6.1 Areas Between Curves

1:00

1. Find the area between $y=x-1$ and $y=9-x^2$ bounded by $x=-2$ and $x=1$

35:00

2. With respect to y, find the area between $x=1-y^2$ and $x=y^2-1$

3. Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find the area of the region bounded by the curves.

$y=x\sin(x^2)$ and $y=x^4$

12/11/2017

MAR BARnes :)

6.1 SoLutions

1

$y = x-1,\ y = 9-x^2,\ x=-2,\ x=1$

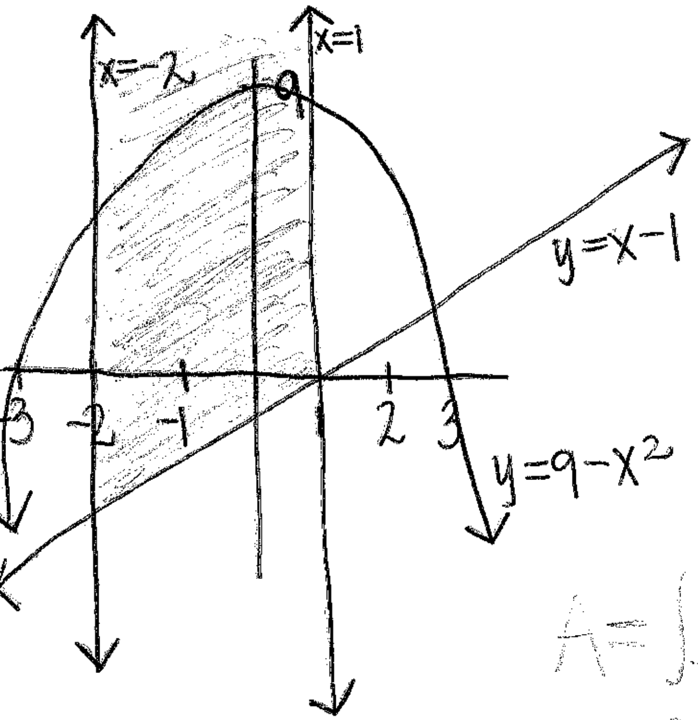
$A = \int_a^b [f(x) - g(x)]\,dx$

$0 = 9 - x^2$

$(3+x)(3-x)$

$3+x=0 \qquad 3-x=0$

$x=-3 \qquad x=3$



$A = \int_{-2}^{1} [9 - x^2 - (x-1)]\,dx$

$A = \int_{-2}^{1} [9 - x^2 - x + 1]\,dx$

$A = \int_{-2}^{1} [-x^2 - x + 10]\,dx$

$-\frac{x^3}{3} - \frac{x^2}{2} + 10x \Big]_{-2}^{1}$

$\left[-\frac{1^3}{3} - \frac{1^2}{2} + 10(1)\right] - \left[-\frac{(-2)^3}{3} - \frac{(-2)^2}{2} + 10(-2)\right]$

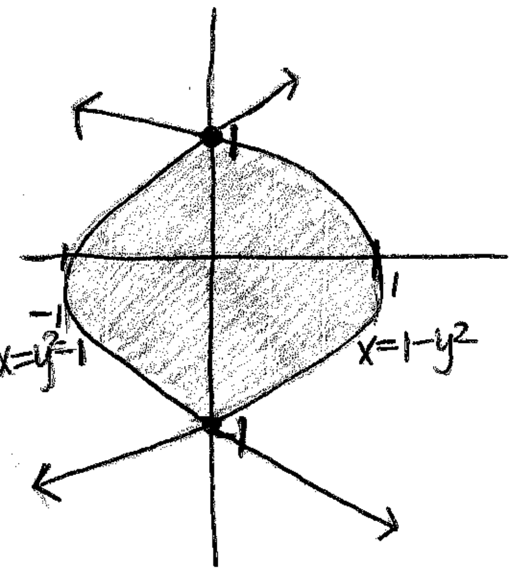
$= 28\frac{1}{2}$

12/12/2017

MAR Barnes :)

