Harrison Mello
Section 1

Section 8.1: Sequences

Determine whether the sequence converges or diverges. If it converges, find the limit.

1. $a_n = \tan\left(\frac{2n\pi}{1+8n}\right)$

2. $a_n = \ln(2n^2+1) - \ln(n^2+1)$

Solutions Section 8.1: Sequences

Harrison Mello
Section 1

1) $a_n = \tan\left(\frac{2n\pi}{1+8n}\right)$

$$\lim_{n\to\infty} \tan\left(\frac{2n\pi \cdot \frac{1}{n}}{1+8n \cdot \frac{1}{n}}\right) = \lim_{n\to\infty} \tan\left(\frac{2\pi}{\frac{1}{n}+8}\right)$$

$$\tan\frac{2\pi}{0+8} = \tan\frac{\pi}{4} = 1$$

Converges to 1

2) $a_n = \ln(2n^2+1) - \ln(n^2+1)$

$$\lim_{n\to\infty} \ln(2n^2+1) - \ln(n^2+1)$$

$$\lim_{n\to\infty} \ln\left(\frac{2n^2+1}{n^2+1}\right)$$

$$\ln\left(\frac{2}{1}\right) = \ln 2$$

Converges to $\ln 2$

8.2 Geometric Series Andrew Messing

1. $\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$

2. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

8.2 Geometric Series Andrew Messina

1. $\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$

$$\sum_{n=1}^{\infty} \frac{12}{(-5)^n} = -\frac{12}{5}\sum_{n=1}^{\infty}\left(-\frac{1}{5}\right)^{n-1} = -\frac{12}{5}, \frac{12}{25}, -\frac{12}{125}, \frac{12}{625}$$

$r = -\frac{1}{5}$ $\quad a = -\frac{12}{5}$ $\quad \frac{a}{1-r} = \frac{-\frac{12}{5}}{1-(-\frac{1}{5})} = \boxed{-2}$

Therefore the series $\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$ converges to -2

2. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty}\left(\frac{1}{3^n} + \frac{2^n}{3^n}\right) = \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n + \left(\frac{2}{3}\right)^n$$

$$\frac{a}{1-r} \quad \frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{\frac{2}{3}}{1-\frac{2}{3}} = \frac{1}{2} + 2 = \boxed{\frac{5}{2}}$$

Therefore the series $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$ converges to $\frac{5}{2}$

Andrew Bennett

Section 8.2 (other than geometric)

1) Determine wether the series $\sum_{n=0}^{\infty} \frac{n-1}{3n-1}$ converges or diverges. And if it converges, to what value?

2) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent, and find its sum.

Andrew Bennett

Section 8.2 (other than geometric)

1) $\sum_{n=0}^{\infty} \frac{n-1}{3n-1}$ $\quad a_n = \frac{n-1}{3n-1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n-1}{3n-1} = \lim_{n\to\infty} \frac{1-\frac{1}{n}}{3-\frac{1}{n}} = \lim_{n\to\infty} \frac{1-0}{3-0} = \frac{1}{3}$$

The series $\sum_{n=0}^{\infty} \frac{n-1}{3n-1}$ diverges due to n^{th} term test for divergence

2) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $\quad \frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$

$$= \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

$$= \left(1-\cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{3}}\right) + \left(\cancel{\tfrac{1}{3}}-\cancel{\tfrac{1}{4}}\right) + \left(\cancel{\tfrac{1}{4}}-\tfrac{1}{5}\right) + \ldots \left(\cancel{\tfrac{1}{n}}-\tfrac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

$$\lim_{n\to\infty} 1 - \frac{1}{n+1} = 1 - 0 = 1$$

Therefore the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges and converges to 1.

