Nick Goodwin

5.1

1. (a) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. Then improve your estimate by using six rectangles.
(b) repeat part (a) using left endpoints
(c) repeat part (a) using midpoints

2. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v(ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

Nick Goodwin

5.1

1\. (a) $f(0) = 1 + 0^2 = 1 \quad f(1) = 1 + 1^2 = 2 \quad f(2) = 1 + 2^2 = 5$

$R_3 = 1(1 + 2 + 5) = 8$

$f(-\frac{1}{2}) = 1 + (-\frac{1}{2})^2 = \frac{5}{4} \quad f(\frac{1}{2}) = 1 + (\frac{1}{2})^2 = \frac{5}{4} \quad f(\frac{3}{2}) = 1 + (\frac{3}{2})^2 = \frac{13}{4}$

$R_6 = \frac{1}{2}(\frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4} + 5) = \frac{55}{8}$

(b) $f(-1) = 1 + (-1)^2 = 2 \quad f(0) = 1 \quad f(1) = 2$

$L_3 = 1(2 + 1 + 2) = 5$

$L_6 = \frac{1}{2}(2 + \frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4}) = \frac{43}{8}$

(c) $M_3 = 1(\frac{5}{4} + \frac{5}{4} + \frac{13}{4}) = \frac{23}{4}$

$f(-\frac{3}{4}) = 1 + (\frac{3}{4})^2 = \frac{25}{16} \quad f(-\frac{1}{4}) = 1 + (-\frac{1}{4})^2 = \frac{17}{16} \quad f(\frac{1}{4}) = 1 + (\frac{1}{4})^2 = \frac{17}{16}$

$f(\frac{3}{4}) = 1 + (\frac{3}{4})^2 = \frac{25}{16} \quad f(\frac{5}{4}) = 1 + (\frac{5}{4})^2 = \frac{41}{16} \quad f(\frac{7}{4}) = 1 + (\frac{7}{4})^2 = \frac{65}{16}$

$M_6 = \frac{1}{2}(\frac{25}{16} + \frac{17}{16} + \frac{17}{16} + \frac{25}{16} + \frac{41}{16} + \frac{65}{16}) = \frac{95}{16}$

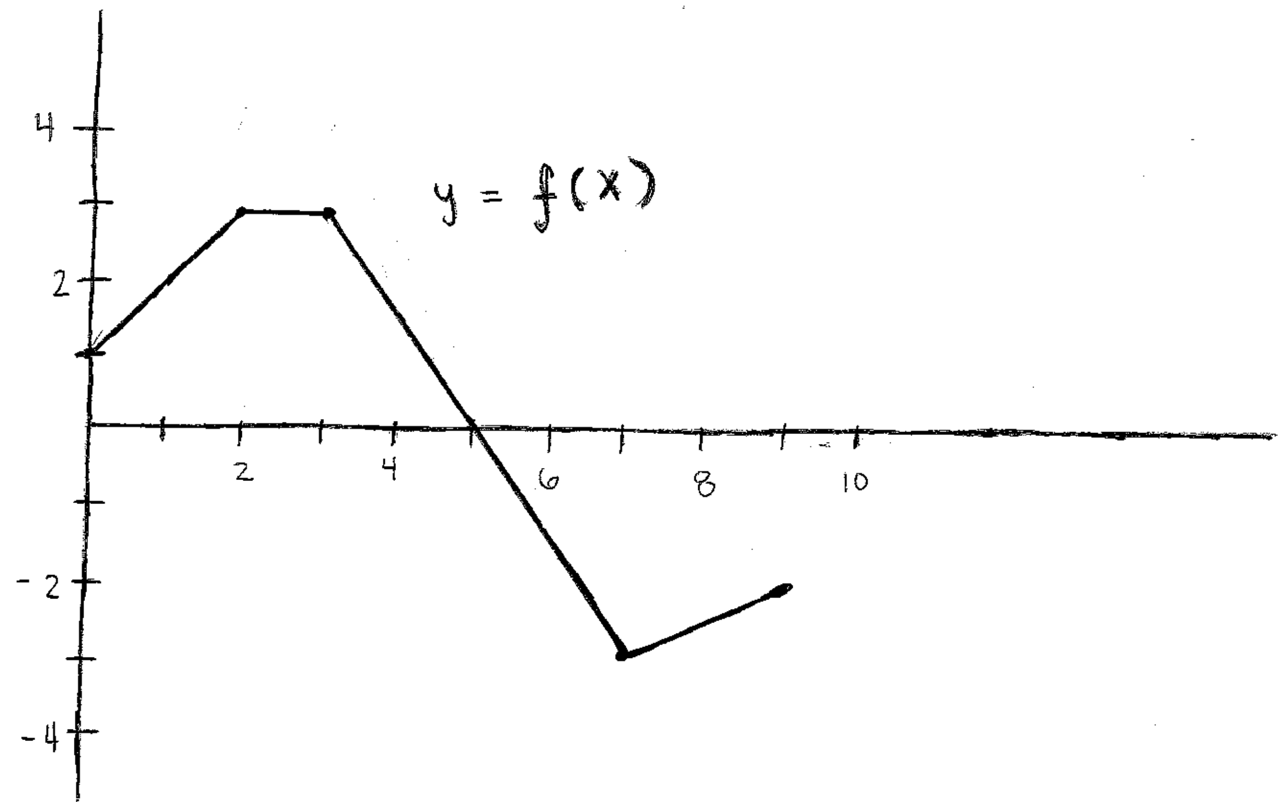
2\. Lower: $L_6 = 0.5(0 + 6.2 + 10.8 + 14.9 + 18.1 + 19.4) = 34.7$ ft.

Upper: $R_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2) = 44.8$ ft.

