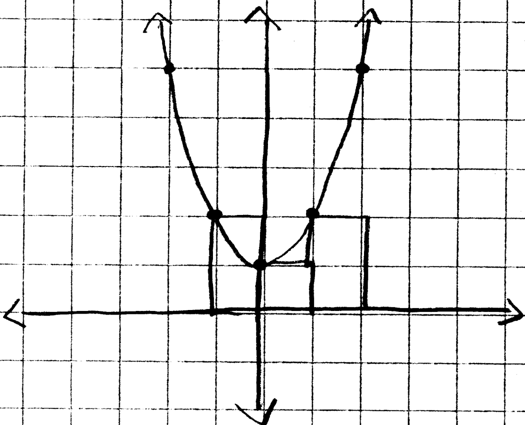
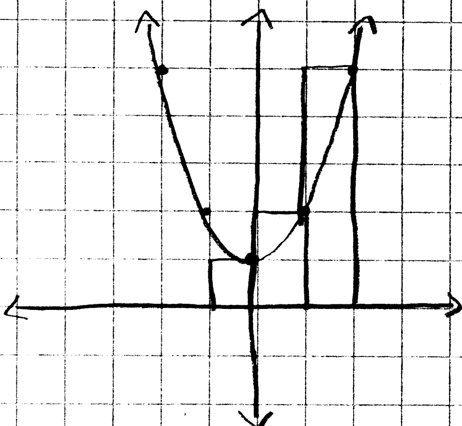
Sections 5.1-5.2 Kyler Wood

1. $f(x) = x^2 + 1$



$R_n = 2 + 1 + 2$
$= 5$
underestimate



$L_n = 1 + 2 + 5$
$= 8$
overestimate

3. $\lim\limits_{n \to \infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2) \Delta x \quad [2, 6]$

$$\int_2^6 x \ln(1 + x^2)\, dx$$

4. $\int_1^4 x^2 + 2x - 5\, dx = \left. \frac{x^3}{3} + x^2 - 5x \right]_1^4$

$$\left[\frac{(4)^3}{3} + (4)^2 - 5(4)\right] - \left[\frac{1}{3} + 1 - 5\right]$$

$$\left(\frac{64}{3} + 16 - 20\right) - \left(\frac{1}{3} - 4\right)$$

$$\left(\frac{63}{3} + 0\right) = \boxed{21}$$

Michael Bonacci

Sections 5.3 & 5.4
Practice Answers

① $\int_1^2 \frac{v^3 + 3v^6}{v^4}\, dv$

$= \int_1^2 \frac{v^3}{v^4}\, dv + 3\int \frac{v^6}{v^4}\, dv$

$= \int_1^2 \frac{1}{v}\, dv + 3\int v^2\, dv$

$= \ln|v|\Big|_1^2 + v^3\Big|_1^2$

$= (\ln|2| - \ln|1|) + (8 - 1)$

$\Rightarrow \ln|2| + 7$

② $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta}\, d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\, d\theta$

$= \int_0^{\frac{\pi}{4}} \sec^2\theta + 1\, d\theta$

$= \tan(\theta) + \theta\Big|_0^{\frac{\pi}{4}}$

$= \tan\left(\frac{\pi}{4}\right) + \frac{\pi}{4}$

$\Rightarrow 1 + \frac{\pi}{4}$

③ $g(y) = \int_2^y t^2 \sin(t)\, dt$

$g'(y) = y^2 \sin(y)$

Yes, its that simple.

④ $h(x) = \int_2^{\frac{1}{x}} \arctan(t)\, dt$

Hint: notice the $\left(\frac{1}{x}\right)$ $\longrightarrow$ cannot simply restate the equation inside of the integral

Kellen Maichen

Solutions for 5.5-5.6

$\boxed{5.5}$

24. $\int \frac{\sin(\ln x)}{x}\,dx$ let $u = \ln x$ $du = 1/x\,dx$

$\int \frac{\sin(u)}{x}\,dx \to \int \sin(u)\,du$

$= -\cos(u) + C \to$ plug u back in $= \boxed{-\cos(\ln x) + C}$

44. $\int_0^{\sqrt{\pi}} x\cos(x^2)$

let $u = x^2$ $du = 2x\,dx$

change limits of integration $\to$ $u = (\sqrt{\pi})^2 = \pi$

$u = 0^2 = 0$

$1/2 \int_0^{\pi} 2x\cos(u)\,dx$

$1/2 \int_0^{\pi} \cos(u)\,du = 1/2\left[\sin(u)\right]_0^{\pi} = 1/2[0] = \boxed{0}$