6.1 Solutions

2. $x = 1 - y^2 \quad x = y^2 - 1$



$$A = \int_c^d [f(y) - g(y)]\,dy$$

$$1 - y^2 = y^2 - 1$$

$$2 - y^2 = y^2$$

$$2 = 2y^2$$

$$y = \pm 1$$

$$A = \int_{-1}^{1} [(1-y^2)-(y^2-1)]\,dy$$

$$A = \int_{-1}^{1} [1-y^2-y^2+1]\,dy$$

$$A = \int_{-1}^{1} (2-2y^2]\,dy$$

$$2y - \frac{2y^3}{3}\Big]_{-1}^{1}$$

$$= \left\{2(1) - \frac{2(1)^3}{3}\right\} - \left\{2(-1) - \frac{2(-1)^3}{3}\right\}$$

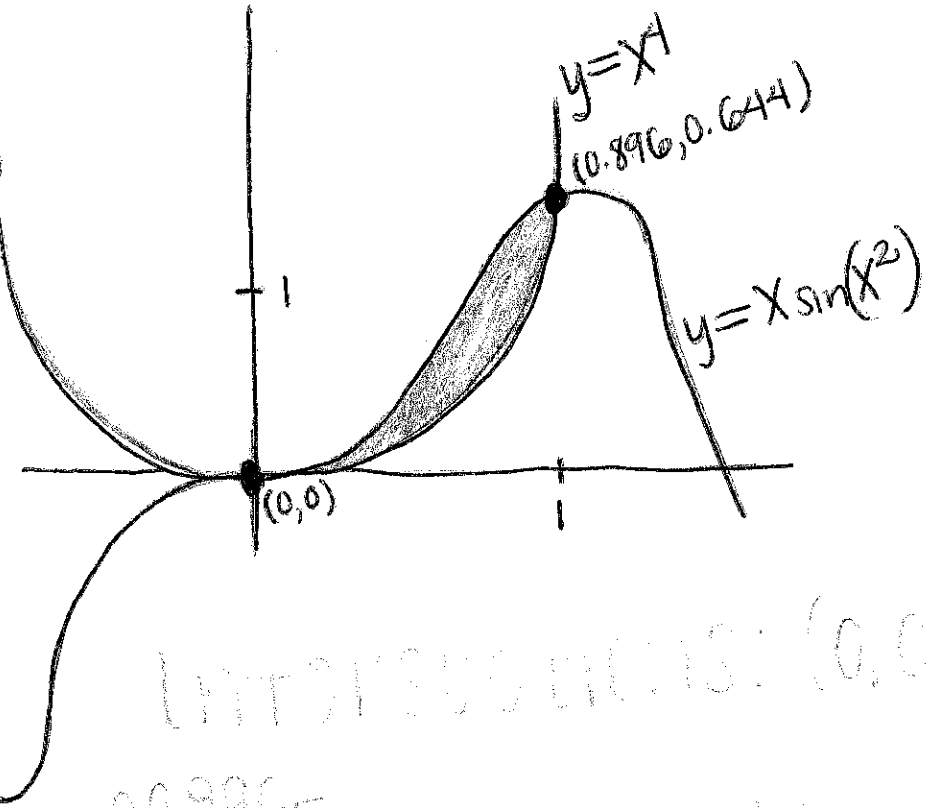
$$= \left\{2 - \frac{2}{3}\right\} - \left\{-2 + \frac{2}{3}\right\} = \frac{8}{3} \text{ or } 2\frac{2}{3}$$

12/1/2017

MAR BARNES!!

6.1 Solutions

3. $y = x\sin x^2,\ y = x^4$



Intersections: (0,0) & (0.896, 0.644)