5.2

The graph of f is shown. Evaluate the integral by interperting in terms of areas.

a.) $\int_0^2 f(x)$ b.) $\int_0^5 f(x)\,dx$ c.) $\int_5^7 f(x)$ d.) $\int_0^9 f(x)\,dx$



Jacqueline Puga

5.2

A.) $\int_0^2 f(x)dx =$ Area of rec. + Area of Δ

$A_1 + A_2 = \int_0^2 f(x)$

$(1)(2) + \frac{1}{2}(2)(2) = 2+2$

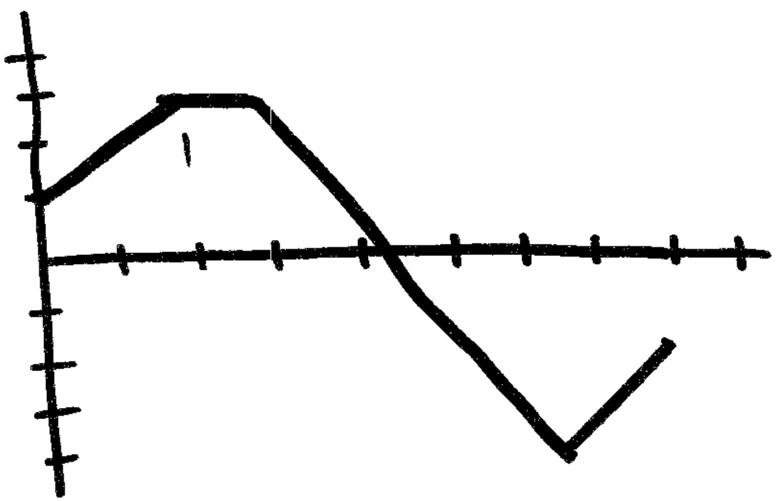
$\boxed{\int_0^2 f(x)dx = 4u^2}$

C.) $\int_5^7 f(x)dx =$ Area of Δ

$= \frac{1}{2}(2)(-3)$

$\boxed{\int_5^7 f(x)dx = -3u^2}$

Graph



(B.) $\int_0^5 f(x)dx =$ Area of rec + Area of Δ + Area of rec + Area of Δ

$\int_0^5 f(x)dx = (2)(1) + \frac{1}{2}(2)(2) + (1)(3) + \frac{1}{2}(2)(3)$

$= 2 + 2 + 3 + 3$

$\boxed{\int_0^5 f(x)dx = 10u^2}$

D.) $\int_0^9 f(x)dx = \int_0^5 f(x)dx + \int_5^7 f(x)dx + \int_7^9 f(x)dx$

$\int_7^9 f(x)dx =$ Area of rec + Area of Δ

$= -4 - 1 = -5$

so

$\int_0^9 f(x)dx = 10 - 3 - 4 - 1$

$\boxed{\int_0^9 f(x)dx = 2u^2}$

Jacqueline Puga

5.2 $\int_{10}^{30} f(x)\,dx$

x	10	14	18	22	26	30
f(x)	-12	-6	-2	1	3	8

$$\int_{10}^{30} f(x)dx = -12\int_{10}^{14} dx + (-6)\int_{14}^{18} dx + (-2)\int_{18}^{22} + 1\int_{22}^{26} dx + 3\int_{26}^{30} dx$$

$$\int_{10}^{30} f(x)dx = -12x\Big|_{10}^{14} + (-6)x\Big|_{14}^{18} + (-2)x\Big|_{18}^{22} + x\Big|_{22}^{26} + 3x\Big|_{26}^{30}$$

$$\int_{10}^{30} f(x)dx = -48 - 24 - 8 + 4 + 12 = \underline{-64} = \text{lower value}$$

$$\int_{10}^{30} f(x) = -6\int_{10}^{14} dx + (-2)\int_{14}^{18} dx + 1\int_{18}^{22} dx + 3\int_{22}^{26} dx + 8\int_{26}^{30} dx$$

$$\int_{10}^{20} f(x) = -6x\Big|_{10}^{14} dx + (-2)x\Big|_{14}^{18} + x\Big|_{10}^{22} + 3x\Big|_{23}^{26} + 8x\Big|_{26}^{30}$$

$$\int_{10}^{30} f(x) = -24 - 8 + 4 + 12 + 32 = \underline{16} \quad \text{upper value}$$

Jacqueline Puga

5.2

5.2

A table of values of an increasing function f is shown. Use the table to find lower and upper estimates for $\int_{10}^{30} f(x)\,dx$.

x	10	14	18	22	26	30
$f(x)$	-12	-6	-2	1	3	8

Jacqueline Puga

Althea Fastidio
Section 5.2

① If $\int_1^5 f(x)\,dx = 12$ and $\int_4^5 f(x)\,dx = 3.2$, find $\int_1^4 f(x)\,dx$.

② Evaluate the integral by interpreting it in terms of areas

$$\int_{-4}^{3} \left|\frac{1}{2}x\right| dx$$

Althea Fastidio
Section 5.2

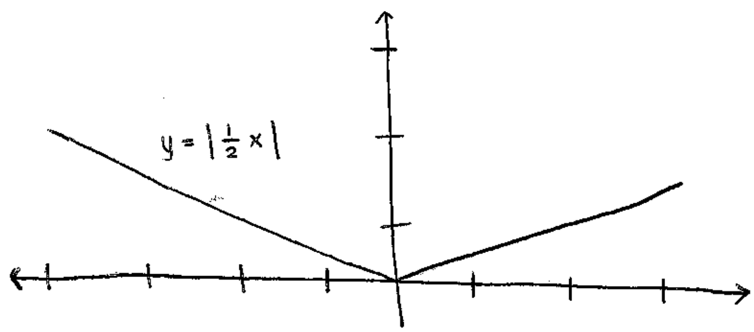
① If $\int_1^5 f(x)\,dx = 12$ and $\int_4^5 f(x)\,dx = 3.2$, find $\int_1^4 f(x)\,dx$

$$\int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = \int_1^4 f(x)\,dx$$

$$12 - 3.2 = \boxed{8.8}$$

② Evaluate the integral by interpreting it in terms of areas

$\int_{-4}^{3} \left|\frac{1}{2}x\right| dx$



$$\int_{-4}^{3} \left|\frac{1}{2}x\right| dx = \frac{1}{2}\cdot 4\cdot 2 + \frac{1}{2}\cdot 3\cdot \frac{3}{2}$$

$$= 4 + \frac{9}{4}$$

$$= \boxed{\frac{25}{4}}$$

Andrew Ghattass
Section 5.3

1) $$\int_0^{\frac{\pi}{3}} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\, d\theta$$

2) $$\int_1^2 \frac{(x-1)^3}{x^2}\, dx$$

Andrew Ghattass
Section 5.3

Answers

1) $$\int_0^{\frac{\pi}{3}} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\, d\theta$$

$$= \int_0^{\pi/3} \frac{\sin\theta + \sin\theta(\sec^2\theta - 1)}{\sec^2\theta}\, d\theta$$