John Martinez
Section 8.3

Direct Comparison

1) Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$$

2) Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$$

Direct Comparison

John Martinez
Section 8.3

1) Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n-1}{n4^n} \rightarrow \frac{n-1}{n4^n} \leq \frac{n}{n4^n} \longrightarrow \frac{n}{n4^n} = \frac{1}{4^n} = \left(\frac{1}{4}\right)^n$$

$r = \frac{1}{4}$ $\quad \frac{1}{4} < 1$ The series converges by GST, thus $\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$ converges by comparison test

2) Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}} \longrightarrow \frac{2+(-1)^n}{n\sqrt{n}} \leq \frac{3}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3}{n^{3/2}}$$

$p = 3/2$
$3/2 > 1$

The series converges by p-series, thus $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$ converges by comparison test

The Comparison Test

- Suppose than $\sum a_n$ and $\sum b_n$ are series with positive terms

(i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent

(ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent

Integral Test 8.3 Jon Woodward

Let a_n be a positive sequence

Suppose $f(x)$ is a continuous, positive, and eventually decreasing function on X with $f(n) = a_n$

Then the series

$\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges

i.e. they either both will converge or diverge

Examples

Find whether the following series converge or diverge via the integral test.

1) $\sum_{n=1}^{\infty} \frac{1}{n^3\sqrt{n}}$

2) $\sum_{n=1}^{\infty} \frac{n^3}{5n^8}$

3) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

4) $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+18}$

Solutions to 8.3 Integral Test

Jon Woodward

1) $\sum_{n=1}^{\infty} \frac{1}{n^3\sqrt{n}} \rightarrow \sum_{n=1}^{\infty} \frac{1}{n^{7/2}}$ +, decreasing & continuous

let $f(x) = \frac{1}{x^{7/2}} = x^{-7/2}$

$f'(x) = -\frac{7}{2}x^{-9/2} < 0$ when $x > 0$

$\int_1^{\infty} \frac{1}{x^{7/2}}\,dx$

converges by the p-series test $p = 7/2 > 1$

so $\sum_{n=1}^{\infty} \frac{1}{n^3\sqrt{n}}$ converges by the integral test

2) $\sum_{n=1}^{\infty} \frac{n^3}{5n^8}$ let $f(x) = \frac{x^3}{5x^8} = \frac{1}{5x^5} = \frac{1}{5}x^{-5}$

$f'(x) = -x^{-6} < 0$ when $x>0$ $f(x)$ is decreasing, positive, and continuous

$\int_1^{\infty} \frac{1}{5x^5}\,dx$ - converges by the p-series test $p = 5 > 1$ so $\sum_{n=1}^{\infty} \frac{n^3}{5n^8}$ converges by the integral test

3. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ let $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$ + & continous

$f'(x) = -\frac{1}{2}x^{-3/2} < 0$ when $x > 0$ so $f(x)$ is decreasing and positive & continuous

$\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$ - divergent by the p-series test $p = 1/2 < 1$

so $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges via the integral test

4. $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+18}$ let $f(x) = \frac{3x^2}{x^3+18}$ + & continuous

$f'(x) = \frac{6x(x^3+18) - 3x^2(3x^2)}{(x^3+18)^2}$

$f'(x) = \frac{6x^4 + 108x - 9x^4}{(x^3+18)^2}$

$f'(x) = \frac{-3x^4+108x}{(x^3+18)^2}$ - decreasing

So via the integral test $\sum_{n=1}^{\infty} \frac{3n^2}{n^3+18}$ diverges

$\int_1^{\infty} \frac{3x^2}{x^3+18}\,dx$

$u = x^3+18$

$du = 3x^2\,dx$

$\lim_{b\to\infty} \int_1^{b} \frac{1}{u}\,du$

$\lim_{b\to\infty} \ln|x^3+18| \Big]_1^{b} = \lim_{b\to\infty} \ln|b^3+18| - \ln|19| = \infty - \ln|19|$ so $\int_1^{\infty} \frac{3x^2}{x^3+18}$ diverges

8.3 Limit Comparison

Samantha Avera

1) $\sum_{n=4}^{\infty} \frac{n^2}{n^3-3}$

2) $\sum_{n=2}^{\infty} \frac{7}{n(n+1)}$

8.3 Limit Comparison Test Solutions

Samantha Avera

1) $\sum_{n=4}^{\infty} \frac{n^2}{n^3-3}$

$\sum_{n=4}^{\infty} \frac{1}{n}$ = divergent by P-series test

$a_n = \frac{n^2}{n^3-3}$ $\quad b_n = \frac{n^2}{n^3} = \frac{1}{n}$

$n^3 > n^3 - 3$ $\quad \frac{n^2}{n^3} < \frac{n^2}{n^3-3}$

$\sum_{n=4}^{\infty} \frac{n^2}{n^3} = \sum_{n=4}^{\infty} \frac{1}{n}$ = divergent

