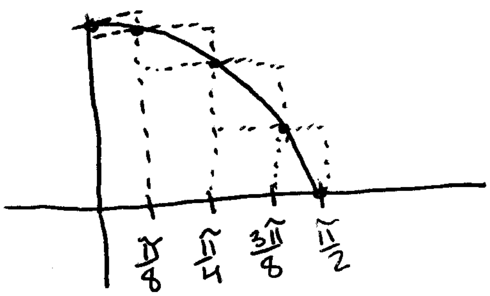
Sections 5.1 - 5.3

Aimee Steen

(1)



a) $\frac{\pi}{8}\left(\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{3\pi}{8}\right)+\cos\left(\frac{\pi}{2}\right)\right)$

$= .7908$ underestimate

b) $\frac{\pi}{8}\left(\cos(0)+\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{4}\right)+\cos\left(\frac{3\pi}{8}\right)\right)$

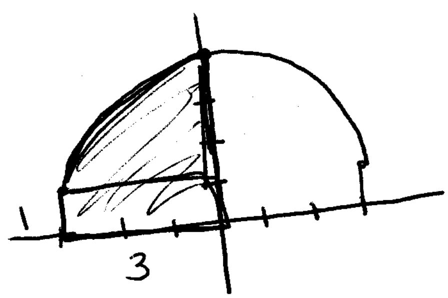
$= 1.1835$ overestimate

c) $\frac{\pi}{8}\left(\cos\left(\frac{\pi}{16}\right)+\cos\left(\frac{3\pi}{16}\right)+\cos\left(\frac{5\pi}{16}\right)+\cos\left(\frac{7\pi}{16}\right)\right)$

$= 1.006$ overestimate

(2)

$$\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx$$



$$\int_{-3}^{0} 1\,dx + \int_{-3}^{0}\sqrt{9-x^2}\,dx$$

↑ its a semi-circle

$$3 + \frac{1}{4}\pi(3)^2$$

$$3 + \frac{9\pi}{4}$$

Sections 5.1-5.3

Aimee Steen

3) $$\int_0^{\frac{1}{\sqrt{3}}} \frac{(t^2-1)}{(t^4-1)}\,dt$$

$$= \int_0^{\frac{1}{\sqrt{3}}} \frac{(t^2-1)}{(t^2+1)(t^2-1)}\,dt$$

$$= \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{(t^2+1)}\,dt$$

✓ continuous on $[0, \frac{1}{\sqrt{3}}]$

✓ differentiable on $(0, \frac{1}{\sqrt{3}})$

* Fundamental Theorem of Calculus!

$$= \arctan(t) \Big|_0^{\frac{1}{\sqrt{3}}}$$

$$= \arctan\left(\tfrac{1}{\sqrt{3}}\right) - \arctan(0)$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6}$$

Stephanie Katz - 5.4, The Fundamental Theorem of Calculus

Solution:

14. $h(x) = \int_0^{x^2} \sqrt{1+r^3}\, dr$

$u = x^2 \qquad du = 2x$

$h'(x) = \frac{d}{dx} \int_0^{x^2} \sqrt{1+r^3}\, dr$

$h'(x) = \frac{d}{dx} \int_0^{u} \sqrt{1+r^3}\, dr$

$h'(x) = \frac{d}{du} \int_0^{u} \sqrt{1+r^3}\, dr \cdot \frac{du}{dx}$

$= \sqrt{1+u^3} \cdot \frac{du}{dx}$

$= \sqrt{1+(x^2)^3}\; 2x$

$= \sqrt{1+x^6} \cdot 2x$

Stephanie Katz - 5.5 The Substitution Rule, - U-Sub

Solutions

22. $\int \frac{\tan^{-1}x}{1+x^2}\,dx$

$u = \tan^{-1}x$

$\frac{du}{dx} = \frac{1}{1+x^2}$

$du = \frac{1}{1+x^2}\,dx$

$= \int u\,du$

$= \frac{u^2}{2} + C$

$= \frac{(\tan^{-1}x)^2}{2} + C$

48. $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\,dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\int_0^{\pi/2} \sin(\sin x) \cos x\,dx$

$\frac{\pi}{2} \rightarrow u$ number $\quad u = \sin\frac{\pi}{2} = 1$

$0 \rightarrow u$ number $\quad u = \sin 0 = 0$

$\int_0^1 \sin u\,du$

$\cos u \Big|_0^1$

$\cos(1) - \cos(0)$

$\cos(1) - 1$

Stephanie Katz - 5.6 - Integration by Parts

Solutions:

18. $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$

$u = \ln y \qquad v = 2\sqrt{y}$

$du = \frac{1}{y}\,dy \qquad dv = y^{-1/2}\,dy$

$= \ln y \cdot 2\sqrt{y}\,\Big|_4^9 - \int_4^9 2\sqrt{y} \cdot \frac{1}{y}\,dy$

$\int_4^9 2y^{-1/2}\,dy$

$2 \cdot 2y^{1/2}\,\Big|_4^9$

$= 2\sqrt{y}\ln y\,\Big|_4^9 - 4\sqrt{y}\,\Big|_4^9$

$= 2\sqrt{9}\ln 9 - 2\sqrt{4}\ln 4 - 4\sqrt{9} + 4\sqrt{4}$

$6\ln 9 - 4\ln 4 - 12 + 8$

$6\ln 9 - 4\ln 4 - 4$

Trig Substitutions for Integrals / Integration with Partial Fractions 5.7

Colin Whitney

1.

$$\int \frac{x^3}{\sqrt{16-x^2}}\,dx$$

Let $x = 4\sin\theta$
$dx = 4\cos\theta\, d\theta$

$$= \int \frac{(4\sin\theta)^3 \cdot 4\cos\theta\, d\theta}{\sqrt{16-(4\sin\theta)^2}}$$

$$= \int \frac{4^4 \sin^3\theta \cos\theta \; d\theta}{\sqrt{16(1-\sin^2\theta)}}$$

Use:
$1-\sin^2 x = \cos^2 x$

$$= \int \frac{4^4 \sin^3\theta \cos\theta\, d\theta}{4\sqrt{\cos^2\theta}}$$

$$= 4^3 \int \frac{\sin^3\theta\cos\theta}{\cos\theta}\, d\theta$$

$$= 4^3 \int \sin^3\theta\, d\theta$$

$$= 4^3 \int \sin^2\theta \sin\theta\, d\theta$$

Use:
$\sin^2 x = 1-\cos^2 x$

$$= 4^3 \int (1-\cos^2\theta)\sin\theta\, d\theta$$

Let $u = \cos\theta$
$-du = \sin\theta\, d\theta$

$$= -4^3 \int (1-u^2)\,du$$

$$= -64\left(u - \frac{u^3}{3}\right) + C$$

$$= -64\left(\cos\theta - \frac{\cos^3\theta}{3}\right) + C$$

Trig Substitutions for Integrals / Integration with Partial Fractions 5.7

Colin Whitney

2. $\int \sqrt{x^2+9}\, dx$

Let $x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$= \int \sqrt{(3\tan\theta)^2+9} \cdot 3\sec^2\theta\, d\theta$

$= \int \sqrt{9(\tan^2\theta+1)} \cdot 3\sec^2\theta\, d\theta$

Use: $\tan^2 x + 1 = \sec^2 x$

$= 9\int \sec^3\theta\, d\theta$

Let:

$u = \sec\theta \quad dv = \sec^2\theta\, d\theta$

$du = \sec\theta\tan\theta\, d\theta \quad v = \tan\theta$

$= 9\int \sec^2\theta \cdot \sec\theta\, d\theta$

$= 9(\sec\theta\tan\theta) - \int \sec\theta\tan^2\theta\, d\theta$

Use: $\tan^2 x = \sec^2 x - 1$

$= 9(\sec\theta\tan\theta) - \int \sec\theta(\sec^2\theta - 1)\, d\theta$

$= 9(\sec\theta\tan\theta) - \int \sec^3\theta - \sec\theta\, d\theta$

$9\int \sec^3\theta = 9(\sec\theta\tan\theta) - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$

$2\left(9\int \sec^3\theta\, d\theta\right) = 9(\sec\theta\tan\theta) + \int \sec\theta\, d\theta$

$2\left(9\int \sec^3\theta\, d\theta\right) = 9(\sec\theta\tan\theta) + \ln|\sec\theta + \tan\theta) + C$

$= \frac{9}{2}(\sec\theta\tan\theta) + \frac{1}{2}\ln|\sec\theta\tan\theta| + C$

Trig Substitutions for Integrals / Integration with Partial Fractions 5.7

Colin Whitney

3.