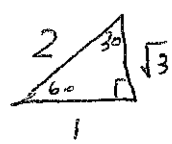
$$= \int_0^{\frac{\pi}{3}} \frac{\sin\theta + \sin\theta\sec^2\theta - \sin\theta}{\sec^2\theta}\, d\theta$$

$$= \int_0^{\pi/3} \sin\theta \, d\theta$$

$$= -\cos\theta \Big|_0^{\pi/3}$$

$$= -\cos\left(\tfrac{\pi}{3}\right) - (-\cos(0))$$

$$= -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$$



2) $$\int_1^2 \frac{(x-1)^3}{x^2}\, dx$$

$$= \int_1^2 \frac{(x^2-2x+1)(x-1)}{x^2}\, dx$$

$$= \int_1^2 \frac{x^3 - 3x^2 + 3x - 1}{x^2}\, dx$$

$$= \int_1^2 \left(\frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{3x}{x^2} - \frac{1}{x^2}\right) dx$$

Andrew Ghattass
Section 5.3

$$= \int_1^2 x - 3 + \frac{3}{x} - \frac{1}{x^2}\, dx$$

$$= \frac{1}{2}x^2 - 3x + 3\ln|x| + \frac{1}{x}\Big|_1^2$$

$$= \frac{1}{2}(2)^2 - 3(2) + 3\ln|2| + \frac{1}{2} - \frac{1}{2}(1)^2 + 3(1) - 3\ln|1| - \frac{1}{1}$$

$$= 2 - 6 + 3\ln|2| + 3 - 1$$

$$= 3\ln 2 - 2$$

Arrion Arcwe

Evaluate 5.3

1. $\int_0^1 x^{4/5}\,dx$

2. $\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta$

Arrion Arewe

1. $\int_0^1 x^{4/5}\,dx = \dfrac{x^{\frac{4}{5}+1}}{\frac{4}{5}+1} = \dfrac{x^{\frac{9}{5}}}{\frac{9}{5}}\Big|_0^1$

$= \frac{5}{9} x^{\frac{9}{5}}\Big|_0^1 = \frac{5}{9}\left(1^{\frac{9}{5}} - 0^{\frac{9}{5}}\right) = \frac{5}{9}(1) = \frac{5}{9}$

2. $\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta = \sec\theta \Big|_0^{\pi/4}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\Rightarrow \dfrac{1}{\cos\theta}\Big|_0^{\pi/4}$

$= \dfrac{1}{\cos(\pi/4)} - \dfrac{1}{\cos(0)} = \dfrac{1}{\frac{\sqrt{2}}{2}} - \dfrac{1}{1} = \dfrac{2}{\sqrt{2}} - 1$

$= -1 + \sqrt{2} = \sqrt{2} - 1$

Bailey Gillis
Section 2
review 5.4

Use part I of the fundamental theorem of calculus to find the derivative of the function:

1) $g(x) = \int_3^x e^{t^2-t}\, dt$

2) $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\, dt$

Bailey Gillis
Section 2
review 5.4
Solution

use part I of the fundamental theorem of calculus to find the derivative of the function:

1) $g(x) = \int_3^x e^{t^2-t}\,dt$

$g'(x) = e^{x^2-x}$

2) $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\,dt$

$= -\int_{\pi}^{x} \sqrt{1+\sec t}\,dt$

$F'(x) = -\sqrt{1+\sec x}$

5.4 Victoria Henson

11. $F(x) = \int_x^{\pi} \sqrt{1 + \sec t}\ dt$

[Hint $\int_x^{\pi} \sqrt{1+\sec t}\ dt = -\int_{\pi}^{x} \sqrt{1+\sec t}\ dt$]

16. $y = \int_{e^x}^{0} \sin^3 t\ dt$

5.4 Victoria Henson

11. $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\, dt$ FTC: $f(x) = \frac{d}{dx}\int_a^x f(t)\, dt$

$F(x) = -\int_{\pi}^{x} \sqrt{1+\sec t}\, dt$

$F'(x) = -\frac{d}{dx}\int_{\pi}^{x} \sqrt{1+\sec t}\, dt = -\sqrt{1+\sec x}$

$\boxed{F'(x) = -\sqrt{1+\sec x}}$

16. $y = \int_0^{e^x} \sin^3 t\, dt$

$u = e^x \quad \frac{du}{dx} = e^x$

$y'(x) = -\frac{d}{dx}\int_0^{u} \sin^3 t\, dt = -\frac{d}{du}\int_0^{u} \sin^3 t\, dt \; \frac{du}{dt}$

$= -\sin u\, u'$

$= -e^x \sin^3(e^x)$

$\boxed{= -e^x \sin^3(e^x)}$

Mikaela Foehr
Section 2
5.5

1. Evaluate the indefinite integral.

$$\int \frac{\sec^2\left(\frac{1}{x^3}\right)}{x^4}\,dx$$

2. Evaluate the definite integral

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

Mikaela Foehr
Section 2
5.5

1. Evaluate the indefinite integral.

$$\int \frac{\sec^2\left(\frac{1}{x^3}\right)}{x^4}\,dx$$

$$= \int x^{-4}\sec^2(x^{-3})\,dx$$

$u = x^{-3}$

$du = -3x^{-4}\,dx$

$-\frac{1}{3}du = x^{-4}dx$

$$= \int \sec^2(u)\cdot -\frac{1}{3}\,du$$

$$= -\frac{1}{3}\int \sec^2(u)\,du$$

$$= -\frac{1}{3}\tan(u) + C$$

$$= \boxed{-\frac{1}{3}\tan\left(\frac{1}{x^3}\right) + C}$$

Mikaela Foehr
section 2
5.5

2. Evaluate the definite integral.

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

solution:

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \frac{dx}{x}$

When $x = e^4$

$u = \ln(e^4)$

$u = 4$

when $x = e$

$u = \ln(e)$

$u = 1$

$$= \int_1^4 \frac{1}{\sqrt{u}}\,du$$

$$= \int_1^4 u^{-\frac{1}{2}}\,du$$

$$= 2u^{1/2}\Big|_1^4$$

$$= 2(4)^{1/2} - 2(1)^{1/2}$$

$$= 2(2) - 2(1)$$

$$= 4 - 2$$

$$= \boxed{2}$$

Sam Ferguson

Sect: 01

Topic : 5.5 Questions

Problem 1: Evaluate the integral by making the given Substitution: $\int x^2\sqrt{x^3+1}\,dx$, $u=x^3+1$

Problem 2: Evaluate the definite integral: $\int_1^2 x\sqrt{x-1}\,dx$

Sam Ferguson

Sect: 01

topic: 5.5 Answers

Problem 1: $\int x^2 \sqrt{x^3+1}\, dx$. $u = x^3+1$

$du = 3x^2\, dx$

$\frac{1}{3} du = x^2\, dx$

so $\int \frac{1}{3} \sqrt{u}\, du$

$= \frac{1}{3} \int u^{1/2} du = \frac{1}{3} \frac{u^{3/2}}{3/2} + C$

$= \frac{2}{9} u^{3/2} + C \quad = \frac{2}{9} (x^3+1)^{3/2} + C$

Problem 2: $\int_1^2 x\sqrt{x-1}\, dx$

$u = x-1 \implies u+1 = x$

$du = dx$

$\int_0^1 (u+1)(\sqrt{u})\, du$

$= \int_0^1 (u^{3/2} + u^{1/2})\, du = \left(\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right)\Big|_0^1$

$= \left(\frac{2}{5}(1) + \frac{2}{3}(1)\right) - 0 = \frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \frac{16}{15}$

Shayne Wells

Review 5.6

Monday, December 11, 2017 8:35 AM

5.6 1.

