# Math 320 Linear Algebra
# Assignment # 7

1. Find the basis for the column space of $A$ where

$$A = \begin{bmatrix} 3 & 6 & -12 & -3 & 1 & 6 \\ 0 & 0 & 5 & 2 & 10 & -1 \\ 0 & 0 & 0 & 1 & 3 & -2 \\ -3 & -6 & 12 & 3 & -1 & -5 \\ 0 & 0 & 10 & 4 & 20 & -2 \end{bmatrix}$$

2. Let $V$ be a vector space. Suppose $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in V$ and $4\vec{v}_1 + 2\vec{v}_2 - 2\vec{v}_3 = \vec{v}_1 - 2\vec{v}_2 + 6\vec{v}_3$. Prove $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is not a basis for $V$.

3. Find the dimension of each of these vector spaces by finding a basis (with proof):
   (a) $\mathbb{R}^{2\times 2}$.
   (b) $W = \left\{ \begin{bmatrix} a+2b \\ 0 \\ 3a+b+c \\ c+d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$

4. Suppose $V$ is a finite dimensional vector space and $W \leq V$. Show if the $\dim(V) = \dim(W)$ then $V = W$.

5. Consider $W = \{f \in P_3 : f(2) = 0\} \leq P_3$.
   (a) Show $W \neq P_3$ by finding an element in $P_3 \setminus W$.
   (b) Find a set of three linearly independent functions $\{p_1, p_2, p_3\}$ in $W$.
   (c) Argue that this set is a basis. (Hint: What is the largest the dimension that $W$ could be?)