$\boxed{x = 0, 0.9}$

$\int_0^{0.896} [x\sin(x^2) - x^4]\,dx$

$\int_0^{0.896} x\sin(x^2)\,dx - \int_0^{0.896} x^4\,dx$

$u = x^2$
$du = 2x\,dx$
$dx = \frac{du}{2}$ or $\frac{1}{2}du$

$\frac{1}{2}\int_0^{0.896^2} \sin u\,du - \int_0^{0.896} x^4\,dx$

$-\frac{1}{2}\cos u\Big|_0^{0.8[illegible]} - \frac{1}{5}x^5\Big|_0^{0.896}$

$= -0.3[illegible] - \frac{1}{2}$ [illegible]55

$= 0.04$

12112017

MAR BARNES ツ

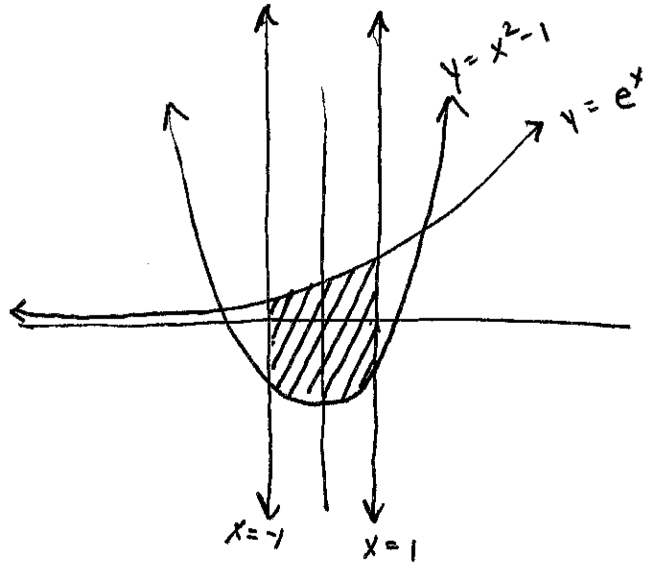
Trevor Denney
Section 1
Topic: 6.1 (More About Areas)

Problems:

#5) $y = e^x$, $y = x^2 - 1$, $x = -1$, $x = 1$

#11) $x = 2y^2$, $x = 4 + y^2$

#5)



$$\int_{-1}^{1} (\text{Top curve} - \text{Bottom curve})\, dx$$

$$\int_{-1}^{1} \left[e^x - (x^2 - 1) \right] dx$$

$$\int_{-1}^{1} e^x - x^2 + 1$$

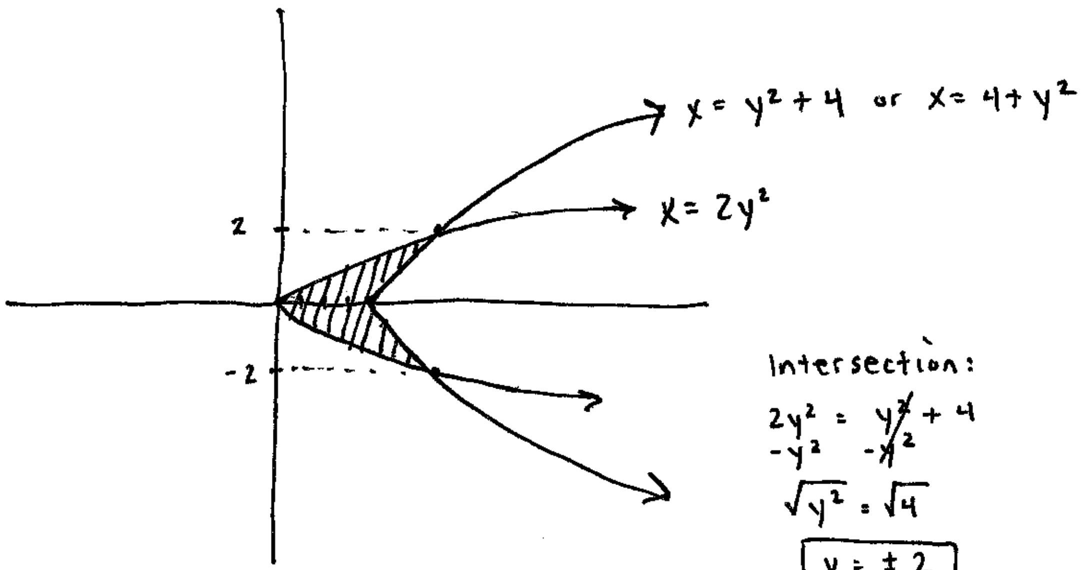
$$\left[e^x - \frac{1}{3}x^3 + x \right]_{-1}^{1}$$

$$e^1 - \frac{1}{3} + 1 - \left[\frac{1}{e^1} + \frac{1}{3} - 1 \right]$$

Simplify =>

$$\Rightarrow \boxed{e - \frac{1}{e} + \frac{4}{3}}$$

11)