Integrate by parts using the given choices for "u" & "dv"

$$\int x^2 \ln x \, dx$$

$$u = \ln x \qquad dv = x^2 dx$$

5.6 17

$$\int_1^2 \frac{\ln x}{x^2} \, dx$$

Shayne Wells

Review 5.6 Answers

Monday, December 11, 2017 8:39 AM

$\boxed{5.6 \quad 1}$

$$\int x^2 \ln x \, dx$$

$$u = \ln x \qquad dv = x^2 dx$$

$$du = \frac{1}{x} dx \qquad v = \frac{x^3}{3}$$

$$\Rightarrow \int u\,dv = uv - \int du\,v$$

$$\Rightarrow \int x^2 \ln x \, dx = \ln x \left(\frac{x^3}{3}\right) - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3}$$

$$\boxed{= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C}$$

Shayne Wells

$\boxed{5.6 \quad 17}$

$$\int_1^2 \frac{\ln x}{x^2}\,dx$$

Let $u = \ln x \quad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx \quad v = -\frac{1}{x}$

So

$$\int_1^2 \frac{\ln x}{x^2}\,dx = \ln x\left(-\frac{1}{x}\right)\Big]_1^2 - \int_1^2 \left(-\frac{1}{x}\right)\frac{1}{x}\,dx$$

$$\Rightarrow -\frac{\ln x}{x}\Big]_1^2 + \int_1^2 \frac{1}{x^2}\,dx$$

$$= -\frac{\ln(2)}{2} + 0 - \left(\frac{1}{2} - 1\right)$$

$$= -\frac{\ln 2}{2} + \frac{1}{2}$$

$$= \boxed{\frac{1}{2} - \frac{\ln 2}{2}}$$

5.6 - Integration by Parts
Shelby Proulx

1. Evaluate the integral using integration by parts with the indicated choices of u and dv.

$\int \theta \cos\theta \, d\theta \qquad u = \theta \quad dv = \cos\theta \, d\theta$

2. Evaluate the integral

$\int p^5 \ln p \, dp$

5.6 - Integration by Parts

Shelby Proulx

1. Evaluate the integral using integration by parts with the indicated choices of u and dv.

$\int \theta \cos\theta \, d\theta$ $\qquad u = \theta \quad dv = \cos\theta \, d\theta$

$\qquad du = d\theta \quad v = \sin\theta$

$\int u\,dv = uv - \int v\,du$

$\int \theta\cos\theta\,d\theta = \theta\sin\theta - \int \sin\theta\,d\theta$

$= \theta\sin\theta - (-\cos\theta)$

$= \theta\sin\theta + \cos\theta$

2. Evaluate the integral

$\int p^5 \ln p \, dp$

$u = \ln p \qquad dv = p^5\,dp$

$du = \frac{1}{p}\,dp \qquad v = \frac{p^6}{6}$

$\int u\,dv = uv - \int v\,du$

$\int p^5 \ln p\,dp = \ln p \cdot \frac{p^6}{6} - \int \frac{p^6}{6} \cdot \frac{1}{p}\,dp$

$= \frac{p^6}{6}\ln p - \int \frac{p^5}{6}\,dp$

$= \frac{p^6}{6}\ln p - \frac{p^6}{6 \cdot 6}$

$= \frac{p^6}{6}\left(\ln p - \frac{1}{6}\right)$

5.7 trig integrals

Diane Castellanos

3) $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x\,dx$

$\sin^2 x = 1-\cos^2 x$

$\int_{\pi/2}^{3\pi/4} (1-\cos^2 x)^2 \sin x \cos^3 x\,dx$

$u = \cos x$

$-du = +\sin x\,dx$

$\int_{\pi/2}^{3\pi/4} (1-u^2)^2 u^3 (-1)\,du$

$\int_{\pi/2}^{3\pi/4} (1-2u^2+u^4)(u^3)(-1)\,du$

$\int_{\pi/2}^{3\pi/4} u^3-2u^5+u^7(-1)\,du$

$\int_{\pi/2}^{3\pi/4} -u^3+2u^5-u^7$

$\left.\frac{-u^4}{4}+\frac{2u^6}{6}-\frac{u^8}{8}\right]_0^{-\sqrt{2}/2}$

$u=\cos(3\pi/4)$

$u=-\sqrt{2}/2$

$u=\cos(\pi/2)$

$u=0$

$\left(-\frac{1}{4}\right)\left(\frac{-\sqrt{2}}{2}\right)^4+\frac{1}{3}\left(\frac{-\sqrt{2}}{2}\right)^6-\frac{1}{8}\left(\frac{-\sqrt{2}}{2}\right)^8-0$

$=\frac{-11}{384} = -0.02864$

5.7 trig integrals

Diane Castellanos

3) $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx$

5.7 trig integrals

Diane Castellanos

2) $\int_0^{\pi/2} \cos^5 x\,dx$

$\int_0^{\pi/2} \cos^4 x \cdot \cos x\,dx \qquad \cos^2 x = 1 - \sin^2 x$

$\int_0^{\pi/2} (1-\sin^2 x)^2 \cos x\,dx \qquad u = \sin x$

$du = \cos x\,dx$

$\int_0^{\pi/2} (1-u^2)^2\,du$

$\int_0^{\pi/2} 1 - 2u^2 + u^4$

$x - \frac{2}{3}x^3 + \frac{x^5}{5} \Big]_0^1 \qquad u = \sin(\pi/2) \quad u = \sin(0)$

$u = 1 \quad u = 0$

$1 - \frac{2}{3}(1)^3 + \frac{1^5}{5} - 0$

$= \frac{8}{15}$

5.7 trig integrals

Diane Castellanos

2) $\int_0^{\pi/2} \cos^5 x \, dx$

Table of Integrals 5.8 Diego Munoz

- Used when an Integral is too difficult to solve by hand
- Not used often
- Usually, we would need to manipulate a current Integral, such as Substitution rule or algebraic manipulation to transform the given Integral