$\boxed{5.6}$

4. $\int xe^{-x}\,dx$ let $u = x$ $dv = e^{-x}\,dx$

$du = dx$ $v = -e^{-x}$

$\int u\,dv = uv - \int v\,du$

$\int xe^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x} + C$

$= \boxed{-e^{-x}(x+1) + C}$

Erin Smith
5.7

1. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = \int_0^{\pi/2} \left(\frac{1-\cos 2x}{2}\right)\left(\frac{\cos 2x+1}{2}\right) dx$

$\frac{1}{4}\int_0^{\pi/2} (1-\cos 2x)(\cos 2x+1)\, dx$

$1/4 \int_0^{\pi/2} \cos 2x + 1 - \cos^2 2x - \cos 2x \, dx$

$1/4 \int_0^{\pi/2} 1-\cos^2 2x \, dx = 1/4 \int_0^{\pi/2} \sin^2 2x \, dx$

$1/4 \int_0^{\pi/2} \sin^2 2x \, dx$ $\quad u=2x \quad du = 2\,dx \quad u=2(0)=2$

$1/8 \int_0^{\pi} \sin^2 u \, du$ $\quad u = 2(\frac{\pi}{2}) = \pi$

$1/8 \int_0^{\pi} \frac{1-\cos 2u}{2} du \Rightarrow 1/16 \int_0^{\pi} 1-\cos 2u \, du$

$1/16 \left[u - 1/2 \sin 2u\right]_0^{\pi}$

$1/16 \left[(\pi - 1/2 \sin 2\pi) - (0 - 1/2 \sin 2(0))\right]$

$1/16 [\pi - 0]$

$\pi/16$

$\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = \pi/16$

Erin Smith
5.7

2. $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\,dx$ $\quad x = 3\sin\theta \quad \theta = \sin^{-1}\left(\frac{x}{3}\right)$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{\sqrt{9-(3\sin\theta)^2}}{(3\sin\theta)^2} \cdot 3\cos\theta\, d\theta = \int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta} \cdot 3\cos\theta\, d\theta$$

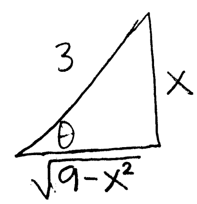
$$\int \frac{3\cos\theta \cdot 3\cos\theta}{9\sin^2\theta}\, d\theta = \int \frac{9\cos^2\theta}{9\sin^2\theta}\, d\theta$$

$$\int \frac{\cos^2\theta}{\sin^2\theta} = \int \cot^2\theta\, d\theta$$

$$\int \cot^2\theta\, d\theta = \int \csc^2\theta - 1\, d\theta$$

$$-\cot\theta - \theta + C$$

$$-\frac{\sqrt{9-x^2}}{x} + \sin^{-1}\left(\frac{x}{3}\right) + C \qquad \theta = \sin^{-1}\left(\frac{x}{3}\right)$$



$\tan = \dfrac{\text{opposite}}{\text{adjacent}}$

$\cot = \dfrac{\text{adjacent}}{\text{opposite}}$

$\cot\theta = \dfrac{\sqrt{9-x^2}}{x}$

Erin Smith
5.7

3 $\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

$\frac{x-4}{x^2-5x+6} \Rightarrow \frac{x-4}{(x-3)(x-2)} \Rightarrow \frac{A}{x-3} + \frac{B}{x-2}$

so $x-4 = A(x-2) + B(x-3)$

let $x=2$ let $x=3$

$-2 = -B$ $-1 = A$

$B = 2$ $A = -1$

$\int_0^1 \frac{-1}{x-3} + \frac{2}{x-2}\,dx \Rightarrow \int_0^1 \frac{-1}{x-3}\,dx + 2\int_0^1 \frac{1}{x-2}\,dx$

$-\ln|x-3|\Big|_0^1 + 2\ln|x-2|\Big|_0^1$

$(-\ln|1-3| + \ln|0-3|) + 2(\ln|1-2| - \ln|0-2|)$

$-\ln 2 + \ln 3 + 2(\ln 1 - \ln 2)$

$-\ln 2 + \ln 3 - 2\ln 2$

$\ln 3 - 3\ln 2$

$\ln 3 - \ln 2^3$

$\ln 3 - \ln 8$

$\ln 3/8 \approx -0.9808$

Cassandra Cori

Solutions 5.9 + 5.10

#2. Use Trapezoidal Rule to approximate:

$\int_0^4 e^{\sqrt{t}} \sin t \, dt, \quad n=8$

$T_8 = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots f(x_n)\right]$

$T_8 = .25\left[e^{\sqrt{0}}\sin(0) + 2e^{\sqrt{.5}}\sin(.5) + 2e^{\sqrt{1}}\sin(1) + 2e^{\sqrt{1.5}}\sin(1.5) + 2e^{\sqrt{2}}\sin(2) + 2e^{\sqrt{2.5}}\sin(2.5) + 2e^{\sqrt{3}}\sin(3) + 2e^{\sqrt{3.5}}\sin(3.5) + e^{\sqrt{4}}\sin(4)\right]$

$T_8 = .25[0 + 1.945 + 4.575 + 6.789 + 7.480 + 5.818 + 1.596 + (-4.556) + (-5.592)]$

$T_8 = .25[18.055] = 4.514$

#3. Is the integral convergent or divergent?

