Math 160 Logic
Assignment # 12

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Determine the following (with minimal explanation):
   (a) $f([-1,2])$
   (b) $f^{-1}([-1,2])$
   (c) $f(\{3,4,5\})$
   (d) $f^{-1}(\{3,4,5\})$
   (e) Is $3 \in f(\mathbb{Q})$?
   (f) Is $3 \in f^{-1}(\mathbb{Q})$?
   (g) Does the function f^{-1} exist? If so describe it.
   (h) Find three sets, $A \subseteq \mathbb{R}$ such that $f(A) = [5,17]$.

2. Let $f : X \to Y$ be a function and $A_1, A_2 \subseteq X$ and $B_1, B_2 \subseteq Y$. Prove or give a counterexample for each of the following. If the statement is false make a true statement (with proof) by adding either the condition that f is injective or f is surjective.
   (a) If A_1, A_2 are disjoint then $f(A_1)$ and $f(A_2)$ are disjoint.
   (b) If B_1, B_2 are disjoint then $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint.

3. Let $f : X \to Y$ be a function. For each of the following prove or give a counterecample.
   (a) $f(\varnothing) = \varnothing$.
   (b) $f^{-1}(\varnothing) = \varnothing$.
   (c) If $A \subseteq X$ and $f(A) = \varnothing$ then $A = \varnothing$.
   (d) If $B \subseteq Y$ and $f^{-1}(B) = \varnothing$ then $B = \varnothing$.

4. Let S be a nonempty set. Define $f : \wp(S) \to \wp(S)$ by, if $A \in \wp(S)$ then $f(A) = S - A$. Prove f is a bijection and find f^{-1}.

5. Let $R : \mathbb{Z}[x] \to \wp(\mathbb{R})$ defined by for all $p \in \mathbb{Z}[x]$,
$$R(p) = \{x \in \mathbb{R} : p(x) = 0\}$$
   so $R(p)$ is the set of the real roots of p.
   (a) Verify that $R(x^2 - 4x + 3) = \{1,3\}$.
   (b) Find $R(x^3 - x)$.
   (c) Find $p \in \mathbb{Z}[x]$ such that $R^{-1}(p) \neq \varnothing$ but $R^{-1}(p) \cap \mathbb{Q} = \varnothing$.
   (d) Show that $R^{-1}(\{\varnothing\}) \neq \varnothing$.
   (e) Find $R^{-1}(\{[0,1]\})$.