

Math 320 Linear Algebra Assignment # 11

1. Find the matrix of the following transformations:

(a) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by:

$$T \left(\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -8 \\ 14 \end{bmatrix}, \quad T \left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -8 \\ -14 \end{bmatrix}, \quad T \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ -14 \end{bmatrix}$$

(b) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ where $T(\vec{v})$ is \vec{v} after its be rotated by 30° clockwise.

(c) $T : P_3 \rightarrow \mathbb{R}^2$ where $T(p(x)) = \begin{bmatrix} p(3) \\ p(1) \end{bmatrix}$ with respect of the the basis on P_3 $\mathcal{B} = (x^2, x^3 + x^2, x^3 + x^2 + x, 1)$ and the standard basis on \mathbb{R}^2 .

(d) $T : P_3 \rightarrow P_2$ where $T(p(x)) = p'(x)$ with respect of the the standard basis on P_3 (i.e. $\mathcal{B} = (1, x, x^2, x^3)$) and the standard basis on P_2 .

2. Let $T_1 : W \rightarrow V$ and $T_2 : V \rightarrow U$. Show that:

(a) $\mathcal{R}(T_2 \circ T_1) \leq \mathcal{R}(T_2)$

(b) $\mathcal{N}(T_1) \leq \mathcal{N}(T_2 \circ T_1)$