Math 320 Linear Algebra
Assignment # 6

1. For each of the vector spaces V and set of vectors $\mathscr{A}$ below do the following: i) determine if the set of vectors $\mathscr{A}$ is linearly independent in the vector space V (either find a non-trivial relation or show one does not exist). If the set is linearly dependent then also do: ii) remove as few vectors as possible from $\mathscr{A}$ to create a new linearly independent set of vectors called $\mathscr{A}'$ iii) show the new set, $\mathscr{A}'$, is linearly independent.

   (a) $V = \mathbb{R}^{2\times 2}$ and
   $$\mathscr{A} = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$$

   (b) $V = P_4$ and $\mathscr{A} = \{x^2 - x, x^2 + x, x^3, x^2 + x^3, 1\}$

   (c) $V = \mathbb{R}^5$ and
   $$\mathscr{A} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -18 \\ -36 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 6 \\ 12 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 9 \\ 18 \\ -15 \end{bmatrix} \right\}$$

2. Let $V = \mathscr{F}(\mathbb{R}, \mathbb{R})$. Determine with proof if the following are linearly independent.

   (a) $\{\sin(x), \cos(x)\}$
   (Hint: the 0 in V is the zero function call it $\mathbf{0}$, that is the function g so that $g(x) = 0$ for all $x \in \mathbb{R}$. So suppose that $a\sin(x) + b\cos(x)$ is the zero function use appropriate values of x to show that $a = b = 0$.)

   (b) $\{e^x, x\}$

   (c) $\{\sin(x), e^x, \sin^2(x), \cos^2(x), 3\}$ (Note: 3 is the constant function 3.)