$$\int \frac{1}{x^2-4}\,dx$$

$$= \int \frac{1}{(x-2)(x+2)}\,dx$$

$$= \int \left(\frac{A}{x-2}\right)+\left(\frac{B}{x+2}\right) dx$$

$$\frac{1}{(x-2)(x+2)} = \frac{A}{(x-2)} + \frac{B}{(x+2)}$$

$$A(x+2) + B(x-2) = 1$$

Let $x=-2$

$$A(-2+2) + B(-2-2) = 1$$

$$B(-4) = 1$$

$$B = -\frac{1}{4}$$

Let $x=2$

$$A(2+2) + B(2-2) = 1$$

$$A(4) = 1$$

$$A = \frac{1}{4}$$

$$= \int \frac{1/4}{x-2} + \frac{-1/4}{x+2}\,dx$$

$$= \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$$

$$= \frac{1}{4}\left(\ln\frac{|x-2|}{|x+2|}\right) + C$$

Mike Sheldon

5.8 - 5.9: Table of Integrals and Approximate Integration

Table 45: $\displaystyle\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$

5.8: #4.) $\displaystyle\int_2^3 \frac{1}{x^3\sqrt{4x^2-7}}\,dx$

$u=2x \quad u^2=4x^2$

$a=\sqrt{7} \quad a^2=7$

$u=2x$
$du=2dx$

$x=2 \Rightarrow u=4$
$x=3 \Rightarrow u=6$

$$=\frac{1}{2}\int_4^6 \frac{du}{(u/2)^2\sqrt{u^2-a^2}}$$

$$=2\int_4^6 \frac{du}{u^2\sqrt{u^2-a^2}}$$

$$=\frac{2\sqrt{u^2-a^2}}{u\cdot a^2}\Bigg|_4^6$$

$$=\frac{2\sqrt{u^2-7}}{7u}\Bigg|_4^6$$

$$=\frac{2\sqrt{6^2-7}}{7\cdot 6}-\frac{2\sqrt{4^2-7}}{7\cdot 4}$$

$$=\frac{\sqrt{29}}{21}-\frac{\sqrt{9}}{14}$$

$$=\frac{2\sqrt{29}}{42}-\frac{9}{42}$$

$$=\boxed{\frac{2\sqrt{29}-9}{42}}$$

5.8: #7.) $\displaystyle\int_0^\pi x^3\sin x\,dx$

Table 84: $\displaystyle\int u^n\sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$

$u=x$

$$=-x^3\cos x+3\int_0^\pi x^2\cos x$$

Table 85: $\displaystyle\int u^n\cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$

$$=-x^3\cos x+3\left(x^2\sin x-2\int_0^\pi x\sin x\,dx\right)$$

$u=x$

$$=-x^3\cos x+3\left(x^2\sin x-2\left(\sin x-x\cos x\right)\right)\Big|_0^\pi$$

$$=\left[\pi^3+3\left(\pi^2\sin\pi-2\left(\sin\pi-\pi\cos\pi\right)\right)\right]-\left[0+3\left(0-2\left(0-0\right)\right)\right]=\boxed{\pi^3-6\pi}$$

Mike Sheldon

5.8 - 5.9: Table of Integrals and Approximate Integration

5.9: 28.) $\frac{.5}{3}\left[0+4(4.67)+(2\cdot 7.34)+4(8.86)+2(9.73)+4(10.22)+2(10.51)\right.$
$\left.+4(16.67)+2(10.76)+4(10.81)+(10.81)\right]$

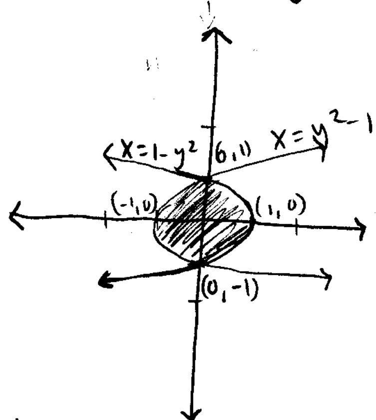
$= \boxed{44.735\ m}$

Shea Ricketis

1. Sketch the region enclosed by the given curves. Find the area of the region (6.1)

$x = 1 - y^2 \qquad x = y^2 - 1$

Note: Intergrate Right to left with this graph.



$$\int_c^d (\text{Right} - \text{Left})\,dy$$

$$\int_{-1}^{1} (1-y^2) - (y^2-1)\,dy$$

$$\int_{-1}^{1} 1 - y^2 - y^2 + 1\,dy$$

$$\int_{-1}^{1} 2 - 2y^2\,dy$$

$$2y - \frac{2y^3}{3}\Big|_{-1}^{1}$$

$$\left(2(1) - \frac{2}{3}(1)^3\right) - \left(2(-1) - \frac{2}{3}(-1)^3\right)$$

$$2 - \frac{2}{3} + 2 - \frac{2}{3}$$

$$4 - \frac{4}{3} = \frac{8}{3}$$

$$\boxed{\text{Area is } \frac{8}{3} \text{ units}^2}$$

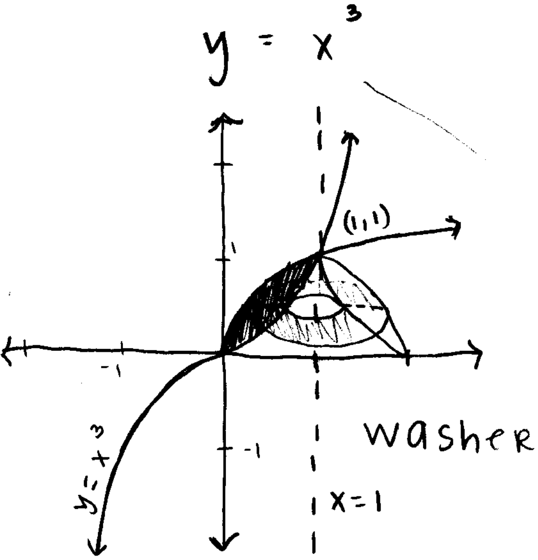
Shea Ricketts

2. Find the volume of the solid obtained by rotating the region bound by the given curves around the specific line. Sketch the region and Label as either disk or washer. (6.2)