Example 1

$$\int_0^2 \frac{x^2+12}{x^2+4}\,dx$$

Example

$$\int \frac{x^2\,dx}{\sqrt{5-4x^2}}$$

Diego Munoz

Answers

$$\int_0^2 \frac{x^2+12}{x^2+4}\,dx$$

* Entry 17

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

* If we perform long division, we get

$$\frac{x^2+12}{x^2+4} = 1 + \frac{8}{x^2+4}$$

* Now we can use formula 17 with $a=2$

$$\int_0^2 \frac{x^2+12}{x^2+4} = \int_0^2 \left(1 + \frac{8}{x^2+4}\right)dx$$

$$= x + 8\cdot\frac{1}{2}\tan^{-1}\frac{x}{2}\Big]_0^2$$

$$= 2 + 4\tan^{-1}1 = \boxed{2+\pi}$$

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx$$

* If we look at the section of the table entitled "Forms involving $\sqrt{a^2-u^2}$," we see that the closest entry is #34

$$\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = \frac{-u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$$

* First make substitution $u=2x$

$$\int \frac{x^2\,dx}{\sqrt{5-4x^2}} = \frac{(u/2)^2}{\sqrt{5-u^2}}\frac{du}{2} = 1/8 \int \frac{u^2}{\sqrt{(5-u^2)}}\,du$$

* Formula 34 with $a^2=5$ (so $a=\sqrt{5}$)

$$\int \frac{x^2\,dx}{\sqrt{5-4x^2}} = 1/8 \int \frac{u^2}{\sqrt{5-u^2}} = \frac{1}{8}\left(-\frac{u}{2}\sqrt{5-u^2} + \frac{5}{2}\sin^{-1}\frac{u}{\sqrt{5}}\right) + C$$

$$= -\frac{x}{8}\sqrt{5-4x^2} + \frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right) + C$$

Paulina Sierra
5.8
Question

Use the Table of Integrals to evaluate the integral.

1. $$\int \frac{\tan^3\left(\frac{1}{z}\right)}{z^2}\,dz$$

2. $$\int \sin^2(x)\cos(x)\ln(\sin(x))\,dx$$

Paulina Sierra

5.8

Answers $\underline{1}$+2

$$\int \frac{\tan^3\left(\frac{1}{z}\right)}{z^2}\, dz$$

Let $u = \frac{1}{z}$

$du = -\frac{1}{z^2}\, dz$

$$-\int \tan^3(u)\, du$$

$$\boxed{\text{Table 69: } \int \tan^3 u\, du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C}$$

$$-\frac{1}{2}\tan^2\left(\frac{1}{z}\right) - \ln\left|\cos\left(\frac{1}{z}\right)\right| + C$$

$$\int \sin^2(x)\cos(x)\ln(\sin(x))\, dx$$

Let $u = \sin(x)$ $\quad du = \cos(x)\, dx$

$$\int u^2 \ln(u)\, du$$

$$\boxed{\text{Table [illegible]: } \int u^n \ln u\, du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C}$$

$$\frac{u^3}{9}(3\ln(u) - 1) + C$$

$$\frac{\sin^3(x)}{9}(3\ln(\sin x) - 1) + C$$

Dana Alghazali / Section 1 / Topic 5.7 (trig Integrals)

2) $\int_0^{\pi/2} \cos^5 x \, dx$

28) $\int \frac{x^2 - x + 6}{x^3 + 3x} \, dx$

2) $\int_0^{\pi/2} \cos^5 x \, dx$

sub $\cos^2 x = 1 - \sin^2 x$, into equation

$$\int_0^{\pi/2} (1 - \sin^2(x))^2 \cos x \, dx$$

Let $u = \sin x$

$du = \cos x \, dx$

Now rewrite in terms of u.

New integration Limits

$\sin(\pi/2) = 1$

$\sin(0) = 0$

$$\int_0^1 (1 - u^2)^2 \, du$$

$$\int_0^1 (u^4 - 2u^2 + 1) du = \left[\frac{1}{5}u^5 - \frac{2}{3}u^3 + u + C \right]_0^1$$

$$= \frac{1}{5}(1)^5 - \frac{2}{3}(1)^3 + 1 - 0$$

$$= \frac{8}{15}$$

Dana Alghazali / section 1 / Topic 5.7 (trig Integrals)

28) $$\int \underbrace{\frac{x^2 - x + 6}{x^3 + 3x}} dx$$

$$\frac{x^2 - x + 6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$

$$x^2 - x + 6 = A(x^2+3) + (Bx + C)x$$

Plug in $x = 0$

$$0^2 - 0 + 6 = A(0^2 + 3) + (B(0) + C)(0)$$

$$6 = 3A$$

$$A = 2$$

Plug in $x = 1$

$$1^2 - 1 + 6 = A(1^2 + 3) + (B(1) + C)(1)$$

$$6 = 4A + B + C \qquad [\text{Plug in } A = 2]$$

$$6 = 4(2) + B + C$$

$$-2 = B + C$$

Plug in $x = -1$

$$(-1)^2 - (-1) + 6 = A((-1)^2 + 3) + (B(-1) + C)(-1)$$

$$8 = 4A + B - C$$

$$0 = B - C$$

$$B = C$$

Since $B = C$ & $B + C = -2$

$\therefore B = -1$ & $C = -1$

And so,

$$\int \frac{x^2 - x + 6}{x^3 + 3} dx = \int \frac{2}{x} dx + \int \frac{-x}{x^2+3} dx + \int \frac{-1}{x^2+3} dx$$

$$= 2\ln|x| - \left(\frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right)\right) - \left(\frac{1}{2}\ln(x^2+3)\right) + C$$