Intersection:

$$2y^2 = \cancel{y^2} + 4$$
$$-y^2 \quad -\cancel{y^2}$$
$$\sqrt{y^2} = \sqrt{4}$$
$$\boxed{y = \pm 2}$$

$$\int_{-2}^{2} \left(\text{Right Curve} - \text{Left Curve}\right) dx$$

$$\int_{-2}^{2} \left(y^2 + 4 - 2y^2\right) dx$$

$$\left[\frac{1}{3}y^3 + 4y - \frac{2}{3}y^3\right]_{-2}^{2}$$

$$\left[4y - \frac{1}{3}y^3\right]_{-2}^{2}$$

$$8 - \frac{8}{3} - \left[-8 + \frac{8}{3}\right]$$

Simplify =>

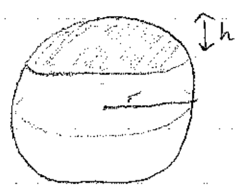
$$\Rightarrow = \boxed{\frac{32}{3}}$$

Nate
Fierstos
Section 2

Review Problems 6.2

1.) Use the washer method to find the region R enclosed by $y=x$ and $y=x^2$. Find the volume of the resulting solid.

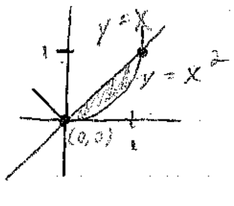
2.) Find the volume of a cap of a sphere with radius r and height h.



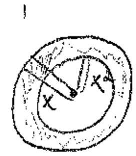
Note
Fierscos
Section 2

6.2 Answers

1.)



The curves intersect at points (0,0) and (1,1). When rotated about the x-axis, the shape of a washer of inner radius x^2 and outer radius x is formed.



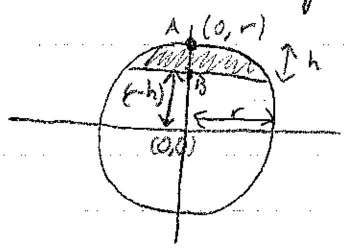
We find the cross-sectional area by subtracting the inner radius from the outer.

$$A(x) = \pi x^2 - \pi(x^2)^2 = \pi(x^2 - x^4)$$

* use points of intersect as bounds

$$V = \int_0^1 \pi(x^2 - x^4)\,dx = \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \boxed{\frac{2\pi}{15}}$$

2.) First, consider a circle w/ radius r and whose center is at the origin.

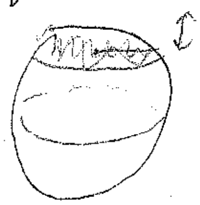


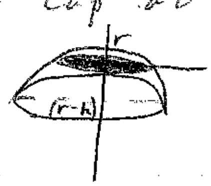
So the y-coordinates are r and $(r-h)$ respectively

eqn. of a circle is $x^2 + y^2 = r^2$

Then $x = \sqrt{r^2 - y^2}$

If we rotate the shaded area about the y-axis we get a ~~get~~ a cap at the top of a sphere





Typical disk

Radius of a disk is $\sqrt{r^2 - y^2}$

$$\text{Area} = \pi\left(\sqrt{r^2 - y^2}\right)^2 \rightarrow \pi(r^2 - y^2)$$

Cont. to back

The region enclosed by the curves $y = \frac{1}{x}$, $x = 1$, $x = 2$, $y = 0$ is rotated about the x-axis. Find the volume of the solid.

Daniels, 6.2 (Q1)

The region enclosed by the curves $x - y = 1$, $y = x^2 - 4x + 3$ is rotated about the line $y = 3$. Find the volume of the solid.

Daniels, 6.2 (Q3)

Bryce Johnson
Section II

6.4 (Not Parametric)

Problems:

① $x = y^{\frac{3}{2}}, \quad 0 \le y \le 1$

Find the exact length of the curve $x = y^{\frac{3}{2}}$

② $y = \ln(\cos x), \quad 0 \le x \le \frac{\pi}{4}$

Find the exact length of the curve $y = \ln(\cos(x))$

Bryce Johnson
Section II

6.4 (Not Parametric)

Solutions:

① $x = y^{\frac{3}{2}}$ $\quad 0 \le y \le 1$

$\frac{dx}{dy} = \frac{3}{2} y^{\frac{1}{2}}$

$L = \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$; $L = \int_0^1 \sqrt{\left(\frac{3}{2}y^{\frac{1}{2}}\right)^2 + 1}\, dy$

$= \int_0^1 \sqrt{\frac{9}{4}y + 1}\, dy$

$= \int_0^1 \sqrt{\frac{9}{4}\left(y + \frac{4}{9}\right)}\, dy$

$= \frac{3}{2} \int_0^1 \sqrt{y + \frac{4}{9}}\, dy$ $\quad$ let $u = y + \frac{4}{9}$, $du = dy$

