Math 320 Linear Algebra
Assignment # 9

1. For each of the following determine if the given function is a linear transformation. Either prove it is or give an example that show it isn't:

   (a) $T_1 : P_3 \to \mathbb{R}^2$ defined by:
   $$T_1(ax^3 + bx^2 + cx + d) = \begin{bmatrix} ab \\ c \end{bmatrix}.$$

   (b) $T_2 : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ defined by:
   $$T_2\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ a-b \\ c \end{bmatrix}.$$

   (c) $T_3 : \mathscr{C}(\mathbb{R}, \mathbb{R}) \to \mathbb{R}$ (where $\mathscr{C}(\mathbb{R}, \mathbb{R})$ is the set of continuous functions on the reals) defined by:
   $$T_3(f) = \int_0^1 f.$$

2. Consider $D : P \to P$ (remember P is the set of all polynomials of any degree) defined by $D(f) = f'$.

   (a) Show that D is a linear transformation

   (b) Show that D is not 1-1.

   (c) Determine (with proof) whether or not D is onto.

3. Let $f : P_2 \to \mathbb{R}^3$ defined by:
   $$f(ax^2 + bx + c) = \begin{bmatrix} a+b \\ a+c \\ a \end{bmatrix}$$

   Show that f is an isomorphism.

4. Suppose that W and V are vector spaces and $f : W \to V$ is an isomorphism. Finish showing $f^{-1} : V \to W$ is an isomorphism but showing that for all $\alpha \in \mathbb{R}$ and $\vec{v} \in V$, $f^{-1}(\alpha\vec{v}) = \alpha f^{-1}(\vec{v})$.