$\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty} \int_3^t (x-2)^{-3/2}$

$= \lim_{t\to\infty} \left.\frac{-2}{\sqrt{x-2}}\right|_3^t = \lim_{t\to\infty}\left[\frac{-2}{\sqrt{t-2}} + \frac{2}{\sqrt{3-2}}\right]$

$= 2$

Converges to 2

Cassandra Cory

solutions 5.9 + 5.10 continued

#4. Is the integral conv or div? If conv. what does it converge to?

$$\int_0^\infty xe^{-5x}\,dx$$

$$= \lim_{t\to\infty} \int_0^t xe^{-5x}\,dx$$ use by parts

$u = x \qquad dv = e^{-5x}\,dx$

$du = dx \qquad v = -1/5\, e^{-5x}$

$$= \lim_{t\to\infty} uv\Big|_0^t - \int_0^t v\,du$$

$$= \lim_{t\to\infty} -1/5\, xe^{-5x}\Big|_0^t + 1/5 \int_0^t e^{-5x}\,dx$$

$$= \lim_{t\to\infty} -1/5\, xe^{-5x}\Big|_0^t + 1/5\left(-1/5\, e^{-5x}\right)\Big|_0^t$$

$$\lim_{t\to\infty} \Big[-1/5\, xe^{-5x} - 1/25\, e^{-5x}\Big]_0^t$$

$$= 1/25$$

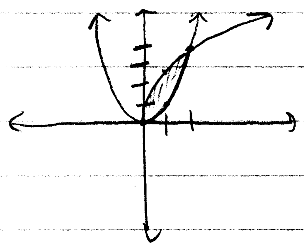
converges to 1/25

Neal Sharma

Problems 1, 2, 3 Solved

1. $x = \sqrt{y}$ $\quad y = \sqrt{8x}$

$y = x^2$ $\quad y = 2\sqrt{2x}$



$$\int_0^2 (2\sqrt{2x} - x^2)\,dx$$

$$\int_0^2 2\sqrt{2x}\,dx - \int_0^2 x^2\,dx$$

$u = 2x$ $\quad \int_0^4 \sqrt{u}\,du$

$du = 2dx$ $\quad \frac{2}{3}u^{\frac{3}{2}}\Big|_0^4 - \frac{1}{3}x^3\Big|_0^2$

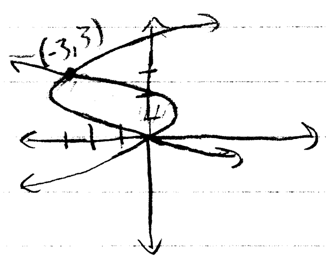
$$\frac{2}{3}(4)^{\frac{3}{2}} - 0 \quad -\left(\frac{1}{3}(2)^3 - 0\right)$$

$$\frac{16}{3} - \frac{8}{3}$$

$$= \frac{8}{3}$$

2. $x = y^2 - 4y$ $\quad x = y(2-y)$

$x = y^2 - 4y$ $\quad x = 2y - y^2$



$$\int_0^3 \left((2y - y^2) - (y^2 - 4y)\right)dy$$

$$\int_0^3 (-2y^2 + 6y)\,dy$$

$$\left(-\frac{2}{3}y^3 + 3y^2\right)\Big|_0^3$$

$$\left(-\frac{2}{3}(3)^3 + 3(3)^2\right) - 0$$

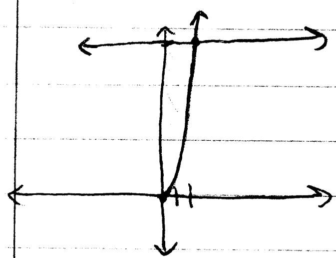
$$-18 + 27$$

$$= 9$$

Neal Sharma

Problems 1,2,3 Solved (Cont.)