Therefore, by the Comparison Test the series given in the problem statement must also diverge

2) $\sum_{n=2}^{\infty} \frac{7}{n(n+1)}$ $\quad a_n = \frac{7}{n(n+1)}$ $\quad b_n = \frac{7}{n(n)} = \frac{7}{n^2}$

$\sum_{n=2}^{\infty} \frac{7}{n^2}$ = convergent by p-series test

$\frac{7}{n(n)} > \frac{7}{n(n+1)}$

So by the Comparison test the given series must converge

8.4 Absolute/Conditional Convergence

Haley Shirley
Section 2

(22) Determine whether the series is absolutely convergent.

$$\sum_{n=1}^{\infty} \frac{n!}{100^n}$$

(26) Determine whether the series is absolutely convergent

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

8.4 Absolute/conditional Convergence

Haley Shirley
Section 2

(22) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

Ratio Test $\lim_{n\to\infty} \left| \frac{(n+1)!}{100^{n+1}} \cdot \frac{100^n}{n!} \right| = \frac{(n+1)\cancel{100^n}}{\cancel{100^n}\cdot 100}$

$\lim_{n\to\infty} \left|\frac{n+1}{100}\right| = \infty$

$\infty > 1$ Therefore by the ratio test $\sum |a_n|$ is divergent. by the Ratio Test.

(26) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

$\lim_{n\to\infty} \left| \frac{(n+1)^2}{2^{(n+1)}} \cdot \frac{2^n}{n^2} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2 \cdot \cancel{2^n}}{\cancel{2^n} \cdot 2 \cdot n^2} \right|$

$\frac{1}{2} \lim_{n\to\infty} \left| \frac{n^2+2n+1}{n^2} \right| = \frac{1}{2} < 1$ Therefore the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ is absolutely convergent by the Ratio Test.

Nora Cheikh

8.4 # 9, 20 Problems

#9. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$$

#20. approximate the sum of the series correct to four decimal places

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n \cdot n!}$$

8.4 #9 and 20 Solutions

Nora Cheikh

#9 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+9}$$

<u>Alternative Series test</u>
converges if
1) $\lim_{n\to\infty} |a_n| = 0$ $\qquad |a_n| = \frac{n}{n^2+9}$

2) $|a_n| \geq |a_{n+1}|$, eventually

$$\lim_{n\to\infty} \frac{n}{n^2+9} \overset{\text{l'Hopital's Rule}}{=} \lim_{n\to\infty} \frac{1}{2n} = 0 \quad \checkmark \ 1)$$

$$y(n) = \frac{n}{n^2+9}$$

$$y'(n) = \frac{(n^2+9) - n(2n)}{(n^2+9)^2} = \frac{n^2 - 2n^2 + 9}{(n^2+9)^2} = \frac{-n^2+9}{(n^2+9)^2}$$

$y'(n)$ negative eventually (when $n > 3$) $\checkmark$ 2)

$\left[\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n}{n^2+9} \right.$ converges by alternative series test since $\lim_{n\to\infty} |a_n| = 0$ and $|a_n|$ decreases eventually. $\left. \right]$

8.4 #9 and 20 Solutions

Nora Cheikh

#20. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!}$ $\frac{(-1)^n}{3^n n!} < \frac{1}{10{,}000}$ at what n?

n=1	n=2	n=3	n=4	n=5
$\frac{(-1)^1}{3 \cdot 1!}$	$\frac{(-1)^2}{9 \cdot 2!}$	$\frac{(-1)^3}{3^3 \cdot 3!}$	$\frac{(-1)^4}{3^4 \cdot 4!}$	$\frac{(-1)^5}{3^5 \cdot 5!}$

$$-\frac{1}{3} + \frac{1}{18} - \frac{1}{162} + \frac{1}{1{,}944} - \frac{1}{29{,}160} \approx -0.28347051$$

$$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!} \approx -0.2835}$$

Adaly Solis 8.4 (ratio test)

* Determine whether the series is absolutely convergent.

1. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

2. $\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$

Adaly Solis 8.4 (ratio test)

*Solutions

1. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

$$\lim_{n\to\infty} \left| \frac{(-3)^{n+1}}{(n+1)^3} \cdot \frac{n^3}{(-3)^n} \right| = \lim_{n\to\infty} \left| \frac{(-3)n^3}{(n+1)^3} \right| = 3 \lim_{n\to\infty} \frac{1}{1+3/n+3/n^2+1/n^3}$$

$$= 3 > 1$$

The $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$, therefore $\sum |a_n|$ is divergent by the Ratio Test.

2. $\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{(-10)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-10)^n} \right| = \lim_{n\to\infty} \left| \frac{-10}{n+1} \right| = 10 \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$$

The $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, therefore $\sum |a_n|$ converges absolutely by the Ratio Test.

Root Test
(8.4)

Robert Jackson
Section 1

38.) $\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1} \right)^{5n}$

39.) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1} \right)^{n}$

38.) $\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$

Root test

$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{1/n}$

if $L<1$; $\sum a_n$ is abs. Convergent
if $L>1$ (or $L=\infty$); $\sum a_n$ is Divergent
if $L=1$; Root Test is inconclusive

$\rightarrow L = \lim_{n\to\infty} \left|\frac{-2n}{n+1}\right|^{5n/n} = \lim_{n\to\infty} \left|\frac{-2n}{n+1}\right|^{5}$

$\rightarrow L = \lim_{n\to\infty} \left|\frac{2(-n)}{n(1+1/n)}\right|^{5}$

$\rightarrow L = \lim_{n\to\infty} \left|\frac{-2}{1+1/n}\right|^{5}$

$\rightarrow L = \left|\frac{-2}{1}\right|^{5} \Rightarrow 2^5$

$\rightarrow L = 32$

$\therefore L > 1$

$\therefore \sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$ is Divergent

Robert Jackson
section 1

8.4 Root Test

Root Test

$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{1/n}$

- if $L<1$; $\sum a_n$ is abs. convergent
- if $L>1$ (or $L=\infty$); $\sum a_n$ is Divergent
- if $L=1$; Root test is inconclusive

39.) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

$\rightarrow L = \lim_{n\to\infty} \left|\frac{n^2+1}{2n^2+1}\right|^{n/n} \Rightarrow \lim_{n\to\infty} \left|\frac{n^2+1}{2n^2+1}\right|^1$

$\rightarrow L = \lim_{n\to\infty} \left| \frac{\frac{n^2}{n^2} + \cancelto{0}{\frac{1}{n^2}}}{\frac{2n^2}{n^2} + \cancelto{0}{\frac{1}{n^2}}} \right|$

$\rightarrow L = \left|\frac{1}{2}\right|$

$\rightarrow \frac{1}{2} < 1 \qquad \cdot L<1$

$\therefore \sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$ is absolutely convergent

Robert Jacson
Section 1

8.4 Root Test

Alexa Rettado
8.5 POWER SERIES
MATH 151-02 Final

1. For the following power series determine the radius and interval of convergence.

1. $$\sum_{n=0}^{\infty} \frac{n+1}{(2n+1)!}(x-2)^n$$

2. $$\sum_{n=0}^{\infty} \frac{1}{(-3)^{2+n}(n^2+1)}(4x-12)^n$$

Alexa Eltodo
8.5 POWER SERIES
MATH 151-02 Final

* * * SOLNS * * *

1. $$\sum_{n=0}^{\infty} \frac{n+1}{(2n+1)!}(x-2)^n$$

① RATIO TEST

$$\lim_{n\to\infty}\left|\frac{(n+2)(x-2)^{n+1}}{(2n+3)!}\cdot\frac{(2n+1)!}{(n+1)(x-2)^n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(n+2)(x-2)}{(2n+3)(2n+2)(2n+1)!}\cdot\frac{(2n+1)!}{(n+1)}\right|$$

$$=|x-2|\lim_{n\to\infty}\frac{n+2}{(2n+3)(2n+2)(n+1)}=0$$

$$\boxed{\text{The interval of convergence is } (-\infty,\infty) \text{ and } R=\infty \text{ because } L=0.}$$