$= \frac{3}{2} \int_{\frac{4}{9}}^{\frac{13}{9}} \sqrt{u}\, du$

$= \frac{3}{2} \left[\frac{2}{3} u^{\frac{3}{2}} \right] \Big|_{\frac{4}{9}}^{\frac{13}{9}}$

$= \left(\frac{13}{9}\right)^{\frac{3}{2}} - \left(\frac{4}{9}\right)^{\frac{3}{2}}$

$= \frac{\sqrt{2197} - 8}{27} = \boxed{\frac{13\sqrt{13} - 8}{27}}$

Bryce Johnson
Section II

6.4 (Not Parametric)

Solutions:

② $y = \ln(\cos x)$, $\quad 0 \leq x \leq \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan(x)$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \,; \quad L = \int_0^{\frac{\pi}{4}} \sqrt{1 + (-\tan(x))^2}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2(x)}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sec x \, dx$$

let $u = \tan(x) + \sec(x)$

$du = \sec(x)\tan(x) + \sec^2(x)dx$

$$= \int_0^{\frac{\pi}{4}} \frac{\sec(x)\tan(x) + \sec^2(x)}{\tan(x) + \sec(x)}\, dx$$

$$= \int_1^{\sqrt{2}+1} \frac{1}{u}\, du$$

$$= \ln u \Big|_1^{\sqrt{2}} = \ln(\sqrt{2}+1) + \ln(1)$$

$$\boxed{= \ln(\sqrt{2}+1)}$$

QUESTIONS

1. Find the exact length of the curve.

$x = 1 + 3t^2 \quad y = 4 + 2t^3 \quad 0 \le t \le 1$

Bennett
Dondoyano
Section II

2. $x = t\cos t \quad y = t\sin t \quad 0 \le t \le 2\pi$

Set up an integral, use calc to find answer

1. $\int_0^1 \sqrt{(6t)^2+(6t^2)^2}\,dt$

$\int_0^1 \sqrt{36t^2+36t^4}\,dt$

$\int_0^1 \sqrt{t^2(36+36t^2)}\,dt$

$\int_0^1 t\sqrt{(36+36t^2)}\,dt$

$u=36+36t^2$

$du=72t\,dt$

$\frac{du}{72}=t\,dt$

$\frac{1}{72}\int_0^1 \sqrt{u}\,du$

$\frac{1}{72}\left[\frac{2}{3}(36+36t^2)^{3/2}\right]_0^1$

ANSWERS

$108(72^{3/2}-36^{3/2})$

$108(432\sqrt{2}-216)$

$\boxed{4\sqrt{2}-2}$

Bennett Dondoyano section I

2.

$\int_0^{2\pi} \sqrt{(t\sin t+\cos t)^2+(\sin t-t\cos t)^2}\,dt$

$\frac{d}{dy}\ t\cos t$

$\downarrow$

$(t\sin t+\cos t)$

$\frac{d}{dx}\ t\sin t$

$\downarrow$

$(\sin t-t\cos t)^2$

$\checkmark$

insert calc

$\checkmark$

21.2563

6.5

Camryn Wallace

1) Find the Average value of the function on the given integral.

$h(x) = \cos^4 x \sin x$, $[0, \pi]$

2) If a cup of coffee has a temperature 95°C in a room where the temperature is 20°C then, according to Newton's law of cooling, the temperature of the coffee after t minutes is $T(t) = 20 + 75e^{-t/50}$. What is the average temperature of the coffee during the first half hour?

6.5

camryn wallace

1)

$h(x) = \cos^4 x \sin x$

$$h_{ave} = \frac{1}{b-a} \int_a^b h(x)\,dx$$

$$= \frac{1}{\pi - 0} \int_0^\pi \cos^4 x \sin x\,dx$$

$$= \frac{1}{\pi} \int_0^\pi \cos^4 x \sin x\,dx$$

Let $\cos x = u$

$-\sin x = du$

when $x = 0,\ u = 1$

$x = \pi,\ u = -1$

$$h_{ave} = \frac{1}{\pi} \int_{-1}^{1} -u^4\,du$$

$$h_{ave} = -\frac{1}{\pi} \int_{1}^{-1} u^4\,du$$

$$= \frac{1}{\pi} \int_{-1}^{1} u^4\,du$$

$$= \frac{1}{\pi} \left[\frac{u^5}{5}\right]_{-1}^{1}$$

$$= \frac{1}{\pi} \left[\frac{1}{5} + \frac{1}{5}\right]$$

$$= \frac{2}{5\pi}$$

2)