$y = x^3$ $\quad y = \sqrt{x}$, about the line $x = 1$

$y = \sqrt{x} \to y^2 = x$

$y^{\frac{1}{3}} = x$



washer

Area of cross section

$\pi\left((R\,out)^2 - R(inner)^2\right)$

$$\pi \int_a^b (R(y)^2 - r(y)^2)\, dy$$

note $R(out) = R(y)$

$R(in) = r(y)$

$R(y) = 1 - y^2$

$r(y) = 1 - y^{\frac{1}{3}}$

$$\pi \int_0^1 (1 - y^2)^2 - (1 - y^{\frac{1}{3}})^2\, dy$$

$$\pi \int_0^1 (1 - 2y^2 + y^4) - (1 - 2y^{\frac{1}{3}} + y^{2/3})\, dy$$

$$\pi \int_0^1 \cancel{1} - 2y^2 + y^4 - \cancel{1} + 2y^{\frac{1}{3}} - y^{\frac{2}{3}}\, dy$$

$$\pi \int_0^1 y^4 - 2y^2 - y^{2/3} + 2y^{1/3}\, dy$$

$$\pi \left[\frac{y^5}{5} - \frac{2}{3}y^3 - \frac{3}{5}y^{2/3} + \frac{3}{2}y^{4/3} \Big|_0^1 \right]$$

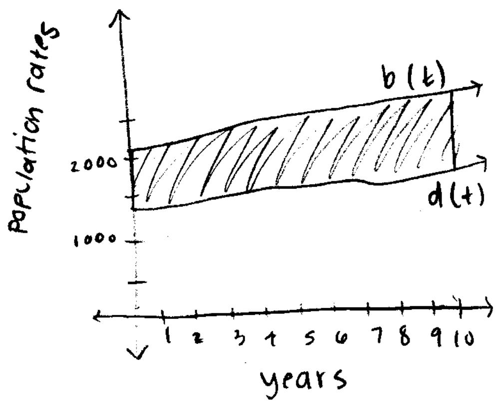
$$\pi \left(\frac{1}{5} - \frac{2}{3} - \frac{3}{5} + \frac{3}{2} \right)$$

$$\pi \left(\frac{6}{30} - \frac{20}{30} - \frac{18}{30} + \frac{45}{30} \right) = \frac{13\pi}{30}$$

$\boxed{\text{washer area is } \frac{13\pi}{30} \text{ units}^3}$

Shea Ricketts

3. If the birth rate of a population is $b(t) = 2200e^{0.024t}$ people per year and the death rate is $d(t) = 1460e^{0.018t}$ people per year, find the area between these curves for 10 years. What does the area represent. (6.2)



$$\int_a^b (\text{TOP} - \text{Bottom})\,dx$$

$$\int_0^{10} 2200e^{0.024t} - 1460e^{0.018t}\,dt$$

can stop here and plug into calculator.

$$\int_0^{10} 2200e^{0.024t} - \int_0^{10} 1460e^{0.018t}\,dt$$

$$2200\int_0^{10} e^{0.024t}\,dt - 1460\int_0^{10} e^{0.018t}\,dt$$

$$\frac{2200}{0.024}\left[e^{0.024}\Big|_0^{10}\right] - \frac{1460}{0.018}\left[e^{0.018}\Big|_0^{10}\right]$$

$$\frac{2200}{0.024}\left(e^{0.024\times10} - e^{0}\right) - \frac{1460}{0.018}\left(e^{0.018\times10} - e^{0}\right)$$

$$24864.50545 - 15996.51945$$

$$8867.989$$

$$\boxed{8868}$$

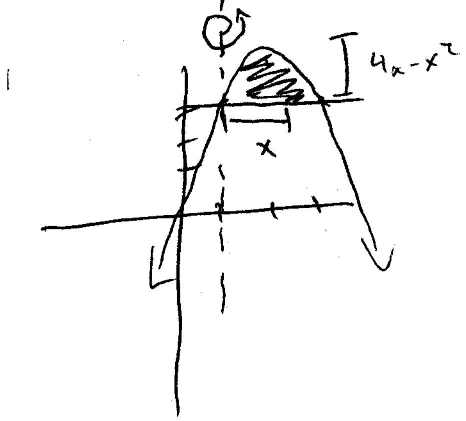
This is the amount of population increase over 10 years.