3. $y=2x^3$ $y=16$ about y-axis



$$\pi \int_0^{16} \left(\sqrt[3]{\frac{y}{2}}\right)^2 dy$$

$$\pi \int_0^{16} \left(\frac{y}{2}\right)^{\frac{2}{3}} dy$$

$$\pi \left(\frac{6}{5}\left(\frac{y}{2}\right)^{\frac{5}{3}}\Big|_0^{16}\right)$$

$$\pi \left(\frac{6}{5}\left(\frac{16}{2}\right)^{\frac{5}{3}} - 0\right)$$

$$\pi \left(\frac{6}{5}(8)^{\frac{5}{3}}\right)$$

$$\pi \left(\frac{6}{5}(2)^5\right)$$

$$\frac{6\pi}{5} \cdot 32$$

$$= \frac{192\pi}{5}$$

Tram Nguyen

① Find the arc length of $y = 2x - 5$ $\quad -1 \leq x \leq 3$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad \frac{dy}{dx} = 2$$

$$= \int_{-1}^{3} \sqrt{1 + (2)^2}\, dx$$

$$= \sqrt{5}\, x \Big|_{-1}^{3} \qquad = 3\sqrt{5} - {}^{-}1\sqrt{5} \quad = 4\sqrt{5}$$

③ find the average value of the function

$f(x) = \cos^4 x \sin x \quad [0, \pi]$

$$f_{avg} = \frac{1}{b-a} \int_a^b f(x)\, dx \qquad u = \cos x$$

$$= \frac{1}{\pi} \int_0^{\pi} \cos^4 x \sin x \qquad du = -\sin x$$

$$- \frac{1}{\pi} \int_0^{\pi} u^4\, du$$

$$- \frac{1}{\pi} \int_0^{\pi} \frac{1}{5} u^5 du \quad = \quad \frac{-1}{\pi} \left[\frac{1}{5} \cos^5 x\right]_0^{\pi}$$

$$\frac{1}{5\pi} - - \frac{1}{5\pi} \quad = \quad \frac{2}{5\pi}$$

Tram Nguyen

(4) (a) find the average value of $f(x) = (x-3)^2$

(b) find c if $f(c) = f(x)$

(c) graph

(a) $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{3}\int_2^5 (x-3)^2\,dx$ $\qquad u = x-3$, $du = dx$

$\frac{1}{3}\int_2^5 u^2\,du$

$= \frac{1}{3}u^2 \Big|_{-1}^{2} = \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1$

(b) $f(c) = f(x)$

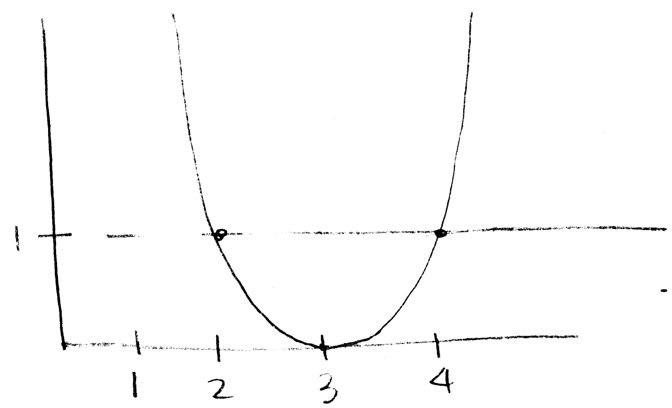
$c^2 - 6c - 9 = 1$

$c^2 - 6c - 8 = 0$

$(c-4)(c-2)$

$c = 4 \qquad c = 2$

(c)



Solutions to 6.6 Problems

Aaron Miller

1) (a) $2 = \int_0^{12\text{ cm}} kx\,dx = \int_0^{.12} kx\,dx$

$$= k\frac{x^2}{2}\Big|_0^{.12} = \frac{k}{2}(.12)^2 = .0072\ k$$

$2 = .0072\,k \qquad k = 277.8$

$$\int_{.05}^{.1} 277.8\ x\ dx = 277.8\ \frac{x^2}{2}\Big|_{.05}^{.1} = \frac{277.8}{2}(.01 - .0025)$$

$$= \boxed{1.04175\ J}$$

(b) $30 = kx \Rightarrow 30 = 277.8\,x$

$$x = \frac{30}{277.8} \approx .108\ m \approx \boxed{10.8\ cm}$$

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2) $\text{weight}(t) = 40 - 0.2t$

$y = 2t$ or $t = \frac{y}{2}$

$\text{weight}(y) = 40 - 0.2\left(\frac{y}{2}\right) = 40 - 0.1y$

$\text{Work} = \int_0^{80} [\text{weight of rope} + \text{weight of bucket} + \text{weight}(y)]\,dy$