Ana Ramos

5.7 Trig Subs (Problems)

17. $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$

11. Use substitution $x = 3\sin\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and the identity $\cot^2\theta = \csc\theta - 1$ to evaluate

$$\int \frac{\sqrt{9-x^2}}{x^2}\, dx$$

5.7 Trig Subs (Solutions) Ana Ramos

17. $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$ $\qquad x = 2\sin\theta$, $\quad dx = 2\cos\theta\, d\theta$

$$= \int \frac{2\cos\theta}{4\sin^2\theta \cdot \sqrt{4-4\sin^2\theta}}\, d\theta$$

$$= \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta \cdot \sqrt{4-4(1-\cos^2\theta)}}$$

$$= \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta \cdot \sqrt{4-4+4\cos^2\theta}}$$

$$= \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta\sqrt{4\cos^2\theta}} = \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta \cdot 2\cos\theta}$$

$$= \frac{1}{4}\int \frac{d\theta}{\sin^2\theta} = \frac{1}{4}\int \csc^2\theta\, d\theta$$

$$= \frac{1}{4}\int (1+\cot^2\theta)\, d\theta$$

$$= \frac{1}{4}\theta - \frac{1}{4}\cot\theta - \frac{1}{4}\theta + C$$

$$= -\frac{1}{4}\cot\theta + C$$

$$= -\frac{1}{4}\left(\frac{\sqrt{4-x^2}}{x}\right) + C = -\frac{\sqrt{4-x^2}}{4x} + C$$

Ana Ramos

11. Let $x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$$= \int \frac{\sqrt{9-(3\sin\theta)^2}}{(3\sin\theta)^2} \cdot 3\cos\theta \, d\theta$$

$$= \int \frac{3\sqrt{1-\sin^2\theta}}{9\sin^2\theta} \cdot 3\cos\theta \, d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \int \cot^2\theta$$

$$= \int (\csc^2\theta - 1) \, d\theta$$

$$= -\cot(\theta) - \theta + C$$

substitute $\theta = \sin^{-1}\left(\frac{x}{3}\right)$ back into the equation

$$-\frac{\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right)}{\sin\left(\sin^{-1}\left(\frac{x}{3}\right)\right)} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

Note:

$\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right) = \sqrt{1-u^2}$

$\sin\left(\sin^{-1}(u)\right) = u$

$$= -\frac{\sqrt{1-\left(\frac{x}{3}\right)^2}}{\frac{x}{3}} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

por $\frac{-\pi}{2} \leq u \leq \frac{\pi}{2}$

Cordelia Miracle

5.7 (partial fractions)

Problem 1:

$$\frac{3x-8}{x^2-4x-5}$$

Cordelia Miracle

5.7 (partial Fractions)

Problem 1 Solution: (Partial fraction decomposition)

$$\frac{3x-8}{x^2-4x-5} = \left(\frac{3x-8}{(x-5)(x+1)}\right)^{(x-5)(x+)} = \frac{A}{x-5} + \frac{B}{x+1}$$

$$= \left(\frac{A}{x-5} + \frac{B}{x+1}\right)(x-5)(x+1) \Rightarrow 3x-8 = A(x+1) + B(x-5)$$

if $x=-1 \Rightarrow -11 = B(-6) \Rightarrow B = \frac{11}{6}$

if $x=5 \Rightarrow 7 = 6A \Rightarrow A = \frac{7}{6}$

plug in values $\Rightarrow \dfrac{7/6}{x-5} + \dfrac{11/6}{x+1}$

Final Solution

$$\boxed{\frac{7/6}{x-5} + \frac{11/6}{x+1}}$$

Cordelia Miracle

5.7 (partial fractions)

Problem 2:

$$\frac{4x+9}{(x-1)(x+1)(x+4)}$$

Cordelia Miracle

5.7 (partial fractions)

problem 2 solution: (partial fraction decomposition)

$$\frac{4x+9}{(x-1)(x+1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+4}$$

$$\left(\frac{4x+9}{(x-1)\cancel{(x+1)}(x+4)}\right)^{(x-1)\cancel{(x+1)}(x+4)} = \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+4}\right)^{(x-1)(x+1)(x+4)}$$

$$4x+9 = A(x+1)(x+4) + B(x-1)(x+4) + C(x-1)(x+1)$$

if $x=-1 \Rightarrow -4+9 = B(-1-1) \Rightarrow \boxed{B = -5/6}$

if $x=-4 \Rightarrow 4(-4)+9 = C(-4-1)(-4+1) \Rightarrow \boxed{C = -7/15}$

if $x = 1 \Rightarrow 4+9 = A(1+1)(1+4) \Rightarrow A = \boxed{13/10}$

Final solution:

$$\boxed{\frac{13/10}{x-1} + \frac{(-5/6)}{x+1} + \frac{(-7/15)}{x+4}}$$

Marielle Krivit | 5.7 ~ Partial fractions

#1. $\int \frac{5x-4}{2x^2+x-1}\,dx$

#2. $\int \frac{2x^2-x+4}{x^3+4x}\,dx$

Marielle Krivit | 5.7 ~ partial fractions

#1. $\int \frac{5x-4}{2x^2+x-1}dx$

$= \int \frac{5x-4}{(2x-1)(x+1)} \Rightarrow \frac{A}{2x-1} + \frac{B}{x+1} = \frac{B(2x-1)+A(x+1)}{(2x-1)(x+1)}$

$B(2x-1) + A(x+1) = 5x-4$

plug in $x=-1 \rightarrow B(-3)+A(0) = -9 \rightarrow \boxed{B=3}$

plug in $x=\frac{1}{2} \rightarrow B(0) + A(\frac{3}{2}) = -\frac{3}{2} \rightarrow \boxed{A=-1}$

$\rightarrow \int \frac{-1}{2x-1} + \frac{3}{x+1} dx$

so

$\int \frac{5x-4}{2x^2+x-1}dx = \int \frac{-1}{2x-1} + \frac{3}{x+1} dx = \int \frac{-1}{2x-1}dx + \int \frac{3}{x+1}dx = \boxed{-\frac{1}{2}\ln|2x-1| + 3\ln|x+1| + C}$

#2. $\int \frac{2x^2-x+4}{x^3+4x}dx$

$= \int \frac{2x^2-x+4}{x(x^2+4)}dx \Rightarrow \frac{A}{x} + \frac{Bx+C}{x^2+4} = \frac{A(x^2+4)+(Bx+C)x}{x(x^2+4)}$

$A(x^2+4) + (Bx+C)x = 2x^2-x+4$

$(A+B)x^2 + Cx + 4A = 2x^2 - x + 4$

$\rightarrow (A+B)x^2 = 2x^2$, $A+B=2$, $A=1$, $\boxed{B=1}$ | $Cx = -1\cdot x$, $\boxed{C=-1}$ | $4A=4$, $\boxed{A=1}$

$\int \frac{1}{x} + \frac{x-1}{x^2+4} dx = \int \frac{1}{x}dx + \int \frac{x}{x^2+4}dx - \int \frac{1}{x^2+4}dx = \boxed{\ln|x| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}(\frac{x}{2}) + C}$

u-sub   arctan

Jillian Cooper
5.7 (Trig Sub)
Questions

Evaluate the integral

$$\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt$$

Use the substitution $x = 3\sin\theta$, $-\pi/2 \le \theta \le \pi/2$, and the identity $\cot^2\theta = \csc^2\theta - 1$ to evaluate