Chris Miller

6.3

15.) Find volume of revolution

$y = 4x - x^2, \quad y = 3 \quad$ about $x = 1$



$2\pi \int_a^b h(r)\,dx \rightarrow$ cylinders

$2\pi \int_1^3 (4x - x^2)(x)\,dx$

$2\pi \int_1^3 4x^2 - x^3$

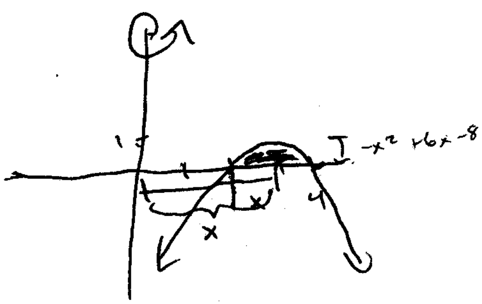
$2\pi \left[\left(\frac{4}{3}\right)x^3 - \frac{1}{4}x^4 \right]_1^3$

$\cancel{\left(\frac{2}{3}\right)\pi}$ or $\frac{8\pi}{3}$

2.667π

29.) Find volume of revolution

$y = -x^2 + 6x - 8 \quad y = 0 \quad$ about: x-axis



$2\pi \int_a^b h(r)\,dx$

$2\pi \int_2^4 (-x^2 + 6x - 8)\,(x-2)$

$\cancel{2\pi \int_2^4 x^3 - 10x^2 + 32x - 32}$

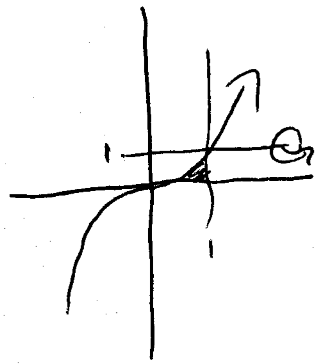
$\cancel{2\pi \int_2^4 [x^3 + 16}$

$2\pi \int_2^4 -x^3 + 4x^2 - 20x + 16$

$= 8\pi$

Find volume of revolution

17.) $y = x^3$ $y = 0$ $x = 1$ about $y = 1$ 6.3



$$2\pi \int_a^b r(h)\, dx$$

$$2\pi \int_0^1 y(\sqrt{y})\, dy$$

$$2\pi \int_0^1 y^{3/2}\, dy$$

$$= .357\pi$$

or

$$\frac{5\pi}{14}$$

Section 6.5 Average Value of a Function

Jeffrey Kadis

Q3: $f(x) = 3x^2 - 2x \quad [1, 4]$

$a = 1,\ b = 4$

Equation $= \frac{1}{b-a} \int_a^b f(x)\,dx$

so... $\frac{1}{4-1} \int_1^4 3x^2 - 2x \, dx$

$= \frac{1}{3} \left[x^3 - x^2 \right]_1^4$

$= \frac{1}{3} \left[\left[(4)^3 - (4)^2 \right] - \left[(1)^3 - (1)^2 \right] \right]$

$= \frac{1}{3} (64 - 16 - 1 + 1)$

$= 48/3$

$\boxed{= 16}$

Section 6.4 Solutions

Jeffrey Kadis

Q1: If we are given a curve with equation $y=f(x)$, $a \le x \le b$, then we can regard x as a parameter.

Use equation: $L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$f(x) = \frac{1}{3}(x^2+2)^{3/2}$

$f'(x) = \frac{1}{\cancel{3}}\frac{\cancel{3}}{2}(x^2+2)^{1/2}\cancel{2}x \Rightarrow x(x^2+2)^{1/2}$

$[f'(x)]^2 = [x(x^2+2)^{1/2}]^2 = x^2(x^2+2) = x^4+2x^2$

$1+[f'(x)]^2 = x^4+2x^2+1$

So... $\int_0^1 \sqrt{x^4+2x^2+1}\, dx \Rightarrow \int_0^1 \sqrt{(x^2+1)(x^2+1)}\, dx \Rightarrow$

$\Rightarrow \int_0^1 \sqrt{(x^2+1)^2}\, dx \Rightarrow \int_0^1 (x^2+1)\, dx \Rightarrow \left[\frac{x^3}{3}+x\right]_0^1$

$\left[\frac{(1)^3}{3}+(1)\right] - \emptyset = \left(\frac{1}{3}+1\right) = \boxed{4/3}$

Section 6.4 solutions

Jeffrey Kadis

Q2: If we are given a curve with equation $x = f(y)$, $a \le y \le b$, then we can regard y as a perameter