$W = \int_0^{80} 0 + 4 + (40 - 0.1y)\,dy$

$\int_0^{80} 44 - 0.1y\,dy = 44y - 0.05y^2\Big|_0^{80}$

$44(80) - 0.05(80)^2 = \boxed{3200\ \text{ft}\cdot\text{lb}}$

6.6 Solutions Cont... Aaron Miller

3) $W = \int_a^b F(x)\,dx$

$12 = \int_0^1 kx\,dx$

$12 = k\frac{x^2}{2}\Big|_0^1$

$k = 24$

$W = \int_0^{.75} 24x\,dx$

$= 12x^2\Big|_0^{.75}$

$\frac{27}{4} = 6.75 \text{ ft} \cdot \text{lb}$

section 6.7-6.8 Answers Corie Neukranz

1. $p = 200 + 0.2x^{3/2}$ find ps? when selling prx = \$400

$p(400) = 200 + 0.2(400)^{3/2}$

$= 1800$

$200 + 0.2x^{3/2} = 0$

$0.2x^{3/2} = -200$

$x^{3/2} = -1000$

$(x^{3/2} = -1000)^2$

$(x^3 = 1000000)^{1/3}$

$x = 100$

$p(100) = 400$

$\int_0^{100} (400 - p)\,dx = \$12{,}000$

2. $400T = \$7.50$

$T = 7.50/400$

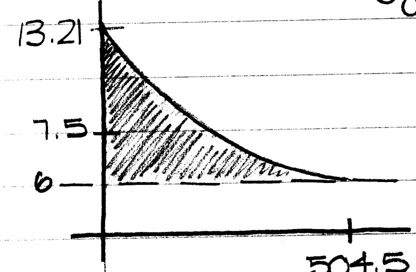
$T = \frac{.5}{35}$

$= .0143$

$y - 7.5 = -.0143(x - 400)$

$y - 7.5 = -.0143x + 5.7143$ → $D(x) = -.0143x + 13.2143$

$\int_0^{504.5} (D(x) - 6)\,dx = \$1819.79$



Cate Neukranz

3. (a) $f(x) \geq 0$ for all x and

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

(b) my intuition tells me that the value of the mean is 5

$$\int_0^{10} .1x\,dx = \frac{1}{20}x^2 \Big]_0^{10}$$

$$= \frac{1}{20}(10)^2 - 0$$

$$= 5$$

4. $f(x) = \begin{cases} kx^2(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x < 0 \text{ or } x > 1 \end{cases}$

(a) $\int_0^1 kx^2(1-x)\,dx = 1$

$k\int_0^1 (x^2 - x^3)\,dx = 1$

$k\left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = 1$

$k\left[\frac{1}{3} - \frac{1}{4}\right] = 1 \quad k\left(\frac{4}{12} - \frac{3}{12}\right) = 1 \quad \left(\frac{k}{12} = 1\right)12 \quad k = 12$

(b) $\int_{1/2}^1 12x^2(1-x)\,dx = 12\int_{1/2}^1 (x^2 - x^3)\,dx = 12\left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_{1/2}^1$

$12\left[\left(\frac{1}{3}(1^3) - \frac{1}{4}(1^4)\right) - \left(\frac{1}{3}\left(\frac{1}{2}^3\right) - \frac{1}{4}\left(\frac{1}{2}^4\right)\right)\right] =$

$12\left[\left(1/3 - 1/4\right) - \left(1/24 - 1/64\right)\right] = 11/16$

(c) $12\int_0^1 x^3(1-x)\,dx = 12\int_0^1 (x^3 - x^4)\,dx$

$12\left[\frac{1}{4}x^4 - \frac{1}{5}x^5\right]_0^1 = \frac{3}{5}$ mean

Thomas Berry

Chapter 7 Review Answers

1. $y = 2/3e^{x} + e^{-2x}$

$y' = \frac{2}{3}e^{x} - 2e^{-2x}$

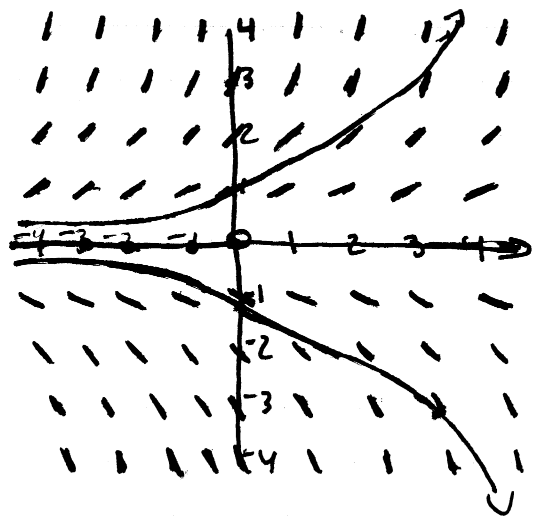
$y' + 2y = \frac{2}{3}e^{x} - 2e^{-2x} + 2\left(\frac{2}{3}e^{x} + e^{-2x}\right)$

$y' + 2y = \frac{2}{3}e^{x} + \frac{4}{3}e^{x} - 2e^{-2x} + 2e^{-2x}$

$y' + 2y = 2e^{x}$ so $y = 2/3e^{x} + e^{-2x}$ is a solution to

$y' + 2y = 2e^{x}$

2.



