11(x-1)^3}{6}$$

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HW #43 8.7: 35, 52, 53, 61, 62, 63, 64

35. $f(x) = \sin^2 x$ Hint: Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$$\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots$$

$$f(x) = \frac{1}{2}(1-\cos 2x) = \frac{1}{2}\left[1-1+\frac{(2x)^2}{2!} - \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!}\right]$$

$$= \frac{1}{2}\left[\frac{2^2x^2}{2!} - \frac{2^4x^4}{4!} + \frac{2^6x^6}{6!} \dots\dots\right] = \frac{2x^2}{2!} - \frac{2^3x^4}{4!} + \frac{2^5x^6}{6!} - \dots$$

$$= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} 2^{2n-1} \frac{x^{2n}}{(2n)!}}$$

52. $\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$

$$\frac{1-\cos x}{1+x-e^x} = \frac{1-1+\frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \dots}{\cancel{1}+\cancel{x}-\cancel{1}-\cancel{\frac{x}{1!}}-\frac{x^2}{2!}-\frac{x^3}{3!}-\frac{x^4}{4!}\dots} = \frac{\frac{\cancel{x^2}}{2!} - \frac{x^{\cancel{4}2}}{4\cdot 3} + \frac{x^{\cancel{6}4}}{6\cdot5\cdot4\cdot3}}{-\frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4\cdot 3}} = \frac{1-\frac{x^2}{4\cdot 3}+\frac{x^4}{6\cdot5\cdot4\cdot3}+\dots}{-1-\frac{x}{3}-\frac{x^2}{4\cdot 3}\dots}$$

$$= \text{Then } \lim_{n\to 0} \frac{1-\cos x}{1+x-e^x} = \frac{1}{-1} = -1$$

53. $\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

$$\sin x - x + \frac{1}{6}x^3 = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right) - x + \frac{x^3}{6}$$

$$= \left(\cancel{x} - \cancel{\frac{x^3}{6}} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots\right) - \cancel{x} + \cancel{\frac{x^3}{6}} = \frac{\cancel{x^5}}{120} - \frac{x^7}{5040} + \dots = \lim_{x\to 0}\left[\frac{1}{120} - \frac{x^2}{5040} + \dots\right]$$

$$= \boxed{\frac{1}{120}}$$

61. $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}\left(\frac{\pi}{4}\right)^{2n+1}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}\left(\frac{\pi}{4}\right)^{2n+1} = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

62. $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!} = \sum_{n=0}^{\infty} \frac{(3/5)^n}{n!}$ Compare it with $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$= \boxed{e^{3/5}}$

63. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \ldots = \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \ldots$

since $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$x = 3$ $e^3 = 1 + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \quad = \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \ldots = \boxed{e^3 - 1}$

64. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots$

$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$

$x = \ln 2$ $e^{-\ln 2} = 1 - \frac{\ln 2}{1!} + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots = 1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots$

$= e^{-\ln 2} = e^{\ln (2)^{-1}} = 2^{-1} = \boxed{\frac{1}{2}}$