2. $$\sum_{n=0}^{\infty}\frac{1}{(-3)^{2+n}(n^2+1)}(4x-12)^n$$

① RATIO TEST

$$\lim_{n\to\infty}\left|\frac{(4x-12)^{n+1}}{(-3)^{3+n}[(n+1)^2+1]}\cdot\frac{(-3)^{2+n}(n^2+1)}{(4x-12)^n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{4x-12}{(-3)[(n+1)^2+1]}\cdot\frac{(n^2+1)}{1}\right|$$

$$=|4x-12|\lim_{n\to\infty}\frac{(n^2+1)}{3(n+1)^2+1}=\frac{1}{3}|4x-12|$$

→

Alex Rltedo
8.5 POWER SERIES
MATH 151-02 Final

*** SOLNS continued ****

(2)

$$\frac{1}{3}\left|4x-12\right|<1$$

$$=\frac{4}{3}\left|x-3\right|<1$$

$$=\left|x-3\right|<\frac{3}{4} \rightarrow \boxed{R=\frac{3}{4}}$$

(3) Interval of convergence

$$-\frac{3}{4}<x-3<\frac{3}{4}$$

$$\frac{9}{4}<x<\frac{15}{4}$$

So...

$x=\frac{9}{4}$ $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(-3)^{2+n}(n^2+1)}(-3)^n=\sum_{n=0}^{\infty}\frac{1}{(-3)^2(n^2+1)}=\sum_{n=0}^{\infty}\frac{1}{9(n^2+1)}$ series converges by direct comparison test to $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n^2}$

$x=\frac{15}{4}$ $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(-1)^{2+n}(3)^{2+n}(n^2+1)}3^n=\sum_{n=0}^{\infty}\frac{1}{(-1)^{3+n}(3)^2(n^2+1)}=\sum_{n=0}^{\infty}\frac{(-1)^{2+n}}{9(n^2+1)}$ series converges by Alternating Series Test.

$$IOC=\boxed{\left[\frac{9}{4},\ \frac{15}{4}\right]}$$

Elli Schroeder

8.6

Question 1

6) $f(x) = \dfrac{1}{x+10}$ → find the power series & determine the interval of convergence

Question 2

30) $\int_0^{0.4} \ln(1+x^4)\,dx$ → Approximate the definite integral to 6 decimal places.

Elli Schroeder

8.6

Answer 1

$$f(x) = \frac{1}{x+10}$$

$$\frac{\frac{1}{10}}{1-(-\frac{x}{10})} = \frac{a}{1-r} \quad \text{where } a = \frac{1}{10} \ \& \ r = \frac{-x}{10}$$

$$f(x) = \sum_{n=0}^{\infty} ar^n$$

$$= \sum_{n=0}^{\infty} \frac{1}{10}\left(\frac{-x}{10}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{1}{10}(-1)^n\left(\frac{x}{10}\right)^n$$

converges when $|r| < 1$

$\left|\frac{-x}{10}\right| < 1$

$x < 10$

ROC = 10

IOC = $(-10, 10)$

$x = -10$ $\quad \lim_{n\to\infty}\left|\frac{1}{10}(-1)^n(-1)^n\right|$

$= \frac{1}{10} \neq 0$ therefore diverges

$x = 10$ $\quad \lim_{n\to\infty}\left|\frac{1}{10}(-1)^n(1)^n\right|$

$= \frac{1}{10} \neq 0$ therefore diverges

Answer 2

$$\int_0^{0.4} \ln(1+x^4)\,dx$$

$$\ln(1-(-x)) = -\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$$

$$\int_0^{0.4} \ln(1+(x^4)) = -\int_0^{0.4}\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n}}{n}$$

$$= -\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n+1}}{n(4n+1)}\Bigg|_0^{\frac{2}{5}}$$

$$= -\sum_{n=1}^{\infty} \frac{(-1)^n 2^{4n+1}}{n(4n+1)5^{4n+1}}$$

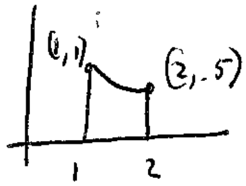
$$= \frac{2^5}{5\cdot 5^5} - \frac{2^9}{2\cdot 9\cdot 5^9} + \frac{2^{13}}{3\cdot 13\cdot 5^{13}} - \frac{2^{17}}{4\cdot 17\cdot 5^{17}}\Bigg] \text{we stop here} \approx 0.002034 \text{ (6dp)}$$