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

Jillian Cooper
5.7 (Trig Sub)
Solutions

Evaluate the integral

$$\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt$$

Bounds:

$t=2$ $\quad \sec\theta = 2$, $\cos\theta=\frac{1}{2}$, $\theta=\frac{\pi}{3}$

$t=\sqrt{2}$ $\quad \sec\theta=\sqrt{2}$, $\cos\theta=\frac{1}{\sqrt{2}}$, $\theta=\frac{\pi}{4}$

let $t=(1)\sec\theta \quad a=1 \quad 0\le\theta\le\frac{\pi}{2}$

$dt=\sec\theta\tan\theta\,d\theta$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta}{\sec^3\theta\sqrt{\sec^2\theta-1}}\,d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta}{\sec^3\theta\,(\tan\theta)}\,d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sec^2\theta}\,d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2\theta\,d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\cos(2\theta)}{2}\,d\theta = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1+\cos(2\theta)\,d\theta = \frac{1}{2}\left[\theta + \frac{\sin(2\theta)}{2}\right]_{\pi/4}^{\pi/3}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{3}+\frac{1}{2}\sin\left(\frac{2\pi}{3}\right)\right)-\left(\frac{\pi}{4}+\frac{1}{2}\sin\left(\frac{\pi}{2}\right)\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)-\left(\frac{\pi}{4}+\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{4\pi+3\sqrt{3}}{12}-\frac{\pi+2}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\pi+3\sqrt{3}-6}{12}\right]$$

$$= \frac{\pi+3\sqrt{3}-6}{24}$$

Jillian Cooper
5.7 (Trig Sub)
Solutions

Use the substitution $x = 3\sin\theta$, $-\pi/2 \le \theta \le \pi/2$, and the identity $\cot^2\theta = \csc^2\theta - 1$ to evaluate

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

$a = 3$ $\quad 3\cos\theta = \sqrt{9-(3\sin\theta)^2}$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$$\int \frac{\sqrt{9-3(\sin\theta)^2}}{(3\sin\theta)^2}\cdot 3\cos\theta\,d\theta = \int \frac{(3\cos\theta)(3\cos\theta)}{9\sin^2\theta}\,d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$= \int \cot^2\theta\,d\theta = \int \csc^2\theta - 1\,d\theta = -\cot\theta - \theta + C$$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$$= \frac{-\cos\theta}{\sin\theta} - \theta + C$$

$$= \frac{-\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right)}{\sin\left(\sin^{-1}\left(\frac{x}{3}\right)\right)} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

$\cos(\sin^{-1}(u)) = \sqrt{1-u^2}$

$\sin(\sin^{-1}(u)) = u$

for our bounds

$$= \frac{\sqrt{1-\left(\frac{x}{3}\right)^2}}{\frac{x}{3}} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$= \frac{3\sqrt{1-\left(\frac{x}{3}\right)^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$= \frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

5.9

Domenic Hull
Section 1

#5: Use a) the midpoint rule and b) Simpsons Rule to aproximate the given integral with the specified value of n.

$$\int_0^2 \frac{x}{1+x^2}\,dx, \quad n = 10$$

20. How large should n be to guarantee that the Simpson's Rule approximation to $\int_0^1 e^{x^2}\,dx$ is accurate to within .00001.

Domenic Hull
Section 1

5. $\int_0^2 \frac{x}{1+x^2}\,dx \quad n=10$

$\Delta x = \frac{2-0}{10} = \frac{1}{5}$

$\int_0^2 \frac{x}{1+x^2} = \frac{1}{5}\left[f(.1) + f(.3) + f(.5) + f(.7) + f(.9) + f(.11) + f(.13) + f(.15) + f(.17) + f(.19)\right]$

$= \frac{1}{5}\left[.099 + .275 + .4 + .469 + .497 + .497 + .483 + .461 + .437 + .412\right]$

$\boxed{=.806}$

$\int_0^2 \frac{x}{1+x^2} = S_n = \frac{1}{15}\left[0 + 4f(.2) + 2f(.4) + 4f(.6) + 2f(.8) + 4f(1) + 2f(1.2) + 4f(1.4) + 2f(1.6) + 4f(1.8) + f(2)\right] \boxed{=.804779}$

$\int_0^2 \frac{x}{1+x^2}\,dx = \frac{1}{2}\left(\ln(1+2^2) - \ln(1)\right) = .804719$

Error A: .00129

Error B: .00006

20.

$f^4(x) = 4(2xe^{x^2}(2x^3+3x)) + e^{x^2}(6x^2+3) = 4e^{x^2}(4x^4+12x^2+3)$

$K = f^4(1) = 206.589$

$|E_s| \le \frac{K(b-a)^5}{180n^4} \le .00001 \rightarrow \frac{206.589(1-0)^5}{180(.00001)} \le n^4$

$n \ge \left(\frac{206.589}{180(.00001)}\right)^{1/4} \ge 18.4$

$\boxed{n=20}$

Tanner Claudio
Section 2
12/11/2017

#1

5.10 - Improper Integrals (Type I)

Problem: Evaluate the integral below.

$$\int_{-\infty}^{0} 4^{\left(\frac{2x}{3}\right)} \, dx$$

Tanner Claudio
Section 2
12/11/2017

#1

5.10 Improper Integrals (Type I)

Answer: Evaluate the integral $\int_{-\infty}^{0} 4^{(\frac{2}{3}x)} dx$

$$\int_{-\infty}^{0} 4^{\frac{2x}{3}} = \lim_{b \to -\infty} \int_{b}^{\emptyset} 4^{\frac{2x}{3}} dx$$

$u = 2/3x$
$du = \frac{2}{3} dx$
$dx = \frac{3}{2} du$

$$= \lim_{b \to -\infty} \int_{b}^{\emptyset} 4^{u} \left(\frac{3}{2}\right) du$$

$$= \lim_{b \to -\infty} \frac{3}{2} \int_{b}^{\emptyset} 4^{u} du$$

$$= \lim_{b \to -\infty} \left[\frac{3}{2} \cdot \frac{4^{\frac{3}{2}x}}{\ln(4)}\right]_{x=b}^{x=\emptyset} = \frac{3}{2} \cdot \frac{4^{(0)(\frac{3}{2})}}{\ln(4)} - \lim_{b \to -\infty} \frac{3}{2} \cdot \frac{4^{\frac{3}{2}b}}{\ln(4)}$$

$$= \frac{3}{2(\ln(4))} - 0$$

* the lim of $4^{\frac{3}{2}b}$ as it goes to negative infinity, goes to zero, so the value can be inferred as zero.

$$\int_{-\infty}^{0} 4^{\frac{2x}{3}} dx = \frac{3}{2(\ln(4))}$$

Tanner Claudio
Dr. Parker / Section 2
12/11/2017

#2

Improper Integrals (Type I)

Problem: Find the area under the curve $y = 1/x^3$ from $x = 1$ to $x = t$ and evaluate it for $t = 10$, 100, and $1{,}000$. Then find the total area under the curve for $x \geq 1$.

