Math 320 Linear Algebra
Assignment # 14

1. Suppose that $A, B, C \in \mathbb{R}^{n\times n}$ with $A \sim B$ and $B \sim C$. Show that $A \sim C$.

2. Let:

$$A = \begin{bmatrix} 2 & 5 & 0 & 3 \\ -1 & a & 2 & 5 \\ 0 & 2 & 0 & -1 \\ -2 & -1 & 0 & 0 \end{bmatrix}$$

   (a) Find $\det(A)$

   (b) Notice that $\det(A)$ does not depend on a. Are there any other values that can be change (while not changing any other values) that does not change the determinate?

3. Suppose that $\lambda \in \mathbb{R}$ is an eigenvalue for $A \in \mathbb{R}^{n\times n}$. (That is there exists $\vec{v_0} \neq 0$ called an eigenvector such that $A\vec{v_0} = \lambda\vec{v_0}$). Let $E_\lambda = \{\vec{v} \in \mathbb{R}^n : A\vec{v} = \lambda v\}$. Show that E_λ is a subspace of $\mathbb{R}^n$.

4. Let

$$A = \begin{bmatrix} 14/3 & -5/3 & 1 \\ 17/3 & -8/3 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

   Suppose that

$$\mathscr{B} = \left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \right\}$$

   is an eigenbasis for A (that is a basis consisting of eigenvectors).

   (a) What are the corresponding eignenvalues?

   (b) Find D and P so that $A = PDP^{-1}$.