Kyle Davis; Section 8.7; Taylor/Maclaurin Series; Questions

a) Find a Taylor series for $f(x) = e^{-6x}$ about $x = -4$

b) Find a Maclaurin series for $f(x) = \cos(2x)$

The curve will look like



The cross sections will be circles,

$$A_{cs} = \pi r^2 = \pi \left(\frac{1}{x}\right)^2$$

Volume of Cross Section $= \pi \cdot \frac{1}{x^2} \cdot \Delta x$

Volume of cone $= \int_1^2 \pi \frac{1}{x^2}\,dx$

$$= \pi \int_1^2 \frac{1}{x^2}\,dx$$

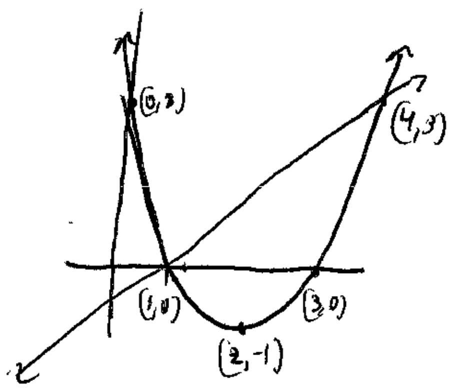
$$= \pi \left(-\frac{1}{x}\right)\Big|_1^2$$

$$= -\frac{\pi}{2} + \pi$$

$$= \frac{\pi}{2}$$

Daniels, 6.2 (Q1)

The curve will look like



The cross-section will be an Annulus (disk).

$$A_{cs} = \pi \left(r_{out}^2 - r_{in}^2\right)$$

$$= \pi \left((x^2-4x+3)^2 - (x^2-1)^2\right)$$

$$= \pi \left(x^4 - 8x^3 + 22x^2 - 24x + 9 - (x^2 - 2x + 1)\right)$$

$$= \pi \left(x^4 - 8x^3 + 21x^2 - 22x + 8\right)$$

$$V_{cs} = \pi \left(x^4 - 8x^3 + 21x^2 - 22x + 8\right)\Delta x$$

$$V = \int_1^4 \pi \left(x^4 - 8x^3 + 21x^2 - 22x + 8\right) dx$$

$$= -\frac{27\pi}{5}$$

Daniels, 6.2 (Q2)

Kyle Davis ; Section 8.7 ; Taylor / Maclaurin Series ; SOLUTIONS

a) $f(x) = e^{-6x}$ about $x = -4$

$\downarrow$

$f(x) = e^{-6x} \quad f'(x) = -6e^{-6x} \quad f''(x) = (-6)^2 e^{-6x} \quad f^3(x) = (-6)^3 e^{-6x}$

$\downarrow$

$f^n(x) = (-6)^n e^{-6x} \Rightarrow f^n(-4) = (-6)^n e^{-6(-4)} \Rightarrow (-6)^n e^{24}$

$\downarrow$

$$e^{-6x} = \sum_{n=0}^{\infty} \frac{f^n(-4)}{n!}(x+4)^n = \boxed{\sum_{n=0}^{\infty} \frac{(-6)^n e^{24}}{n!}(x+4)^n}$$

b) $f(x) = \cos(2x)$

$\downarrow$

Maclaurin for $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

$\downarrow$

$$\boxed{\cos(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!}}$$

8.7

Rosalyn A

1) Evaluate the limit

$$\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

2) Let $f(x) = x^4 e^{-x^7}$

find $f^{(18)}(x)$ and $f^{(21)}(x)$

8.7 Rosalyn A.

1) Evaluate the limit

$$\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\lim_{x\to 0} \frac{\left(x - \frac{x^3}{6} + \frac{x^5}{5!} - \cdots\right) - x + \frac{1}{6}x^3}{x^5}$$

cancel...