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#2

Improper Integrals (Type I)

Answer: Integral:

$$\int_1^t \frac{1}{x^3}\,dx =$$

$$\int_1^t x^{-3}\,dx = \left.\frac{-1}{2x^2}\right|_{x=1}^{x=t} = \frac{-1}{2(t)^2} - \left(\frac{-1}{2}\right) = \frac{1}{2} - \frac{1}{2t^2}$$

Evaluate: $t = 10$

$$\frac{1}{2} - \frac{1}{2(10)^2} = \frac{1}{2} - \frac{1}{200} = \frac{99}{200} = 0.495$$

$t = 100$

$$\frac{1}{2} - \frac{1}{2(100)^2} = \frac{1}{2} - \frac{1}{20{,}000} = \frac{9999}{20{,}000} = 0.49995$$

$t = 1{,}000$

$$\frac{1}{2} - \frac{1}{2(1{,}000)^2} = \frac{1}{2} - \frac{1}{2{,}000{,}000} = \frac{999{,}999}{2{,}000{,}000} = 0.4999995$$

Total Area:

$t = \infty$

$$\frac{1}{2} - \frac{1}{2(\infty)^2} = \frac{1}{2} - \cancel{0} = \frac{1}{2}$$

↳ equal to zero as infinity grows...

* here you can see how the area can be taken as x grows in size.

Anthony DePaul Section II

5.10 [Type II]

Def] a) If f is cont. on [a,b) & discont. at b then $\int_a^b f(x)dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$

b) If f is cont. on (a,b], & discont. at a then $\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$

c) If f has a discontinuity at c, where a<c<b, and both $\int_a^c f(x)dx$ and $\int_c^b f(x)\,dx$ are convergent then, $\int_a^b f(x)\,dx = \int_a^c f(x) + \int_c^b f(x)\,dx$

26.] $\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$

30.] $\int_0^1 \frac{1}{4y-1}\,dy$

Anthony DePaul section II 5.10 [Type II]

Solution

26 $\int_2^3 \frac{1}{\sqrt{3-x}}\,dx = \lim_{t\to 3^-}\int_2^t (3-x)^{-1/2}\,dx = -\lim_{t\to 3^-}\left[2(3-x)^{1/2}\right]_2^t$

$= -2\left[\lim_{t\to 3^-}(3-t)^{1/2} - (3-2)^{1/2}\right] = -2[0-1] = \boxed{+2}$

undefined @ $\frac{1}{4} = y$

39 $\int_{-1}^{1} \frac{1}{4y-1}\,\cancel{dx} = \lim_{a\to (1/4)^-}\int_0^a \frac{1}{4y-1}\,dy + \lim_{b\to (1/4)^+}\int_b^1 \frac{1}{4y-1}\,dy$

$= \lim_{a\to (1/4)^-}\left[\frac{1}{4}\ln|4y-1|\right]_0^a + \lim_{b\to (1/4)^+}\left[\frac{1}{4}\ln|4y-1|\right]_b^1$

$= \lim_{a\to (\frac{1}{4})^-} \frac{1}{4}\left[\ln|4a-1| - \cancel{\ln|-1|}^{0}\right] + \lim_{b\to (1/4)^+} \frac{1}{4}\left[\ln|4(1)-1| - \ln|4b-1|\right]$

$= \underbrace{\lim_{a\to (\frac{1}{4})^-} \frac{1}{4}\left[\ln|4a-1|\right]}_{-\infty} + \lim_{b\to \frac{1}{4}^+} \frac{1}{4}\left[\ln(3) - \ln|4b-1|\right]$

= so Divergent

Gabriel Pellot
Section II

5.10 COMPARISON TEST

1. SHOW THAT $\int_0^{\infty} e^{-x^2}\,dx$ IS CONVERGENT BY COMPARISON TEST FOR INTEGRALS.

2. USE THE COMPARISON TEST TO DETERMINE WHETHER THE INTEGRAL $\int_1^{\infty} \frac{1+e^{-x}}{x}\,dx$ CONVERGES OR DIVERGES.

Gabriel Pellot
Section II

5.10 COMPARISON TEST

① - $\int_0^\infty e^{-x^2} = \int_0^1 e^{-x^2}dx + \int_1^\infty e^{-x^2}dx$

* SINCE THIS IS INFINITE WE DON'T CARE

* SINCE THIS GOES TO INFINITY WE NEED TO WORRY ABOUT THIS

- FIND AN INTEGRAL BIGGER THAN $\int_1^\infty e^{-x^2}dx$...

FOR $X \geq 1$, $X^2 \geq X$
SO $-X^2 \leq -X$
$e^{-x^2} \leq e^{-x}$

- COMPLETE THE NEW INTEGRAL $\int_1^\infty e^{-x}dx$

$\int_1^\infty e^{-x}dx = \lim_{t\to\infty} \int_1^t e^{-x}dx = \lim_{t\to\infty} (-e^{-t} + e^{-1}) = e^{-1}$

- SINCE $\int_1^\infty e^{-x^2} \leq \int_1^\infty e^{-x}$, AND $\int_1^\infty e^{-x}$ CONVERGES,

$\int_1^\infty e^{-x^2}$ ALSO CONVERGES.

② - $\int_1^\infty \frac{1+e^{-x}}{x}dx$ SINCE e^{-x} IS ALWAYS POSITIVE, $\frac{1+e^{-x}}{x} > \frac{1}{x}$

- WE KNOW $\int_1^\infty \frac{1}{x}dx$ DIVERGES, SINCE $P=1$

- SO SINCE $\frac{1+e^{-x}}{x} > \frac{1}{x}$, AND $\int_1^\infty \frac{1}{x}dx$ DIVERGES,

THE INTEGRAL $\int_1^\infty \frac{1+e^{-x}}{x}dx$ MUST ALSO DIVERGE.