Use equation: $L = \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$

$f(y) = y^{3/2}$

$f'(y) = \frac{3}{2} y^{1/2}$

$[f'(y)]^2 = \left[\frac{3}{2} y^{1/2}\right]^2 = \frac{9}{4} y$

$[f'(y)]^2 + 1 = \frac{9}{4} y + 1 \Rightarrow \frac{9y+4}{4} \Rightarrow \frac{1}{4}(9y+4)$

so.. $\int_0^1 \sqrt{\frac{1}{4}(9y+4)}\, dy = \int_0^1 \frac{1}{2}(9y+4)^{1/2 + \frac{2}{2}} dy \Rightarrow \left[\frac{1}{2}\frac{2}{3}\frac{1}{9}(9y+4)^{3/2}\right]_0^1 \Rightarrow$

$= \left[\frac{1}{27}(9(1)+4)^{3/2}\right] - \left[\frac{1}{27}(9(0)+4)^{3/2}\right]$

$= \frac{1}{27}(46.87 - 8) = \frac{38.87}{27} \approx \boxed{1.44}$

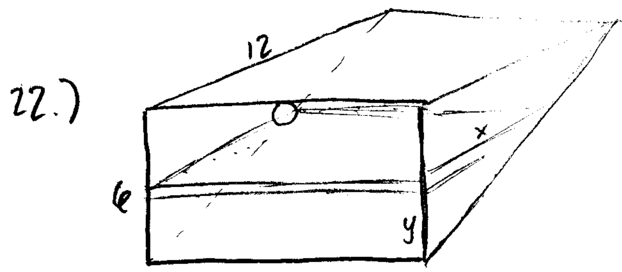
6.6.

6.) $f(x) = kx$ and $W = \int_0^{0.05} 2500x\,dx = 1250x^2 \Big|_0^{0.05}$

$0.01k = 25$ $\qquad = 3.125$ N

$k = 2500$

22.)



$A_{cross} = (12 - x) \cdot 10$

$V_{cross} = (120 - 10x)\,dx$

$V_{solid} = (120 - 10(180 - (6-y)^2)^{\frac{1}{2}} + 120\,)\,dy$

$= (240 - 10(180 - (6+y)^2)^{\frac{1}{2}}\,dy$

$= \int_0^6 240 - 10(180 - (6-y)^2)^{\frac{1}{2}}\,dy$

$W = \int_0^6 240 - 10(180 - (6-y)^2)^{\frac{1}{2}}\,dy\,(6-y)$

$= \frac{1}{2}\int_0^6 240 - 10u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\left(240u - \frac{20u^{3/2}}{3}\right)\Big|_0^6$

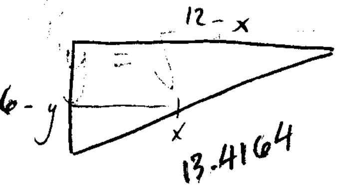
$= 671.01 \cdot 62.5 \frac{lb}{ft^3} = \boxed{41938.14 \text{ ft-lb}}$

$u = 180 - (6-y)^2$

$du = -2(6-y)[-1]dy$

$\frac{1}{2}du = (6-y)\,dy$

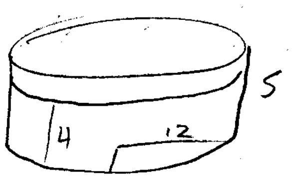
$a^2 + b^2 = c^2$



$y^2 - 12y + 36 + x^2 - 24x + 144 = 180$

$x = \pm\sqrt{180 - (6-y)^2} - 12$

18.)



distance: $5 - y$

$A_{cross} = \pi r^2 = \pi \cdot 144$

$V_{pool} = 144\pi\,dy$

$= \int_0^4 144\pi\,dy$

$\frac{W_{pool}}{\text{weight}} = \int_0^4 144\pi\,dy\,(5-y) = \int_0^4 720\pi - 144\pi y\,dy$

$= 720\pi y - 72\pi y^2 \Big|_0^4$

$= 2880\pi - 1152\pi = 1728\pi \cdot 62.5 \frac{lb}{ft^3}$

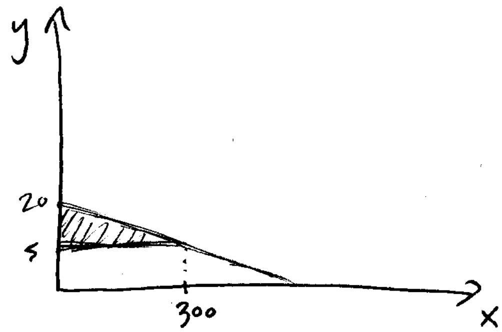
$= 108{,}000\pi$ ft-lb

6.7

4.) $p = 20 - 0.05x$ when $x = 300$

$P = 20 - 0.05(300)$

$= 20 - 15 = 5$



$C_{surp} = 300 \cdot 15/2 =$ $\begin{array}{r} 150 \\ \times 15 \\ \hline 750 \\ 1500 \\ \hline 2250 \end{array}$ $= \$2250$