$$\lim_{x\to 0} \frac{\frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots}{x^5} = \lim_{x\to 0} \frac{1}{5!} - \frac{x^2}{7!} + \frac{x^4}{9!} - \cdots$$

$$= \frac{1}{5!} = \boxed{\frac{1}{120}}$$

2) Let $f(x) = x^4 e^{-x^7}$

find $f^{(18)}(0)$ and $f^{(21)}(0)$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{-x^7} = 1 - x^7 + \frac{x^{14}}{2!} - \frac{x^{21}}{3!} + \cdots$$

$$x^4 e^{-x^7} = x^4 - x^{11} + \frac{x^{18}}{2!} - \frac{x^{25}}{3!} + \cdots$$

$$f(x) = f(x) + \frac{f'(x)x}{1!} + \frac{f''(x)x^2}{2!} + \cdots$$

$$\cdots \frac{f^{(18)}(x)\, x^{18}}{18!} \cdots$$

$$\cdots \frac{f^{(21)}(x)\, x^{21}}{21!} \cdots$$

take the part in front of x^{18} from both

Find $f^{(18)}(0)$

$$\frac{f^{(18)}(0)}{18!} = \frac{1}{2!}$$

$$f^{(18)}(0) = \boxed{\frac{18!}{2!}}$$

Find $f^{(21)}(0)$

Since x^{21} is skipped in the series representation of $f(x) = x^4 e^{-x^7}$, we know that

$$\frac{f^{(21)}(0)}{21!} = 0$$

so $f^{(21)}(0) = 0$

Euler's Formula 8.7

Cammy Manes
MATH 151
section 1

Calculate:

1. $e^{i(\pi/4)}$

Use Euler's Formula to show:

2. $\cos(2x) = \cos^2 x - \sin^2 x$

   $\sin(2x) = 2\cos x \sin x$

Solutions Euler's Formula 8.7

Cammy Manes
MATH 151
Section 1

1. $$e^{i(\pi/4)} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$$

$$e^{i(\pi/4)} = \frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)$$

$$e^{ix} = \cos x + i\sin x$$

2. $$e^{i2x} = \cos(2x) + i\sin(2x)$$

$$(e^{ix})^2 = (\cos x + i\sin x)^2$$
$$= (\cos x + i\sin x)(\cos x + i\sin x)$$
$$= \cos^2 x + \cos x\, i\sin x + i^2\sin^2 x$$
$$= \cos^2 x + i\,2\cos x\sin x - \sin^2 x$$

$$a+bi = c+di$$

$$a = c$$
$$b = d$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$\sin(2x) = 2\cos x\sin x$$

Pedro Gomez

8.8 Taylor Polynomials

1) Find the Taylor polynomial $T_3(x)$ for the function f at the number a.

$$f(x) = \cos(x) \qquad a = \frac{\pi}{2}$$

2) Approximate f by a Taylor Polynomial with degree n at the number a.

$$f(x) = \ln(1 + 2x) \quad , \quad a = 1 \quad , \quad n = 3$$

Pedro Gomez

8.8 Taylor Polynomials
Answer Key

1)

- Find first three derivatives since Taylor polynomial is at T_3.

$f(\frac{\pi}{2}) = 0$

$f'(x) = -\sin(x)$ $\quad f'(\frac{\pi}{2}) = -1$

$f''(x) = -\cos(x)$ $\quad f''(\frac{\pi}{2}) = 0$

$f'''(x) = \sin(x)$ $\quad f'''(\frac{\pi}{2}) = 1$

- Taylor Polynomial T_3 =

$$T_3(x) = -(x-\frac{\pi}{2}) + \frac{1}{3}(x-\frac{\pi}{2})^3$$

2)

- $n = 3$ so take first three derivatives

$f(1) = \ln(3)$

$f'(x) = \frac{2}{1+2x}$ $\quad f'(x) = \frac{2}{3}$

$f''(x) = -\frac{4}{(1+2x)^2}$ $\quad f''(x) = -\frac{4}{9}$

$f'''(x) = \frac{16}{(1+2x)^3}$ $\quad f'''(x) = \frac{16}{27}$

- Taylor Polynomial $T_3(x)$:

$$T_3(x) = \sum_{n=0}^{3} \frac{f^n(0)}{n!}(x-1)^n$$

$$= \frac{\ln(3)}{0!}(x-1)^0 + \frac{2/3}{1!}(x-1)^1 + \frac{-4/9}{2!}(x-1)^2 + \frac{16/27}{3!}(x-1)^3$$

$$= \ln(3) + \frac{2}{3}(x-1) - \frac{2}{9}(x-1)^2 + \frac{8}{81}(x-1)^3$$