3. solve $dy/dx = xy^2$

$\int dy/y^2 = \int x\,dx$

$-1/y = x^2/2 + C$

$-y = \frac{2}{x^2} + \frac{1}{C}$

$y = \frac{-2}{x^2} - \frac{1}{C}$

or

$y = \frac{-2}{x^2} + D$

Mckenna Wray

7.4-7.5: Exponential Growth/Decay; Logistic Growth

1. Population of bacteria quadruples every 60 minutes. $P(1)=20$, $P(2)=80$.

a) $\frac{dP}{dt}=kP$

$\frac{1}{P}dP=kdt$

$\ln|P|=kt+c$

$|P|=e^{kt+c}$

$P=\pm e^c e^{kt} \quad c>0$

$P=Ae^{kt}$

$80=20e^{k(2)}$

$\ln 4=\ln e^{2k}$

$\ln(4)=2k\ln e$

$k=\frac{1}{2}\ln(4)$

$k=\ln\sqrt{4}$

$\boxed{k=\ln 2}$

b) using $P=Ae^{kt}$ from part a:

$P(0)=5$

$P=Ae^{kt}$

$\boxed{P=5e^{(\ln 2)t}}$

3. $P(0) = 100$; carrying capacity $= 1{,}000$ McKenna Wray
$P(1) = 250$; $P(5) = ?$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

$$\int \frac{dP}{P(1 - P/M)} = \int k\,dt$$

$$\frac{1}{P(1-P/M)} = \frac{M}{P(M-P)} = \frac{A}{P} + \frac{B}{M-P}$$

$M = A(M-P) + BP$

$P=0$ $M = AM + 0$

$A = 1$

$M=0$ $0 = -AP + BP$

$0 = -(1)P + BP$

$P = BP$

$B = 1$

$$\int \frac{1}{P} + \frac{1}{M-P}\,dP = \int k\,dt$$

$$\ln\left|\frac{P}{M-P}\right| = kt + C$$

$$\ln\left|\frac{M-P}{P}\right| = -kt - C$$

$$\frac{M-P}{P} = e^{-C}e^{-kt}$$

$$\frac{M-P}{P} = Ae^{-kt}$$

$$\frac{M}{P} - 1 = Ae^{-kt}$$

$$\frac{M}{P} = Ae^{-kt} + 1$$

$$\frac{P}{M} = \frac{1}{Ae^{kt} + 1}$$

$$P = \frac{M}{Ae^{-kt} + 1}$$

$$250 = \frac{1000}{100e^{-k(1)} + 1}$$

$250(100e^{-k} + 1) = 1000$

$100e^{-k} + 1 = 4$

$100e^{-k} = 3$

$\ln e^{-k} = \ln 3/100$

$-k \ln e = \ln(3/100)$

$\boxed{k = -\ln(3/100)}$

$$P = 1000 \Big/ 100e^{-(-\ln(3/100))5} + 1$$

$P = 1000 / 1.00000243$

$\boxed{P = 999.997}$

SOLUTION TO PROBLEMS OF SECTIONS 8.1 - 8.2

(1) Since $0 \le \cos^2 n \le 1$, we know that $0 \le a_n \le \frac{1}{2^n}$.

But, since $\lim_{n\to\infty} 0 = 0$ and $\lim_{n\to\infty} \frac{1}{2^n} = 0$, by the squeeze theorem, $\lim_{n\to\infty} a_n = 0$.

(2) Note that we can express $0.\overline{73}$ as a geometric series:

$$0.\overline{73} = \sum_{n=1}^{\infty} 73 \cdot \left(\frac{1}{100}\right)^n$$

since $|r| = \frac{1}{100} < 1$, by the geometric series test, the series converges and its sum is:

$$\sum_{n=1}^{\infty} 73 \cdot \left(\frac{1}{100}\right)^n = \frac{0.73}{1 - \frac{1}{100}} = \frac{0.73}{\frac{99}{100}} = \frac{73}{99}$$

(3) We claim that the series is convergent.