$$\text{Average} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

$$f_{ave} = \frac{1}{30-0} \int_0^{30} 20 + 75e^{-t/50}\,dt$$

$$= \frac{1}{30}\left[20t - 3750e^{-t/50}\right]_0^{30}$$

$$= \frac{1}{30}\left[(600 - 3750e^{-3/5}) - (0 - 3750e^0)\right]$$

$$= \frac{1}{30}\left[4350 - 3750e^{-3/5}\right]$$

$$= 76.4\,^\circ C$$

Kevin Gillies 6.6 (Springs / Cables)

p472

#5 A force of 10 lb is required to hold a spring stretched 4 in. beyond its natural length. How much work is done in stretching it from its natural length to 6 in. beyond its natural length?

p473 #13 A Cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.

Kevin Gillies 6.6 (springs / cables)

p472

#5 Hooke's Law

$f(x) = kx$

$f(\frac{1}{3}\text{ ft}) = 10\text{lbs}$

$k(\frac{1}{3}) = 10$

$k = 30$

$f(x) = 30x$

Integrate

$$\int_0^{1/2} 30x\,dx = 30 \cdot \frac{x^2}{2}\Big|_0^{1/2} = \frac{15}{4}\text{ ft lbs}$$

p473

#13 Work done on Segment

$$W = \int_0^{500} 2x\,dx$$

$$= x^2\Big|_0^{500}$$

$$= 250{,}000\text{ ft-lb}$$

work done on Coal

$$W = (800\text{ lb})(500\text{ ft})$$

$$= 400{,}000\text{ ft-lb}$$

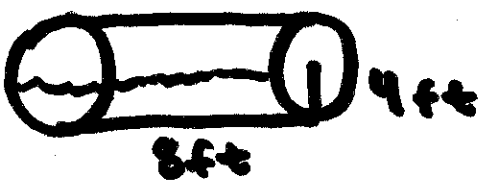
Total Work

$$W = (250{,}000\text{ ft-lb})(400{,}000\text{ ft-lb})$$

$$= 650{,}000\text{ ft lb}$$

1 A cylindrical tank is half filled with water. The tank has a legnth of 8ft and a radius of 4ft. The water weighs $62.5 \frac{lb}{ft^3}$

Write an integral (do not evaluate) that gives the amt of work that is done in pumping the water out over the top edge to empty the tank



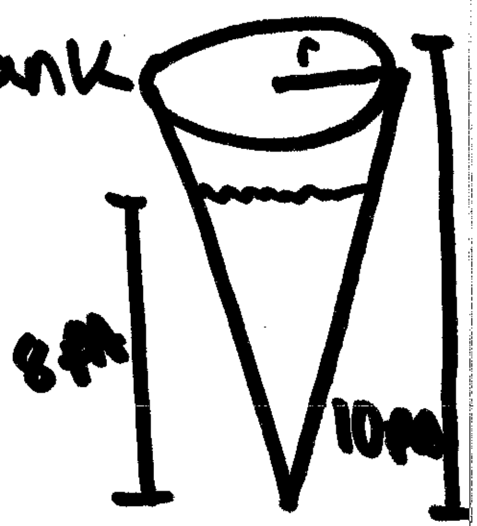
2 Find the Work needed to empty this tank

radius at top = 4m

height = 10 m

height of water = 8 m

Density of water = $1000 \frac{Kg}{m^3}$



6.6

Coel Miller

1

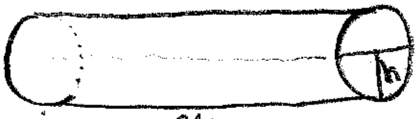
$W = \int F \cdot d \Rightarrow F = ma$

$= \int m \cdot a \cdot d \Rightarrow m = (\text{volume})\,\text{density} \Rightarrow \text{density} = \rho$

$= \int \rho \cdot V \cdot a \cdot d$

$\rho = 62.4 \frac{lb}{ft^3}$ = density of water

Coel Miller
6.6



8ft

1) Find the volume of a cross section

$V_{cs} = L \cdot w \cdot \Delta h$

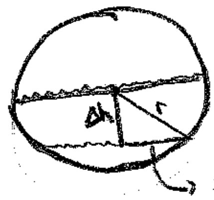
$V_{cs} = 8 \cdot w \cdot \Delta h$

L = Legnth
w = width

h = height of water (not the radius)
*The radius does not change, the height does.

