Math 320 Linear Algebra
Assignment # 8

1. Let A be as follows:

$$A = \begin{bmatrix} 0 & 0 & 2 & -4 & 8 & 2/5 & -4/5 \\ 0 & 2 & -2 & 6 & 4 & 18 & 2 \\ 0 & 4 & -4 & 12 & 9 & 33 & 5 \\ 0 & 4 & -4 & 12 & 8 & 37 & 2 \\ 0 & 0 & 5 & -10 & 20 & 1 & -2 \end{bmatrix}$$

   Find:

   (a) A basis for $\text{col}(A)$.

   (b) A basis for $\text{row}(A)$.

   (c) $\text{rank}(A)$

   (d) if each of $\vec{b}_1, \vec{b}_2, \vec{b}_3$ is in $\text{col}(A)$ or $\text{row}(A)$ (could be in one of them, both or neiither):

$$\vec{b}_1 = \begin{bmatrix} -96/5 \\ 58 \\ 102 \\ 118 \\ -48 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \vec{b}_3 = \begin{bmatrix} 0 \\ 8 \\ -22 \\ 52 \\ -39 \\ 346/5 \\ 43/5 \end{bmatrix}$$

2. Suppose $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_m \in V$ and $\alpha \in \mathbb{R}$ and $1 \leq i, j \leq n$ with $i \neq j$. Show that the

$$\text{span}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_i, \ldots, \vec{r}_j, \ldots \vec{r}_m) = \text{span}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_i + \alpha\vec{r}_j, \ldots, \vec{r}_j, \ldots \vec{r}_m).$$

   This show that a the row space does not change when the third type of elementary row operation is performed. (Notice that you are proving something stronger since you are showing it for every vector space not just $\mathbb{R}^n$.)

3. Determine if the following square matrices are singular or nonsingular.

$$A_1 = \begin{bmatrix} 3 & -2 & -3 \\ 0 & 2 & -3 \\ -1 & 3 & -3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 5 & 2 & 15 \\ -1 & 0 & -1 & 1 \\ 3 & -3 & 1 & -14 \\ 5 & 3 & 0 & -1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2 & 1 & 2 & 0 & 2 & 0 \\ 2 & 0 & 2 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 & 2 & 0 \end{bmatrix}$$

4. Given $a, b, c \in \mathbb{R}$ with $a \neq 0$ find d so that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is singular.