Math 320 Linear Algebra
Assignment # 5

1. Let V be a vector space and let $r \in \mathbb{R}$ and $\vec{v} \in V$. Show $(-r)\vec{v} = -(r\vec{v})$. (Hint: First realize there is something to prove here).

2. Let V be a vector space and $W = \{\vec{0}\}$. Show that W is a subspace of V. This is called the trivial subspace of V.

3. Let V be a vector space. And $\vec{v} \in V$ with $\vec{v} \neq \vec{0}$.
   (a) Show if $r\vec{v} = \vec{0}$ then $r = 0$. (We showed the other direction in class).
   (b) Show if $r_1, r_2 \in \mathbb{R}$ and $r_1\vec{v} = r_2\vec{v}$ then $r_1 = r_2$.
   (c) Show that if W is a subspace of V and W is non-trivial (i.e. $W \neq \{\vec{0}\}$). then W is infinite.

4. For each of the following determine (with proof) if W is a subspace of the vector space V.
   (a) $V = \mathbb{R}^4$ and
   $$W = \left\{ \begin{bmatrix} a+2b \\ 0 \\ 3a+b \\ c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$
   (b) $V = P_4$ and
   $$W = \left\{ ax^3 + bx^2 + 2x + c : a, b, c \in \mathbb{R} \right\}$$
   (c) $V = \mathbb{R}^{2\times 2}$ and
   $$W = \left\{ \begin{bmatrix} a & a^2 \\ b & 0 \end{bmatrix} : a, b \in \mathbb{R} \right\}$$
   (d) $V = \mathbb{R}^3$ and
   $$\left\{ \vec{x} \in \mathbb{R}^3 : A\vec{x} = \vec{b} \right\}$$
   where:
   $$A = \begin{bmatrix} 1 & -2 & 7 \\ 3 & -2 & -1 \\ -1 & 8 & 2 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$
   (e) $V = \mathscr{F}(\mathbb{R}, \mathbb{R})$ (the set of all function from the $\mathbb{R}$ to the $\mathbb{R}$).
   $$W = \{ f \in \mathscr{F}(\mathbb{R}, \mathbb{R}) : f(2) = 0 \}.$$

5. Let
   $$A = \begin{bmatrix} 2 & 0 & 4 & 4 & 0 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 2 & -4 & -2 & 4 \\ 1 & 0 & 2 & 2 & 0 \end{bmatrix}$$
   and
   $$\vec{b} = \begin{bmatrix} 4 \\ 3 \\ 10 \\ 2 \end{bmatrix}$$

and

$$\vec{v_1} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 4 \\ -2 \\ -4 \\ 2 \end{bmatrix}, \quad \vec{v_4} = \begin{bmatrix} 4 \\ -1 \\ -2 \\ 2 \end{bmatrix}, \quad \vec{v_5} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

(a) Does $A\vec{x} = \vec{b}$ have a solution? If so find one.

(b) Is $\vec{b} \in \text{span}(\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}, \vec{v_5})$? If so find $a_1, a_2, a_3, a_4, a_5 \in \mathbb{R}$ so that $b = a_1\vec{v_1} + a_2\vec{v_2} + a_3\vec{v_3} + a_4\vec{v_4} + a_5\vec{v_5}$. How is this related to the previous part?