- We know that $\sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$ is a geometric series.

since $|r| = \frac{1}{e} < 1$, by the geometric series test, the series converges to:

$$\sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{\frac{1}{e}}{1 - \frac{1}{e}} = \frac{\frac{1}{e}}{\frac{e-1}{e}} = \frac{1}{e-1}$$

- $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{A}{n} + \frac{B}{n+1}\right)$

$A(n+1) + Bn = 1 \Rightarrow$
- For $n = 0$, $A = 1$
- For $n = -1$, $-B = 1 \rightarrow B = -1$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$ Note that this is a telescoping sum

Section 8.3 - 8.4 Solutions

① $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \ldots = \sum_{n=0}^{\infty} \frac{1}{2n+1}$

use limit comparison test; let $b_n = \frac{1}{n}$ (which is divergent)

$$\lim_{n\to\infty} \frac{1}{2n+1} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{2n}{n} + \frac{1}{n}} = \lim_{n\to\infty} \frac{1}{2+0} = \frac{1}{2}$$

$\frac{1}{2}$ is a finite # and $\frac{1}{2} > 0$, so $\sum_{n=0}^{\infty} \frac{1}{2n+1}$ $\boxed{\text{is also divergent}}$

② $\frac{2+(-1)^n}{n\sqrt{n}} < \frac{4}{n^{3/2}}$ $\quad \sum_{n=0}^{\infty} \frac{4}{n^{3/2}} = 4 \sum_{n=0}^{\infty} \frac{1}{n^{3/2}}$ converges by p-test

so $\sum_{n=0}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$ also $\boxed{\text{converges}}$ by comparison test

③ i) b_{n+1} must be $\leq b_n$ $\quad b_{n+1} = \frac{n+1}{(n+1)^2+9} = \frac{n+1}{n^2+2n+10}$

$\frac{n+1}{n^2+2n+10} < \frac{n}{n^2+9}$ when $n \geq 3$ ✓

ii) $\lim_{n\to\infty} b_n = 0$ $\quad \lim_{n\to\infty} \frac{n}{n^2+9} = \lim_{n\to\infty} \frac{\frac{n}{n^2}}{\frac{n^2}{n^2} + \frac{9}{n^2}} = \lim_{n\to\infty} \frac{0}{1+0} = 0$ ✓

so $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$ $\boxed{\text{converges}}$ by alternating series test

Andrew Hayes

7. $f(x) = \sin(\pi x)$ Maclaurin series $= \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$

$$F(x) = \sin(\pi x)$$
$$f'(x) = \cos(\pi x)\cdot\pi = \pi\cos(\pi x)$$
$$f''(x) = (0)\cos(\pi x) + -\sin(\pi x)\cdot\pi\cdot\pi = -\pi^2\sin(\pi x)$$
$$f'''(x) = (0)\sin(\pi x) + \cos(\pi x)\cdot\pi\cdot -\pi^2 = -\pi^3\cos(\pi x)$$
$$f^4(x) = (0)\cos(\pi x) + -\sin(\pi x)\cdot\pi\cdot -\pi^3 = \pi^4\sin(\pi x)$$
$$f^5(x) = (0)\sin(\pi x) + \cos(\pi x)\cdot\pi\cdot\pi^4 = \pi^5\cos(\pi x)$$

$f(x) = \sin(\pi x)$	$f(0) = 0$
$f'(x) = \pi\cos(\pi x)$	$f'(0) = \pi$
$f''(x) = -\pi^2\sin(\pi x)$	$f''(0) = 0$
$f'''(x) = -\pi^3\cos(\pi x)$	$f'''(0) = -\pi^3$
$f^4(x) = \pi^4\sin(\pi x)$	$f^4(0) = 0$
$f^5(x) = \pi^5\cos(\pi x)$	$f^5(0) = \pi^5$

$$\sin(\pi x) = \pi x - \frac{\pi^3}{3!}x^3 + \frac{\pi^5}{5!}x^5 - \dots (-1)^n \frac{\pi^{2n+1}}{(2n+1)!}x^{2n+1}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}\pi^{2n+3}x^{2n+3}}{(2n+3)!}\cdot\frac{(2n+1)!}{(-1)^n\pi^{2n+1}x^{2n+1}}\right| =$$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}}{(-1)^n}\cdot\frac{\pi^{2n+3}}{\pi^{2n+1}}\cdot\frac{x^{2n+3}}{x^{2n+1}}\cdot\frac{(2n+1)!}{(2n+3)!}\right|$$

$$\lim_{n\to\infty}\left|-1\cdot\pi^2\cdot x^2\cdot\frac{(2n+1)!}{(2n+3)!}\right|$$

$$\pi^2x^2\lim_{n\to\infty}\frac{(2n+1)!}{(2n+3)!} = \pi^2x^2\lim_{n\to\infty}\frac{\cancel{(2n+1)}}{(2n+3)(2n+2)\cancel{(2n+1)}} =$$

$$\pi^2x^2\lim_{n\to\infty}\frac{1}{(2n+3)(2n+2)} = \pi^2x^2\cdot 0 = \pi^2x^2 < 0$$ Converges for All X $R = +\infty$

$$\sin(\pi x) = \sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{(2n+1)!}x^{2n+1}, \quad R = +\infty$$

