

Math 160 Logic Assignment # 9

Note I will use the symbol \wp to denote the power set. I have never been too happy with the options for power set in latex.

1. Suppose. $A \subseteq \mathbb{R}$ has the property that if $a, b \in A$ then $a + b \in A$ (i.e. A is closed under addition). Use induction to show that if $n \in \mathbb{N}$ and $a_1, a_2, \dots, a_n \in A$ then $a_1 + a_2 + \dots + a_n \in A$.
2. Let A, B, C be sets. Show that $(A \cup B) \times C = (A \times C) \cup (B \times C)$
3. Let $A = \{1, 2, 3, 4, \{3, 4\}\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7, 8, 9\}$. Determine the following:
 - (a) Is $\{3, 4\} \in A$?
 - (b) Is $\{3, 4\} \in \wp(A)$?
 - (c) Is $\{3, 4\} \subseteq \wp(A)$?
 - (d) Is $\{3, \{3, 4\}\} \in \wp(A)$?
 - (e) What is $|\wp(A)|$?
 - (f) Is $\{6, 7\} \in \wp(B \cup C)$?
 - (g) Is $\{6, 7\} \in \wp(B \cap C)$?
 - (h) What is $|\wp(B \cup C)|$?
 - (i) What is $|\wp(B \cap C)|$?
4. Let A, B, C be sets. Show that $C \subseteq A$ and $C \subseteq B$ if and only if $C \subseteq A \cap B$.
5. Let A, B be sets. Prove $\wp(A) \cup \wp(B) \subseteq \wp(A \cup B)$.