

## Math 160 Logic Assignment # 7

You may use the following fact from basic number theory:

Let  $a, b, p \in \mathbb{N}$  with  $p$  prime . If  $p|ab$  then either  $p|a$  or  $p|b$ .

1. Write the contrapositive of the above statement. (You will keep the first sentence the same since it is the setup which remains the same and do the contrapositive of the second which is the conditional part.).
2. Prove the following by induction on  $n$ :  
If  $p, q$  be primes with  $p \neq q$  then for all  $n \in \mathbb{N}$ ,  $p \nmid q^n$ .  
Hint first fix  $p, q$  with  $p \neq q$  and let  $P(n)$  be the statement that:  $p \nmid q^n$ . Use induction to show  $P(n)$  holds for all  $n \in \mathbb{N}$ .
3. Prove  $\frac{1}{3} \notin \mathbb{Q}_2$ .
4. Prove or disprove (i.e. prove the negation)  $\frac{3}{8} \in \mathbb{Q}_3$ .
5. Prove or disprove (i.e. prove the negation)  $\frac{5}{12} \in \mathbb{Q}_6$ .