Math 160 Logic
Assignment # 6

1. Consider the statement:
$$\forall x \in \mathbb{N}, \exists y \in \mathbb{N} \text{ s.t. } x = 2y$$

   (a) Write the negation of this statement.

   (b) Prove the statement or the negation (i.e. the one that is true).

2. Prove the following by induction. Make sure to clearly define the statement you are proving by induction.

   (a) For all $n \in \mathbb{N}$, $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.

   (b) For all $n \in \mathbb{N}$, $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

   (c) For all $n \in \mathbb{N}$, $3|(n^3 + 2n)$.

3. Find n_0 so that for all $n \in \mathbb{N}, n \geq n_0$, $n! > 2^n$. Prove the claim by induction.

4. Determine which postage amounts can be created using 3 and 7 cent stamps. In other words, determine which numbers n can be written in the form $n = 3a + 7b$ where a, b are nonnegative integers.

   (Hint: Check the first few values of n directly, then find an n_0 so that all values $n_0 \leq n$ can be achieved. Prove this result by strong induction.)