7.) $p = 200 + 0.2x^{3/2}$

$P_{surp} = 400 \cdot 100 - \int_0^{100} (200 + 0.2x^{3/2})\,dx$

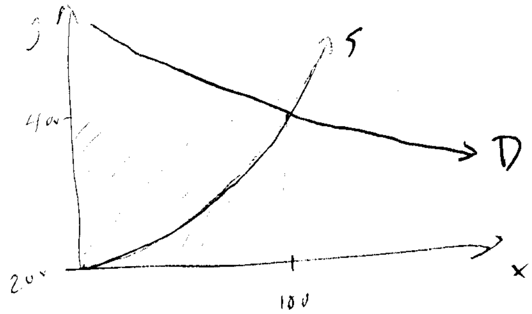
$400 = 200 + 0.2x^{3/2}$

$200 = 0.2x^{3/2}$

$1000 = x^{3/2}$

$1000000 = x^3$

$100 = x$



$P_{surp} = 40000 - \left(200x + \frac{2}{5} \cdot \frac{1}{5} x^{5/2}\right)\Big|_0^{100}$

$= 40000 - 20000 - \frac{2}{25} 100^{5/2}$

$= 20000 - \frac{2}{25} \cdot 100000$

$= 20000 - 8000 = \$12000$

14.) $f(t) = 2200 + 10e^{0.8t}$

pop change $= 2200 + 10e^{0.8(9)} - (2200 + 10e^{0.8(5)})$

$= 10e^{7.2} - 10e^{4} =$

Christopher Sheehan
8.1 - 8.2
Solutions

1. $a_n = \tan\left(\frac{2n\pi}{1+8n}\right)$

$a_1 = \tan \frac{2\pi}{9}$

$a_2 = \tan \frac{4\pi}{17}$

$a_3 = \tan \frac{6\pi}{25}$

$a_4 = \tan \frac{8\pi}{33}$

$\lim_{n\to\infty} \frac{2n\pi}{1+8n} = \tan \frac{2\pi}{8} = \tan \frac{\pi}{4} = \boxed{1}$

2. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

$= \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n}$

$= \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

$= \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{\frac{2}{3}}{1-\frac{2}{3}}$

$= \frac{1}{2} + 2$

$= \boxed{\frac{5}{2}}$

Christopher Sheehan
8.1 - 8.2
Solutions

3. $\sum_{n=2}^{\infty} (1+c)^{-n} = 2$

$$\sum_{n=1}^{\infty} (1+c)^{-(n+1)} = 2$$

$$\sum_{n=1}^{\infty} (1+c)^{(-n-1)} = 2$$

$$\sum_{n=1}^{\infty} \frac{1}{(1+c)^{n+1}} = 2$$

$$\sum_{n=1}^{\infty} \frac{1^n}{(1+c)^{n+1}} = 2$$

$$\sum_{n=1}^{\infty} \frac{1^{-1} \cdot 1^{n+1}}{(1+c)^{n+1}} = 2$$

form: $\sum_{n=1}^{\infty} a r^{n+1}$

$$\sum_{n=1}^{\infty} 1 \cdot \left(\frac{1}{1+c}\right)^{n+1} = 2$$

$r = \frac{1}{1+c}$

$$\boxed{c = \frac{\sqrt{3}-1}{2}}$$

Solutions:

1) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ $f(x) = \frac{1}{x\ln x}$

function is continuous, positive, & decreasing when $x > 0$
$\therefore$ Integral Test applies

$\lim_{B\to\infty} \int_2^B \frac{1}{x\ln x}\,dx$ $u = \ln x$
$du = \frac{1}{x}\,dx$

$\lim_{B\to\infty} \int_{\ln 2}^{\ln B} \frac{1}{u}\,du$

$$\lim_{B\to\infty} \ln u \Big]_{\ln 2}^{\ln B} = \underbrace{\ln(\ln B) - \ln(\ln 2)}$$

as B approaches infinity this approaches infinity so integral $\int \frac{1}{x\ln x}dx$ diverges.
Therefore, $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ diverges by Integral Test

Nicole Potter
Sections 8.3-8.4

2) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$ $b_n = \frac{n}{n^2+9}$ $f(x) = \frac{x}{x^2+9}$

$\lim_{n \to \infty} b_n$

$= \lim_{n \to \infty} \frac{n}{n^2+9}$

$= \lim_{n \to \infty} \frac{\frac{1}{n}}{1 + \frac{9}{n^2}} = 0$

$f'(x) = \frac{(x^2+9) - x(2x)}{(x^2+9)^2}$

$f'(x) = \frac{-x^2+9}{(x^2+9)^2}$

$f'(x) < 0$ when $(-x^2+9) < 0$

when $x > 3$

function is decreasing on interval $(3, \infty)$

so $f(n+1) < f(n)$ or

$b_{n+1} < b_n$ when $n \geq 3$

$\therefore$ series is decreasing

Since b_n is decreasing and $\lim_{n \to \infty} b_n = 0$, the alternating series test applies and the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$ is convergent

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Sections 8.3-8.4

3) $$\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$$

$$\lim_{n\to\infty} \left| \frac{10^{n+1}}{(n+2)4^{2n+3}} \cdot \frac{(n+1)4^{2n+1}}{10^n} \right|$$

$$= \lim_{n\to\infty} \frac{10(n+1)}{(n+1)4^2}$$

$$= \frac{10}{16} \lim_{n\to\infty} \frac{(n+1)}{(n+2)} = \frac{10}{16}$$

Since $\frac{10}{16} < 1$, $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$ is absolutely convergent by ratio test.

