

Math 160 Logic Assignment # 10

1. In homework # 6, you proved that any amount bigger than ¢12 can be made using ¢3 and ¢7 cent stamps. In other words, any $n \geq 12$ can be written in the form $n = 3a + 7b$ where a, b are nonnegative integers. Now do this proof using the well ordered principle.
2. When we proved that $\sqrt{2}$ was irrational we used the following fact:
If $q \in \mathbb{Q}$ there exists $a \in \mathbb{Z}$ and $b \in \mathbb{N}$ such that $q = \frac{a}{b}$ and a and b have no common factors, that is $\nexists c \in \mathbb{N}$ with $c \geq 2$ such that $c|a$ and $c|b$.
Prove this by using the well ordered principle.
(Hint: Let $q \in \mathbb{Q}$ and $B = \{b \in \mathbb{N} : \exists a \in \mathbb{Z} \text{ with } q = \frac{a}{b}\}$. Argue $B \neq \emptyset$. Let $b_0 \in B$ be the least element. Show that $q = \frac{a}{b_0}$ where a and b_0 have no common factors.)
3. In class we proved for all $n \in \mathbb{N}$ there exists $m \in \mathbb{N} \cup \{0\}$ such that either $n = 2m$ or $n = 2m + 1$ by using the well ordered principle. Prove it using induction.