Andrew Hayes

13. $f(x)=e^x \quad a=3$

$$f(x)=e^x \qquad f(3)=e^3$$
$$f'(x)=e^x \qquad f'(3)=e^3$$
$$f''(x)=e^x \qquad f''(3)=e^3$$
$$f'''(x)=e^x \qquad f'''(3)=e^3$$

$$f^n(3)=e^3$$

$$\text{Taylor series} = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x)=e^x \quad a=3 \quad = \sum_{n=0}^{\infty} \frac{f^n(3)}{n!}(x-3)^n = \sum_{n=0}^{\infty} \frac{e^3}{n!}(x-3)^n$$

48. $$\int_0^{.2} \left[\tan^{-1}(x^3) + \sin(x^3)\right]$$

$$\tan^{-1}(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{2n+1}$$

$$\sin(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n+1)!}$$

$$\int_0^{.2} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{2n+1} + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n+1)!}$$

$$\left.\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(2n+1)(6n+4)}\right]_0^{.2} + \left.\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(2n+1)!(6n+4)}\right]_0^{.2}$$

$$\left[\frac{x^4}{4} - \frac{x^{10}}{30} + \frac{x^{16}}{80} - \cdots\right] + \left[\frac{x^4}{4} - \frac{x^{10}}{60} + \frac{x^{16}}{1920}\right]$$

$$\left[\frac{(.2)^4}{4} - \frac{(.2)^{10}}{30} + \frac{(.2)^{16}}{80}\right] + \left[\frac{(.2)^4}{4} - \frac{(.2)^{10}}{60} + \frac{(.2)^{16}}{1920}\right] = .00079983$$

Andrew Hayes

5. $f(x)=\cos(x)$, $a=\pi/2$

$f(x)=\cos(x)$ $\quad f(\pi/2)=\cos(\pi/2)=0$
$f'(x)=-\sin(x)$ $\quad f'(\pi/2)=-\sin(\pi/2)=-1$
$f''(x)=-\cos(x)$ $\quad f''(\pi/2)=-\cos(\pi/2)=0$
$f'''(x)=\sin(x)$ $\quad f'''(\pi/2)=\sin(\pi/2)=1$

$$T_3(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3$$

$$T_3(x)=f(\pi/2)+\frac{f'(\pi/2)}{1!}(x-\pi/2)+\frac{f''(\pi/2)}{2!}(x-\pi/2)^2+\frac{f'''(\pi/2)}{3!}(x-\pi/2)^3$$

$$T_3(x)=0-(x-\pi/2)+\frac{0}{2}(x-\pi/2)^2+\frac{1}{6}(x-\pi/2)^3$$

$$\underline{T_3(x)=-(x-\pi/2)+\frac{1}{6}(x-\pi/2)^3}$$

6. $f(x)=\frac{\ln x}{x}$, $a=1$

$$f(x)=\frac{\ln x}{x}$$

$$f'(x)=\frac{(\frac{1}{x}\cdot x)-(1)(\ln x)}{x^2}=\frac{1-\ln x}{x^2}$$

$$f''(x)=\frac{(-\frac{1}{x})(x^2)-(2x)(1-\ln x)}{x^4}=\frac{-x-2x+2x\ln x}{x^4}$$

$$f'''(x)=\frac{\left[-1-2+2(\ln x)+\frac{1}{x}(2x)\right]\cdot\left[x^4\right]-\left[(4x^3)(-x-2x+2x\ln x)\right]}{x^8}=\frac{11-6\ln x}{x^4}$$

$f(1)=0$
$f'(1)=1$
$f''(1)=-3$
$f'''(1)=11$

$$T_3(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3$$

$$T_3(x)=f(1)+\frac{f'(1)}{1!}(x-1)+\frac{f''(1)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3$$

$$\underline{T_3(x)=(x-1)-\frac{3}{2}(x-1)^2+\frac{11}{6}(x-1)^3}$$

Andrew Hayes

25. $\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5}$ $\quad(|\text{error}| < .05)$

$\frac{x^5}{5} < \frac{x^7}{7}$

$\frac{|x|^7}{7} < .05$

$|x|^7 < .35$

$|x| < \sqrt[7]{.35}$

$|x| < .86$

$\underline{-.86 < x < .86}$