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8.5: # 22

$$\sum_{n=2}^{\infty} \frac{x^2}{n(\ln n)^2}$$

$$\lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^{2n}} \right| = x^2$$

converges if $|x^2|<1$

diverges if $|x^2|>1$

endpoints: $-1, 1$

$$\lim_{n\to\infty} \frac{(-1)^{2n}}{n(\ln n)^2} = 0 \quad \text{converges}$$

$$\lim_{n\to\infty} \frac{1^{2n}}{n(\ln n)^2} = 0 \quad \text{converges}$$

ROC = 1

IOC = $[-1, 1]$

8.6: #4

$\frac{3}{1-x^4}$ is a geometric series: $a=3$, $r=x^4$

$$\frac{3}{1-x^4} = \sum_{n=0}^{\infty} 3(x^4)^n = \sum_{n=0}^{\infty} 3x^{4n}$$

$$\lim_{n\to\infty} \left| \frac{3x^{4(n+1)}}{3x^4} \right| = |x^4|$$

conv when $|x^4|<1$

div when $|x^4|>1$ endpoints: $-1, 1$

$$\sum_{n=0}^{\infty} 3(-1)^{4n} = 3 \quad \text{diverges}$$

$$\sum_{n=0}^{\infty} 3(1)^{4n} = 3 \quad \text{diverges}$$

IOC = $(-1, 1)$

Alyssa Zoppi

8.6 : # 26

$$\int \arctan(x^2)\,dx$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\int \frac{1}{1+x^2} = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

$$\arctan(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2n+1}$$

$$\int \arctan(x^2)\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)}$$

converges when $|x| < 1$

$|-x^2| < 1 \rightarrow |x| < 1$

$|x| < 1$

$|x^2| < 1 \rightarrow |x| < 1$

$|x| < 1$

ROC = 1

13.) $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

i) $f(a) = e^a \longrightarrow e^3$

$f'(a) = e^a \longrightarrow e^3$

$f''(a) = e^a \longrightarrow e^3$

$\downarrow$

ii) $f(x) = e^3 + \frac{e^3}{1!}(x-3) + \frac{e^3}{2!}(x-3)^2 + \frac{e^3}{3!}(x-3)^3 + \frac{e^3}{4!}(x-3)^4 + \ldots$

iii) $f(x) = \sum_{n=0}^{\infty} \frac{e^3}{n!}(x-3)^n$

53) $$\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5} = \lim_{x\to 0} \frac{-x + \frac{1}{6}x^3 + \sin x}{x^5}$$

$$= \lim_{x\to 0} \frac{-x + \frac{1}{6}x^3 + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}}{x^5}$$

$$= \lim_{x\to 0} \frac{-x + \frac{1}{6}x^3 + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\right)}{x^5}$$

$$= \lim_{x\to 0} -x + \frac{x^3}{6} + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\right)$$

$$* \quad \frac{x^3}{3!} = \frac{x^3}{6}$$

$$= \lim_{x\to 0} \frac{\cancel{-x} + \cancel{\frac{x^3}{6}} + \cancel{x} - \cancel{\frac{x^3}{6}} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots}{x^5}$$

$$= \lim_{x\to 0} \frac{\frac{x^5}{5!} - \frac{x^7}{7!} + \ldots}{x^5} = \lim_{x\to 0} \frac{x^5}{5!\,x^5} - \frac{x^7}{7!\,x^5} + \ldots$$

$$= \lim_{x\to 0} \frac{1}{5!} - \frac{x^2}{7!} + \ldots$$

$$= \frac{1}{5!} = \frac{1}{120}$$

29) $f(x) = x\cos\left(\frac{1}{2}x^2\right)$

i) $\longrightarrow \cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

$$\cos\left(\tfrac{1}{2}x^2\right) = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{1}{2}x^2\right)^{2n}}{(2n)!}$$

ii)
$$f(x) = x \cdot \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{1}{2}x^2\right)^{2n}}{(2n)!}$$

$$= x \cdot \sum_{n=0}^{\infty} (-1)^n \frac{\frac{x^{2\cdot 2n}}{2^{2n}}}{(2n)!}$$

$$= x \cdot \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{2^{2n}(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{2^{2n}(2n)!}$$