a) Find width

w = width of a cross section, which is always changing



r = radius

$= \frac{w}{2}$

Pythagorean thm = $a^2 + b^2 = c^2$

$$\Delta h^2 + \left(\frac{w}{2}\right)^2 = r^2$$

$$w = \sqrt{4(r^2 - \Delta h^2)}$$

$$w = \sqrt{64 - 4\Delta h^2}$$

$$V_{cs} = \Delta h \cdot 8 \cdot \sqrt{64 - 4\Delta h^2}$$

2) Using $F = ma = \rho \cdot V \cdot a$

$$F = \Delta h \cdot 8 \sqrt{64 - 4\Delta h^2} \cdot 62.4$$

$= \frac{lbs}{ft^3}$ which is already accounting for the acceleration due to gravity on earth.

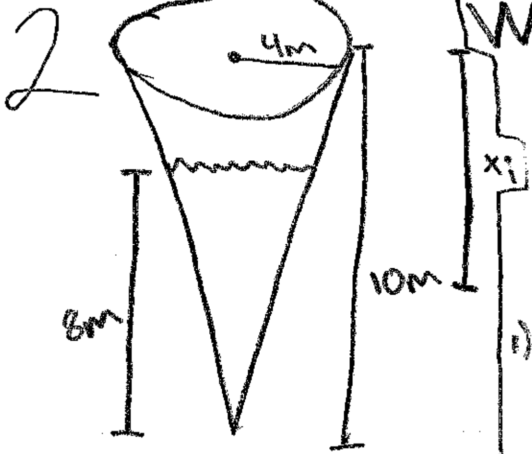
3) $W = \int_0^4 F \cdot d$

a) Distance = $8 - h$ or $2r - h$

$$W = \int_0^4 8 \cdot 62.5 \cdot \sqrt{64 - 4\Delta h^2} \cdot (8 - h)$$

$$W = 499.2 \int_0^4 \sqrt{64 - 4\Delta h^2} \cdot (8 - h)$$

2



Coel Miller
6.6

$$\text{Work} = \int F \cdot d = \int \text{mass} \cdot \text{acceleration} \cdot \text{distance}$$

$$= \int \text{density} \cdot \text{volume} \cdot \text{acceleration} \cdot \text{distance}$$

$$1000\ \frac{kg}{m^3} \qquad ? \qquad 9.8\ \frac{m}{s^2} \qquad x_i$$

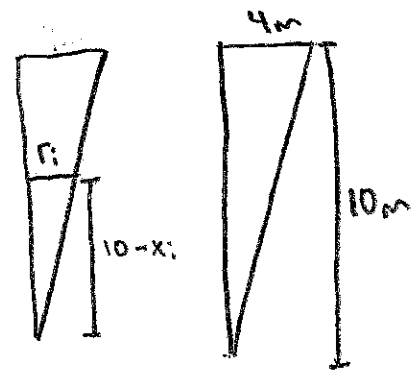
x_i = the distance between the top of the tank and the water line (which is constantly changing as you drain the tank).

r_i = the radius at x_i

1) Volume of a cross section $= \pi r_i^2 \cdot \Delta x$

$r_i = ?$

Using similar triangles since they will have the same proportions.



$$\frac{r_i}{10 - x_i} = \frac{4}{10}$$

$$r_i = \frac{2}{5}(10 - x_i)$$

So...

$$V_{cs} = \frac{4\pi}{25}(10 - x_i)^2 \Delta x$$

2) $\text{mass} = 1000\ \frac{kg}{m^3} \cdot \frac{4\pi}{25}(10 - x_i)^2 \Delta x = 160\pi (10 - x_i)^2 \Delta x$

3) $F = ma = 160\pi(10 - x_i)^2 \Delta x \cdot 9.8\ \frac{m}{s^2} = 1568\pi(10 - x_i)^2 \Delta x$

4) $W = \int F \cdot d \qquad \text{distance} = x_i$

$$W = \int_2^{10} 1568\pi (10 - x_i)^2 \Delta x \cdot x_i = 1568\pi \int_2^{10} (100x - 20x^2 + x^3)dx$$

$$= 1568\pi \left[50x^2 - \frac{20x^3}{3} + \frac{x^4}{4}\right]_2^{10} = 3.4 \times 10^6\ J$$