Alyssa Beamish

7.1

① Show that $y = \frac{2}{3}e^{x} + e^{-2x}$ is a solution of the differential equation $y' + 2y = 2e^{x}$.

②

a) What can you say about a solution of the equation $y' = -y^2$ just by looking at the differential equation?

b) verify that all members of the family $y = 1/(x+c)$ are solutions of the equation in part (a).

c) can you think of a solution of the differential equation $y' = -y^2$ that is not a member of the family in part (b)?

d) Find a solution of the initial-value problem

$$y' = -y^2 \qquad y(0) = 0.5$$

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1.1 Solutions

① $y' + 2y = 2e^x \Rightarrow y' = 2e^x - 2y$

consider $y = \frac{2}{3}e^x + e^{-2x}$ is a solution

$y' = \frac{2}{3}e^x + (-2)e^{-2x}$

$y' = \frac{2}{3}e^x - 2e^{-2x}$

$y' = 2e^x - 2(\frac{2}{3}e^x + e^{-2x})$

$y' = 2e^x - \frac{4}{3}e^x - 2e^{-2x}$

$y' = \frac{2}{3}e^x - 2e^{-2x}$

Therefore $y = \frac{2}{3}e^x + e^{-2x}$ is a solution to the differential equation $y' + 2y = 2e^x$.

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②

a) we can say it is 0 or decreasing

b) consider $y = \frac{1}{x+c}$ as a solution

$y' = -\frac{1}{(x+c)^2} = -\left(\frac{1}{x+c}\right)^2$

$y' = -y^2$ so therefore $y = \frac{1}{x+c}$ is a solution

c) $y = 0$

d) $y = \frac{1}{x+c}$

$y(0) = \frac{1}{0+c} = \frac{1}{2}$ $\quad \frac{1}{c} = \frac{1}{2}$

$c = 2$

$\boxed{y = \frac{1}{x+2}}$

SECTION 7.3
SEPERABLE EQUATIONS

SHANE BYRNE-SLEPICKA

1. Find the solution for the differential equation that satisfies the given initial condition

$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u} \quad , \; u(0) = -5$$

2. Solve the differential equation

$$\frac{du}{dt} = 2 + 2u + t + tu$$

SECTION 7.3 SOLUTION

SHANE BYRNE-SLEPICKA

SEPERABLE EQUATIONS

1. $$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u} \quad , u(0) = -5$$

$$2u\,du = 2t + \sec^2 t\,dt$$

$$\int 2u\,du = \int 2t + \sec^2 t\,dt$$

$$u^2 = t^2 + \tan(t) + C$$

$$(-5)^2 = 0 + \tan(0) + C$$

$$C = 25$$

$$\boxed{u^2 = t^2 + \tan(t) + 25}$$

2. $$\frac{du}{dt} = 2 + 2u + t + tu$$

$$\frac{du}{dt} = (1+u)(2+t)$$

$$\frac{1}{1+u}\,du = 2+t\,dt$$

$$\int \frac{1}{1+u}\,du = \int 2+t\,dt$$

$$\ln|1+u| = \frac{1}{2}t^2 + 2t + C$$

$$|1+u| = e^{\frac{1}{2}t^2 + 2t + C}$$

$$1+u = k\,e^{\frac{1}{2}t^2 + 2t} \quad , k = \pm e^C$$

$$\boxed{u = -1 + k e^{\frac{1}{2}t^2 + 2t}}$$